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An International Review of Spectroscopy and
Astronomical Physics

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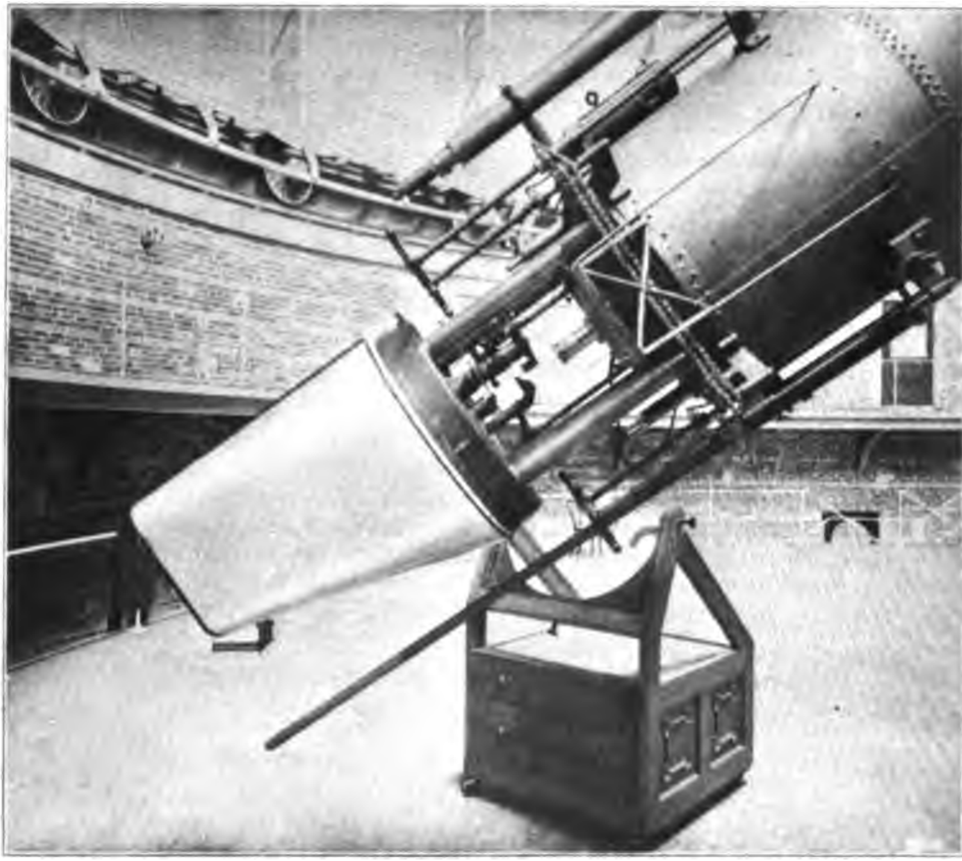

Bruce Spectrograph Attached to Forty-inch Yerkes Telescope.

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AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XV JANUARY 1902 NUMBER 1

THE BRUCE SPECTROGRAPH OF THE YERKES OBSERVATORY.

By EDWIN B. FROST.

THE original equipment of the Yerkes forty-inch refractor included, as an important accessory, a universal spectrograph of the general type designed by the late Professor Keeler for the Allegheny Observatory. This instrument has been described in a paper by Professor Hale and Mr. Ellerman, on "The Spectra of the Fourth Type Stars,"[1] in which its efficiency for that work is clearly shown. On the inauguration of this Observatory it was intended that this spectrograph should be employed for the determination of stellar motions in the line of sight, a research to which the program of the director assigned a considerable part of the time of the great refractor. Professor Campbell's remarkable work with the Mills spectrograph of the Lick Observatory, described in his classical paper[2] in this JOURNAL, made it clearly evident, however, that a highly specialized instrument, designed for this research alone, with particular emphasis on stability and permanence of adjustment, would be needed for the best results. When the writer began

[1] ASTROPHYSICAL JOURNAL, 10, 93, 1899. [2] *Ibid.*, 8, 123, 1898.

his connection with the Observatory, in 1898, for the special purpose of undertaking this line of work, it was hoped that circumstances would soon permit the design and construction of such a spectrograph. Pending the procurement of the necessary funds, observations were begun with the first-mentioned spectrograph, many plates of stellar spectra were obtained, with the regular coöperation of Mr. Ellerman, and much practical experience was gained of value in planning the new instrument. A method of reducing the plates was also developed, and about one hundred plates of stellar spectra were measured and reduced. During the academic year 1899–1900 the writer had the regular assistance of Mr. W. S. Adams, then Fellow in Astrophysics, in this work. After some changes in the instrument, in the summer of 1899, the spectra were considerably improved, and the measurements of the plates taken after that time showed an excellent internal agreement as indicated by the results from different lines. The velocity obtained from different plates of the same star differed, however, more than would be expected with a stable instrument. It should be mentioned that during all the use of this spectrograph for line of sight work, it was also employed on certain nights of the week for securing spectra of stars of the fourth type, which involved a change of most of the adjustments and a different position of the prisms, with its obvious dangers. Furthermore, the spectrograph was not provided with any protection from changes of temperature, so that progressive variations in the deviation and dispersion were to be expected during the exposures. The results of the measurements of plates taken with the spectrograph have, for the most part, been left unpublished, as they cannot reach the new standard of accuracy set by the work of the Mills spectrograph.

Under such circumstances, great satisfaction was felt on the receipt from Miss Catherine W. Bruce, in the autumn of 1899, of a gift of twenty-three hundred dollars for a new spectrograph—one of the last of her many benefactions to astronomy. This was soon afterward supplemented by a generous grant of

five hundred dollars from the Rumford Fund of the American Academy of Arts and Sciences. It is a pleasure to acknowledge our deep indebtedness to the Committee on the Rumford Fund for this grant, without which the instrument could not have been completed. Lenses were at once ordered, and a number of samples of prisms furnished by Mr. Brashear were carefully tested for their suitability. The selection was made of a clear flint from Mantois, and an order for a sufficient quantity of similar glass was given by Mr. Brashear.

The design of the instrument represents the ideas of Professor Hale and myself, carried out with the valuable assistance of Mr. Ritchey, who made most of the drawings, planned much of the mechanical construction and superintended the execution of the work in the instrument shop of the Observatory.

Among the principles kept in view in the design were: (*a*) rigidity, (*b*) permanence of adjustment, (*c*) freedom from mechanical strain, (*d*) adaptability to temperature control. The first named is a prime necessity in case of an instrument for this kind of work. The exposure time must necessarily be considerably prolonged to reach the fainter stars, and the flexure of the parts of the spectrograph must be reduced to a minimum by solid and stiff construction. With a mounting as massive as that of the forty-inch refractor, the weight of the spectrograph is of little significance, and accordingly the castings and metallic parts generally were made of such size as to provide an ample margin of safety from flexure. (*b*) It seems practically essential for an instrument designed for a specific purpose like this that the prisms and lenses should be maintained in fixed, invariable positions, except in so far as the lenses require a change of focus at different temperatures. The necessity of any movable device for keeping the prisms at minimum deviation is avoided, and when once correctly in position they should require no further adjustment. (*c*) The various parts of the apparatus were to be so fitted together that mechanical strain would be avoided at normal temperatures, and so far as was feasible steel and iron were to be employed, to escape the effects due to the combina-

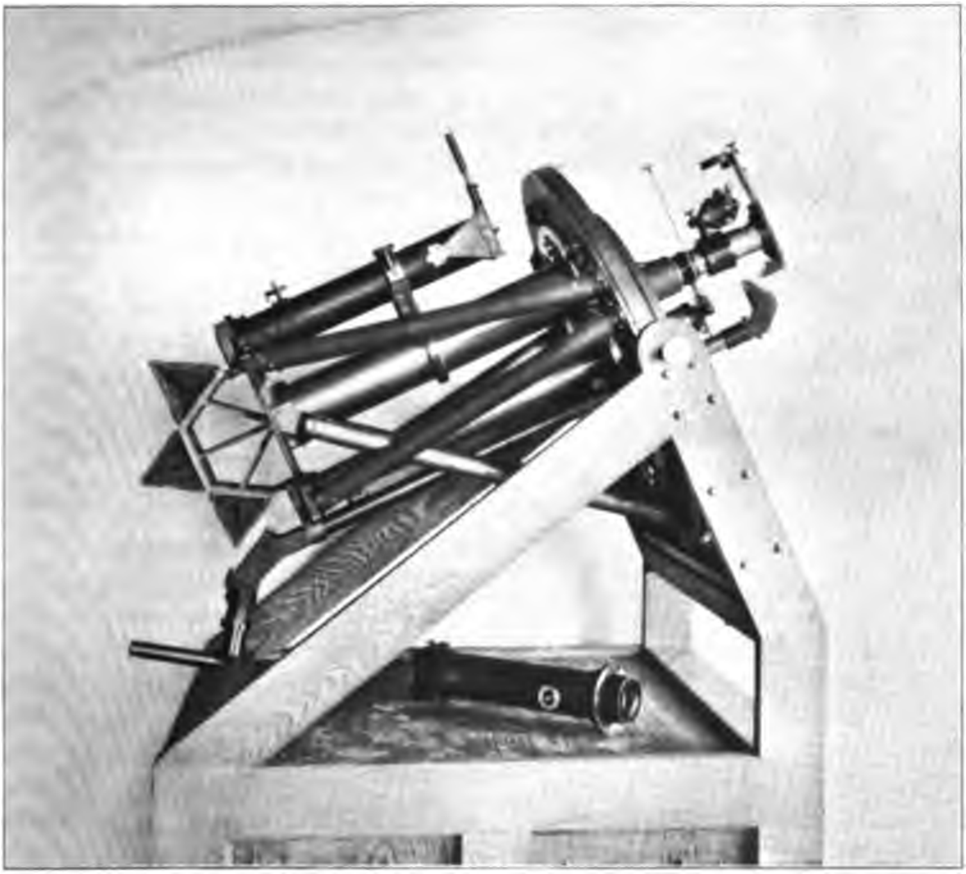
tion of metals of different coefficients of expansion. (*d*) Our own experience, as well as that of others, sufficiently indicated the necessity of so designing the instrument that it could be easily inclosed and maintained at a steady temperature, and so that the exposed portions could be insulated where possible.

The dimensions of the spectrograph of course depended upon the size of the optical parts, in particular of the prisms. After careful consideration, and in accordance with the advice of others having experience with large prisms, we decided that the diameter of the beam to be transmitted by the prisms should not exceed two and one-half inches, and then we further limited it to two inches (51 mm). Inasmuch as the correcting lens which Professor Hastings was designing for the forty-inch telescope was not to change the angular aperture of the great objective of $\frac{1}{19}$, this at once fixed the focal length of the collimator at thirty-eight inches. We desired to obtain all the advantage possible from a long-focus collimator, in permitting the use of a wider slit and consequent shorter exposure, without loss of purity as compared with a shorter collimator. The conditions of rigidity and compactness seemed best satisfied by a total deviation of 180° for the ray passing through the prisms at minimum. With these points settled, the mechanical construction could be planned accordingly.

MECHANICAL CONSTRUCTION.

Plate II shows an elevation of the spectrograph resting on its carriage, on a scale of about 1 : 10. Mechanically considered, it consists essentially of two castings rigidly connected by a framework of steel tubes. The massive circular, ribbed iron casting, at the right in the figure, serves as the foundation of the construction. It weighs 200 pounds, and may be called the ring casting. From a point about 7 inches above its center projects a very stiff hot-rolled tube of steel, $5\frac{1}{2}$ inches in diameter, $\frac{1}{4}$ inch in thickness, and about 28 inches long. This is very firmly imbedded in the ring casting, and bears at its lower end the casting of annealed iron to which the cells containing the

PLATE II.



Bruce Spectrograph on Carriage.

prisms are attached. The thickness of the sides and webs of this, which may be called the prism casting, is $\frac{5}{16}$ inch. At its upper and lower ends, with centers 14 inches apart, apertures are provided for the passage of the beam of light from the collimator and to the camera seen above. Six oblique braces of cold-drawn steel tubing, of 50 per cent. carbon, firmly screwed to lugs on this casting, give additional support to it, connecting it with the ring casting. The collimator and the cameras are of similar tubing, of diameters respectively $2\frac{3}{4}$ and $3\frac{3}{4}$ inches. The collimator is of course centrally situated with respect to the ring casting, and projects through it for about 9 inches. This tube actually supports nothing but the two-inch lens at the one end and the slit at the other. It has been turned true internally and externally at all significant points, and is attached by flanges to the two castings. A rack and pinion permits a sufficient motion of the lens in the tube for purposes of focusing. The head of the pinion is divided to millimeters, and can be easily estimated to the tenth. The camera tubes are attached to the prism casting by three bayonet joints at their very accurately surfaced flanges, and are further supported by a solid bracket, with hinged clamp, attached at about the center of the length of the large $5\frac{1}{2}$-inch tube. The twenty-four-inch camera is seen in position, the eighteen-inch lies on the carriage. Each is well provided with metallic diaphragms, as is the collimator tube. The rear end of each camera is provided with a brass frame, into which the plate-holder slides and is clamped by two screws. This frame can be tilted by a measurable amount to permit an improvement of the focus over an extended range of spectrum. A microscope with diagonal prism can be inserted in place of the plate-holder, for visual observations, and is shown in position in the figure. Ten brass plate-holders, to carry plates 2×4 inches in size, are provided. They are numbered with raised numerals, so that they can be distinguished in the dark.

In reference to the mounting of the prisms something of a departure was made from recent instruments, although a similar arrangement has been employed by others. It seemed desirable

to avoid the use of springs to retain the prisms in place, on account of the possibility of changes of pressure with the reversal of the telescope and inversion of the spectrograph. Accordingly each prism was placed in an iron mounting composed of two parallel brackets of triangular area, 63 mm apart, rising from a plane plate of 15mm thickness, all forming one continuous casting. The brackets are ribbed on the outside surface, and are further stiffened by a short iron rod connecting them just outside of the refracting edge of the prism. One of these brackets has its inside surface very accurately worked to a plane, and upon it rests the smooth under surface of the glass prism, also ground plane, separated only by a thickness of blotting paper. A triangular plate of iron of the size of the prism rests upon the other surface of the prism, with a similar intervention of blotting paper, and is held firmly in place by pressure from three screws through the upper bracket. The pressure from these screws, being distributed over a plane area of not less than 58 sq. cm, or 9 sq. in., may be presumed to be sufficiently uniform, and is regulated to as low a value as is consistent with the safe maintenance of the prism in position. The position of the prism between the two surfaces is defined by small strips of metal which give two points of contact on the back and on each face. Thus the back of the prism does not come in contact with the iron base of the cell, but is separated from it by about 2 mm, thereby reducing thermal conduction from the prism to the heavier iron parts.

The back side of the prism mounting and the outside of the prism casting, upon which it rests, have been given an accurately plane surface. Adjusting screws hold these two surfaces in contact, but permit them to be separated should it be found necessary to slightly vary the incidence on any prism. A light aluminium case, in two parts, completely incloses the prisms and nearly the whole of the prism casting, to which it is attached by six thumb-screws, permitting its removal in a moment when necessary. This case, practically light-tight, tends to maintain the steadiness of the temperature of the prisms, and also similarly protects the collimator and camera lens.

Slit.—The slit is of simple construction, with one jaw fixed (but adjustable) and the other moved by a screw of ¼mm pitch, with the head divided in one hundred parts. The slit as a whole may be turned in position angle by a tangent-screw for purposes of adjustment. The use of the Huggins reflecting slit had been found so convenient in the former spectrograph that we had resolved to employ it; but we felt it open to the suspicion, which has been expressed by others, that possibly disturbing effects might arise from the different inclination of the two jaws to the line of collimation ($90°-i$, and $90°+i$) when they are in the same plane. Hence we inclined the jaws separately, symmetrically away from the line of collimation, so that their speculum surfaces made an angle of 185° 47′. The adjustment was accurately made by screws which operate through the plates carrying the jaws. This arrangement involves the separate treatment of the light cones from the two jaws until they can be united in the guiding apparatus. The jaws have very sharp edges and are beveled rather more than usual to compensate for the inclination of the faces. The slit and the attachments for the comparison spectrum were furnished by William Gaertner & Co., of Chicago, who gave valuable suggestions in its design.

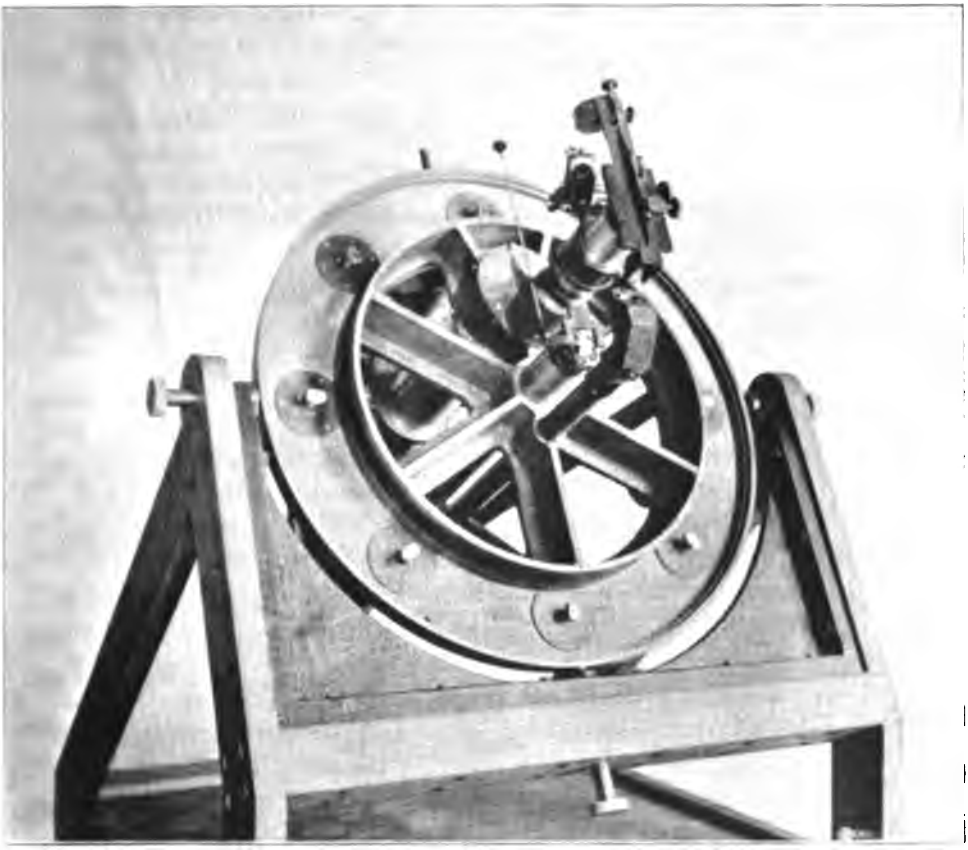
The length of the slit used in a given exposure is determined by windows in a small bar which slides just behind the jaws, and is moved by a rack and a pinion with a long shaft. A hardened steel spring catch entering notches on the under side of the rack defines the position of the windows. The position of the catch can be accurately adjusted by screws. The support for the bar and its mechanism is entirely independent of the slit or collimator tube, and is derived from the steel pillar which carries the spark apparatus. Double windows are used for the spark light, situated on either side of the single window for the star's light. Two of each are now in use, wider windows giving longer star and spark lines when the shorter camera is employed.

Spark apparatus.—This is supported on a steel tube of 2¾ in. diameter and 13 in. height, which is solidly attached to the ring casting. It carries a brass sleeve, movable up and

down the pillar by threaded collars, which supports a rotating drum having places for four pairs of electrodes, which can be successively brought into action. A small special vacuum tube takes the place of one pair of electrodes. A rack and pinion permits a motion of the whole drum parallel to its axis so that the spark can be properly adjusted in the direction of the length of the slit. Ebonite bushings insulate the drum and electrodes at the necessary points. The spring contact leading to the pair of electrodes in position for use also defines the place of the drum as to rotation. The top of the pillar is surmounted by a small brass casting with ways in which slides, by rack and pinion, a brass plate carrying at one end a concave (ellipsoidal) silvered mirror, and at the other a small plane 45° mirror or diagonal prism (the former is shown in the figure). The spark is at one focus of the concave mirror (of two inches diameter) and the slit at the other. The pillar has an aperture at the right height to allow the beam to pass, in which, if preferred, a lens may be introduced as a substitute for the concave mirror. When the spectrum of the spark has been obtained, the plate carrying the diagonal prism and concave mirror is racked back out of the way of the light from the star. The mirror can be moved toward or from the spark by a screw, thus giving an image out of focus if desired; and the mirror may be centered and adjusted by three screws operating against springs at its back.

Guiding device.—That portion of the light of the star which does not pass through the slit is reflected from the speculum jaws in two cones diverging on either side of the cone from the great objective. Each is caught by a diagonal prism situated within the "goose-neck," which may be seen projecting above the slit. The distance between these prisms is sufficient to permit the unobstructed passage of the light cones from the great objective and from the spark. After another reflection by a single larger diagonal prism, the light falls upon two small lenses at the upper end of the tube, which runs parallel to the collimator and is seen below it in the figure. Two further lenses

PLATE III.



REFLECTING SLIT AND COMPARISON SPECTRUM APPARATUS OF BRUCE SPECTROGRAPH.

of the diameter of the tube, deliver the two beams of parallel light to the guiding telescope at the lower left hand extremity of the instrument, after reflection at a mirror in front of this telescope, and making with it and with the long tube equal angles. In the eyepiece the images of the two jaws are in their natural juxtaposition, and the star image is seen as well as if the jaws were in the same plane. This mirror can be turned about an axis in its own plane, so as to leave a free path to the guiding telescope for the beam from the collimator reflected from the first surface of the first prism. A turn of a small projecting shaft which moves the mirror thus makes it possible to pass instantly from viewing the front of the slit to viewing the back; or, in other words, to pass from guiding by that portion of the light which does not go through the slit, to guiding by the light which has gone through the slit and is reflected by the first prism.

This simple arrangement has proven very satisfactory in practice. We guide most of the time by the light from in front of the slit, but the direct view of the collimator is very convenient in testing the adjustment of the slit and windows. The guiding telescope is unscrewed, and the eye used directly for controlling the full illumination of the collimator lens by the light from the spark. The optical combination of the guiding telescope and forty-inch objective gives a power of nearly 800 in observing the star's image on the slit.

The observer sits in his chair at the guiding telescope and holds a stout rod projecting from the telescope tube, springing the tube slightly (ordinarily not more than a small fraction of a millimeter) to correct for small departures of the telescope from uniform motion. Switches controlling very slow-moving motors in hour-angle and declination are also at the observer's hand, and can be accurately used for bringing the star to the center of the slit. This is marked by two parallel diamond rulings, 0.5 mm apart, on the speculum surfaces, perpendicular to the length of the slit, and the star's image is held between these rulings during the exposure. The angular field of the reflecting jaws is necessarily rather small, about $1'.5 \times 2'$, so that the

short finder of the telescope is not always adequate for promptly bringing a star upon the slit, on account of the flexure of the tube of the forty-inch. We have accordingly attached a six-inch photographic lens of 62 feet focal length near the great objective and use this as an additional (tubeless) finder. When the star is brought to the intersection of the cross wires of this finder, the star is found at its proper position on the slit.

Attachment to refractor.—When not in use the spectrograph rests on the carriage as shown in Plate II. Three powerful taper screws pass through a heavy iron casting in the carriage and enter three holes in the rim of the large ring casting of the spectrograph, two at the extremities of a horizontal diameter and one at the foot of a vertical diameter. No support is required at the back part of the spectrograph, which lies at the most convenient angle for attachment to the forty-inch refractor, when the latter is set on the meridian, and at — 25° declination, and the rising floor is at its highest level. With the carriage at its proper place on the floor, the heavy ring on the tube of the forty-inch is racked out about two feet (the micrometer and adapter having been previously removed), until it is in contact with the ring casting of the spectrograph. Then the two rings are firmly locked together by turning a pinion which simultaneously rotates eight bolts through the 90° necessary to make them effective. Thermal radiation from the spectrograph to the large mass of metal of the telescope tube is diminished by thin plates of ebonite at the surfaces of contact of the two rings just mentioned. While the process of transfer from micrometer to spectrograph is comparatively simple and can be done by one person if necessary, it is desirable that an assistant should be present. In order to restore the balance of the telescope after the attachment of the spectrograph, iron weights to the amount of about 500 pounds are removed from rods at the lower end of the tube. The slit is brought into the proper focal plane of the 40-inch objective, for the given central ray and for the given temperature, according to the color curve, by racking the whole spectrograph in or out. The setting is read by a millimeter

scale on one of the powerful sliding tubes carrying the telescope ring.

Temperature case.—The temperature of the spectrograph is maintained by the large aluminium case shown in Plate I, where the spectrograph is attached to the telescope. The case is of ample dimensions to inclose the whole spectrograph, its diameter at the point of attachment to the spectrograph being thirty-six inches, and its longest slant height about forty-five inches. It has double walls of sheet aluminium, separated by a space of half an inch, which is filled with felt. Access to the spectrograph is gained through two oak doors, the upper of which is opened when a plate-holder is inserted in the camera. The focusing pinion of the camera is reached from the lower door, which contains a plate-glass window through which the thermometers are read. When both doors are opened the space is sufficient for exchanging the cameras. A conical turret of aluminium was also constructed for inclosing the whole upper part of the instrument, from the slit and the spark apparatus to the correcting lens, but it has not yet been found necessary to use this. The temperature case is attached by screws to the rim of the ring casting of the spectrograph, and is only removed when there is need of access to the prisms or lenses, or for changing the focus of the collimator.

The heating current is led in directly from the 110-volt mains on the telescope tube, and passes through about forty feet of No. 24 German-silver wire strung around the inside of the temperature case. With this length of wire no rheostat is necessary. The thermometer in the outer case responds to the passage of the current by a rise of 1° C. in about two minutes.

The two thermometers are placed beside each other and about 40 mm apart, the one entering the inner prism case, and lying very close to the third prism, while the other is just outside of the inner case and thus records the air temperature within the outer case.

The thermometers are 48 cm long, and are graduated to fifths of degrees C., $0\overset{\circ}{.}2$ being equal to 1 mm. They can be

easily read to tenths of a degree C. through the window in the lower door. Our experience with the temperature case has so far been very satisfactory. A change of more than 0°.1 in the temperature of the air near the prisms seldom occurs during an exposure of an hour or two; and usually the temperature of the inner case can be maintained within 0°.1 or 0°.2 C. through the whole night. Ordinarily a change of over 1° C. in the outer case is necessary to make an appreciable change in the inner case. If our experience in extreme cold weather indicates its necessity, we can introduce an electric fan to keep in circulation the air of the outer case; or an automatic thermostat regulator may be employed; but during the present summer and autumn the above simple arrangement has been amply sufficient.

OPTICAL DETAILS.

The lenses.—The correcting lens, collimator, and 24-inch camera lens were made by Brashear after specifications by Professor C. S. Hastings. Our first order was for the "isokumatic" quadruple construction described by Professor Hastings in this JOURNAL.[1] It subsequently appeared, however, that one of the varieties of glass used in his experiments could no longer be procured; nor were Mr. Brashear's efforts to have it especially made in Paris successful. Consequently our order was changed to a triple construction, with cemented lenses. These lenses were delivered during the autumn of 1900.

1. The correcting lens is of 57 mm aperture, and is designed, in accordance with the data of the color curve of the forty-inch objective, to cover as great a range on either side of λ 4500 as was feasible. It was to be placed 100 cm in front of the focus of the telescope for λ 4500, for which wave-length it was not to change the angular aperture. The lens is carried within a stiff steel tube, which has a mounting like that of the adapter for the micrometer; the lens is 21 inches inside the end of the telescope tube, the best position found by experiment corresponding closely to the designer's specifications.

[1] ASTROPHYSICAL JOURNAL, 9, 162, 1899.

The performance of the correcting lens is very satisfactory. A star spectrum is of practically the same width for nearly 200 tenth-meters on either side of λ 4500.

2. The triple cemented collimator is of 51 mm aperture and 958 mm focal length, corresponding closely to the angular aperture of the forty-inch objective. It was first focused by Schuster's method and then by Newall's method, with entirely accordant results. The focusing will be repeated at lower temperatures in the approaching cold weather.

3. Camera A. This is a Zeiss Anastigmat, No. 9 of Series II. It was selected from a group of lenses of that character kindly furnished by Bausch and Lomb for experimental examination in the summer of 1900. It has an aperture of 71 mm, and a focal length, as determined by very sharp star trails, of 449 mm, giving an angular aperture of 1 : 6.3. Its performance has been satisfactory: metallic spectra taken with it are sharp over an angular field of nearly 4°, and measurable over 8°. Our experience has shown it to be a great convenience to have two different cameras for use in a spectrograph, even if their focal lengths are not greatly different. In cases where it is difficult to discriminate between defects of a lens and the prisms, the question may be promptly settled by the trial with another camera.

4. Camera B. This triple lens, of Professor Hastings' design, has an aperture of 76 mm and a focal length of 607 mm as determined by star trails; its angular aperture is therefore 1 : 8. After some weeks of use this lens developed a serious astigmatism, and it has been twice recemented by Brashear during the present season. The difficulty may have arisen from a strain at some point in the lens cell, which has been enlarged. It now seems to be cured of this trouble. The lines in metallic spectra taken with this lens are not quite as sharp as is desired, but as they appear to be now entirely symmetrical, the accuracy of setting does not seem to be impaired.

The central ray.—The choice of λ 4500 as the ray which should pass the prisms at minimum deviation, and for which the

lenses should be corrected, was made, after the satisfactory experience in that part of the spectrum with the former spectrograph, for the following reasons: (1) The maximum sensitiveness of the fast plates we have used (Cramer's Crown and Seed's Gilt Edge) is somewhat beyond this point toward the red for light that has passed through the forty-inch lenses and the flint prisms of the spectrograph. The difference in sensitiveness for a point near λ4600 and near λ4300 is quite marked; an estimate is difficult, but probably a ratio of 2 : 1 would not be too large. This would mean a considerable saving in exposure time—an important consideration in case of the faint stars which are on our observing lists. (2) Most of the other spectrographs now engaged in line of sight work are set for the region of $H\gamma$: hence it was desirable that a different portion of the spectrum should be used here. (3) In spectra of the *Orion* and *Sirian* types the lines at λ4471 and λ4481 are very important, so that they should be near the minimum ray. (4) The spectrum of the titanium spark, which we have found more convenient than the iron spark, has a large number of excellent lines on either side of λ4500.

The prisms.—The prisms, of glass from Mantois, No. 3814, were received from Mr. Brashear in April 1900. They were 57 mm high, with faces of 117, 127 and 135 mm respectively. They were quite free from striae and bubbles, and of an excellent degree of transparence. The indices of the glass were : for λ4227, $n = 1.6418$; λ4405, $n = 1.6374$; λ4861, $n = 1.6287$. The angles were 66° 1′, which would give a deviation of 180° for the ray λ4480. The mean path in the glass of the beam of that wave-length was 206 mm. After measurement of their angles and indices and a brief visual examination, the prisms were mounted for use in a prismatic camera and taken to North Carolina for the eclipse of May 28. Several of the spectra then obtained have been reproduced in this JOURNAL. Allusion is made in my paper[1] to a tendency to diffuseness toward the violet side of the strongest bright lines, which was not then explained

[1] ASTROPHYSICAL JOURNAL, 12, 315–323, 1900.

It now seems probable that this was at least partly due to the lack of homogeneity of the glass.

After the return from the eclipse the prisms were carefully tested, both visually and photographically, with the use of a variety of auxiliary lenses. While there appeared to be no variation of the definition, when different portions in a vertical direction of the prism face were used, a marked change was always noticeable when the beam of light (of reduced aperture) was first sent through the half of the prism near the thin edge and then through the half near the thick base. The discrimination between effects due to the prisms and the lenses was rendered difficult by the astigmatism of some of the lenses used. One lens of 38 inches focus employed as a collimator had so marked an astigmatism that a rotation of the lens in its cell through 60° blurred the visual image so much as to make the region of spectrum wholly unrecognizable. The collimator was focused by various methods, including the use of a grating, so that the imperfections of prisms could be eliminated from the determination of focus. When the lenses corrected for $\lambda 4500$ were received, in the autumn, the tests were repeated visually and photographically, with the prisms individually and collectively, with a similar result.

Finally I took the prisms to Allegheny, and with the kind assistance of Professor Wadsworth and Mr. Brashear, was able to clearly exhibit the effect. As it was thought possible that refiguring the surface might diminish this, Mr. Brashear retained the prisms and Mr. McDowell perfected a new method and refigured the surfaces with the greatest possible care.

When the prisms were again received at the Observatory, however, it was unfortunately found that the difficulty was still present, so that the use of the full aperture of 51 mm was precluded.

Under these circumstances it seemed best that new glass should be ordered, of the most perfect homogeneity procurable. Our experience in this matter is given in detail for the possible benefit of others requiring large prisms for spectroscopes. We

were surprised to find that it is not the general practice among opticians to use for prisms as high a quality of glass as is employed for telescope objectives; or, specifically, that prism glass is not commonly subjected to the process of "fine annealing." Inasmuch as the requirements are no less exacting in case of a prism than of an objective, we at once decided that the new glass should be of a quality equal to that of the best telescope objectives; and this was a principal specification in the order sent to Schott & Co. of Jena on December 31, 1900.

Meantime numerous stellar and experimental plates were taken with various apertures of the collimator beam; by Mr. Ellerman during the latter part of the winter, by myself during the spring. But no really satisfactory stellar plates were obtained with practicable apertures.

The shape in which the material is cast would also seem of distinct importance in respect to the homogeneity of the resulting glass. It seemed a reasonable presumption that a prism cut from a disk would be more likely to be free from variations of homogeneity along its face, and this shape was requested. It was also requested that the disks should be considerably larger than the inscribed prisms. The kind of glass ordered was the dense silicate flint (O 102), which was used in the new spectrograph of the Potsdam Observatory, and highly commended for its transparency and freedom from color. The shipment of the material (designated as O 998 by Schott & Co.) for the two larger prisms, amounting to six kilograms, was received in March 1901 by Mr. Brashear, who had agreed to figure the prisms without expense to the Observatory. The block for the smallest prism (about 2 kg), which had to be fine annealed separately, was not received until June. The sizes of the three prisms as sent from Jena were: faces, 150, 150, 170 mm; 140, 140, 145 mm; and, equilaterally, 125 mm; height, 80 mm. They had been ground and polished at top and bottom to permit of the polariscopic tests. At Mr. Brashear's they were cut down to the dimensions desired for the spectrograph, viz., faces, 114, 123, and 133 mm; bases, 120, 130, and 140 mm, and height, 57 mm; and were carefully

figured. The angle required to give a deviation of 180° for λ4500 was 63° 36.′ The actual angles, as measured here, were found to be: I (smallest), 63° 35′ 40″; II, 63° 31′ 35″; III, 63° 36′ 20.″ On account of the small angle of No. II the central ray, or wave-length at minimum, will be about λ4480. The indices are as follows: $H\gamma$, 1.6766; $H\beta$, 1.6647; λ4300, 1.6778; λ4500, 1.6724; λ4700, 1.6678. The mean path of the rays in the glass is 195 mm. The most favorable angle for this glass in respect to loss by reflection is 61° 45′, to which the above prism-angles are sufficiently near.

No alteration was required in the prism-cells on changing from the Mantois to the Jena prisms beyond slightly shifting the small strips which fix each prism's position.

In the examination and measurement of the prisms and the subsequent work with the spectrograph from June of the present year, I have had the valuable assistance of Mr. W. S. Adams, assistant at the Observatory, whose whole time is devoted to this instrument, and to measuring and reducing the plates.

The examination of the new prisms showed a distinct improvement in the definition as compared with the others. The effect of change of focus when the rays pass through the prism near the edge and then near the base is not wholly eliminated, but is less marked than before, both when the prisms are examined separately and together. A comparison of the best solar spectra taken with the two trains showed a sharpness in case of the Jena prisms, when the whole aperture was used, equal to that when only half aperture was employed with the Mantois prisms. An improvement in definition is still obtained by reducing the aperture to one half — partly due to the reduction of the aperture of the lenses — but the actual gain in sharpness is hardly sufficient to make it worth while to lose any light.

This somewhat disappointing result from the new prisms is perhaps due to the moulding of the prisms, for it would appear that they were not cut from disks as requested, but were moulded in triangular shape by Messrs. Schott & Co. We do not despair of securing prisms that will give good definition

over their whole face, which Mr. Newall's similar experience has shown to be possible with large prisms; but the accuracy of the measurements for stellar velocities with the present prisms may be considered as quite satisfactory.

THE SPECTRA.

The dispersion and scale of the plates for the two cameras are indicated in the following table:

Wave-length	Angular dispersion for one tenth-meter	Tenth-meters per mm	
		A	B
4300	45'.7	10.0	7.4
4500	33.8	13.5	10.1
4700	26.0	17.6	13.1

For comparison, similar data for the Mills spectrograph and the new Potsdam spectrograph are collected in the following table, valid for the wave-length of $H\gamma$.

Spectrograph	Camera focus	Dispersion for one tenth-meter	Tenth-meters per mm
Bruce	A 449 mm	42'.8	10.7
Bruce	B 607	42.8	7.9
Mills............	406	40.5	12.6
Potsdam III	1 560	39.7	10.2
Potsdam III	2 410	39.7	13.8

The dispersion of the three instruments is thus practically the same for this region. The Bruce spectrograph, however, has, of course, less angular dispersion at λ 4500 than at $H\gamma$— by about one fifth. The scale of the spectra with camera B, at λ 4500, is the same as that of the longer Potsdam camera at $H\gamma$, and a trifle greater than that of the Mills spectrograph at $H\gamma$. The range over which our measures of stellar spectra are commonly made is with camera A from about λ 4300 to λ 4650; with camera B from about λ 4340 to λ 4700. On metallic and solar spectra the measures may be extended over a wider range.

In the practical separation of lines on fine-grained photographic plates, the performance of the Bruce spectrograph is

almost identical with that described by Professor Vogel in his paper in the ASTROPHYSICAL JOURNAL (11, 393, 1900) for his spectrograph III; and this was found to correspond closely to the results of the Mills spectrograph. On stellar spectra on the most rapid plates an equal sharpness is not to be expected. We have not yet obtained such plates on which separations of much less than 0.20 tenth-meters can be made in the region for which the prisms are at minimum, although 0.22 tenth-meters are often separated.

A comparison of the three spectrographs named would thus indicate very similar capabilities. The Bruce spectrograph does not fully develop, with either the Mantois or Jena trains of prisms, the power that the length of the bases of the prisms would imply, doubtless because of lack of homogeneity of such large masses of glass. But with an equal resolution, it has the advantage of the other instruments in respect to its length of collimator, permitting the use of a wider slit for the same purity, and shortening the time of exposure, which, with equal atmospheric conditions, should be equivalent to increasing the light power of the instrument, so that fainter stars should be obtainable.

For the same purity the photographic efficiency, as regards exposure time should be as the ratios of the squares of the focal lengths of collimator and cameras, which would be about as follows: Bruce, camera A, 4.6; Mills, 3.1; Bruce, camera B, 2.5; Potsdam III, 1.4. The apparent advantage of speed of camera A over camera B is not fully realized, however, on account of the reflections and absorption in the anastigmat, which has five component lenses.

As has been pointed out by Professor Campbell the actual loss of stellar light in a slit spectrograph with a train of prisms is serious in the extreme. An exact statement is not possible for lack of adequate data as to the absorption and reflections in the lenses involved. In the present case probably at least 40 per cent. of the light is lost at the forty-inch objective; and further amounts at the correcting lens and collimator; the

prism train transmits only about 30 per cent. of the beam incident upon it, and finally the camera lens exacts its tribute. Thus a conservative estimate would assign a loss of hardly less than 90 per cent. of the star's light falling upon the forty-inch object-glass, without considering the waste of light at the slit. Under average conditions of the atmospheric unsteadiness, probably less than one half of the light falling upon the jaws actually enters the slit; with bad "seeing" this may become as low as one fifth. Hence, taking into account the inaccuracy of guiding it would seem that with good conditions of the atmosphere, not over one twentieth of the star's light falling upon the object-glass is effective on the plate in the camera, and under bad conditions this may be as low as one one-hundredth.

Exposure times.—The exposure time required for a stellar spectrum obviously varies greatly with the atmospheric conditions of steadiness and transparency. With a slit-width of 0.025 mm ($=0\overset{\prime\prime}{.}25$) and camera A we should commonly give an exposure of about 20 minutes to a star of the solar type like *α Arietis*, whose magnitude is 2.0 in the Harvard Photometry and 4.1 in the Draper Catalogue. I have obtained a strong impression of *Polaris* in 8 minutes with a slit-width of $0\overset{\prime\prime}{.}5$. With that slit-width an exposure of two hours gives a spectrum of the actual photometric magnitude 5.5, which is as faint a star as we have yet photographed.

Comparison spectrum.—The rotating drum of the spark apparatus carries titanium and iron electrodes and a helium tube, which can be used successively without any disturbance of the adjustments. We use a small coil of self-induction in the secondary circuit for the suppression of the air lines developed by the iron spark. It also tends to increase the sharpness of the titanium lines.

The windows behind the slit give for camera A a width of 0.16 mm or 0.34 mm, as preferred, to the star spectrum and to the portions of the comparison spectrum immediately above and below it. For camera B these values are one third larger.

We have found the titanium spark more satisfactory than

that from iron for the region we cover. When a long exposure is required for the star, two or more brief exposures, symmetrical in respect to time, are given to the comparison spectrum.

An occulting plate can be turned into position in front of the sensitive film so that a shorter exposure may be given to the most intense comparison lines.

MEASUREMENT OF PLATES.

The plates were until recently measured on Zeiss comparators, with millimeter silver scale graduated to fifths of millimeters read by a second microscope. As the use of the second microscope with the weaker illumination of the scale tended to strain the eyes and took considerable time, two new screw micrometers were made, according to specifications by Professor Hale and myself, by William Gaertner & Co., of Chicago. The screw has a pitch of 0.5 mm, and the large head permits a reading to 0.001 mm. The stage holding the plate slides in ways for the purpose of quickly aligning the spectrum, and to enable any desired reading on the head to correspond with the position of any line on the plate. The screws have not yet been investigated, but a preliminary series of measures on a glass scale shows that the error of a division is not likely to be much over 1 μ, or about the same as the errors occurring in the scale of the Zeiss comparator. A description of these micrometers and of their performance is reserved for a later time.

The schedule of measurement of a plate consists in making a series of four settings on each star line taken, two when approaching the wire from the right, and two from the left. Two settings are made on the upper and two on the lower part of each selected comparison line.

Only the best star lines are measured, if the star has numerous lines to choose from; and comparison lines lying near the star lines are selected; without reference to direct coincidences of the two. The plate is turned around on the stage after it has been measured with the violet toward the left, and the settings are repeated with the violet toward the right. The second set

of readings are subtracted from some convenient number and the mean of the two sets is taken.

This measurement in both directions is considered essential, as an observer may have a large systematic difference between his mode of setting the cross-wire on a black line and a white line (on the negative). The writer, for instance, had a systematic difference amounting to several kilometers in velocity on plates taken with the old spectrograph. On measuring a positive copy of such a plate it was found that the systematic difference was the same in amount but opposite in sign from that on the negative. There seems to be no reason to think that this systematic difference is not eliminated by measuring the plate in both directions.

REDUCTION OF PLATES.

The method of reduction which I adopted two years ago, and to which we still adhere, is based on the principle that each plate is to be reduced for itself, independently of any standard plate of either a metallic or solar spectrum. This can be readily done with the aid of the Hartmann formula in its simple form $\lambda = \lambda_0 + \frac{c}{s - s_0}$, which perhaps should be called the Cornu formula. Three of the best comparison lines, one near the middle and the other two at the ends of the region of the star spectrum to be included, are selected as standards for the plate in question, and the constants are derived in a matter of fifteen minutes with the aid of a calculating machine. The corrections for curvature are applied directly to the mean of the settings on the comparison lines, and the wave-lengths of all lines measured on the plate are computed from the constants of the formula. Then the differences are taken between the wave-lengths of the comparison lines as thus computed and as given by Rowland. This indicates the departure of the simple formula from an exact expression of the dispersion, together with the error of setting on the comparison line. This can be smoothed out by a small supplementary curve, but usually the quantities involved are so small that the corrections to the formula at the positions of the

star lines can be applied mentally. The differences are then formed between the wave-lengths in the star and in the Sun, when the lines occur in the Sun; if the lines are not solar, laboratory determinations are taken, as for helium. These differences, which are the displacements due to motion in the line of sight, are converted into velocities, and the mean is taken. In spectra of the solar type when two lines are so close as to blend into one in the stellar spectrum, the normal wave-length in the Sun is taken as the mean of the two lines in Rowland's table, weighted according to their intensities in that table. In spectra of types containing but few lines, weights are assigned to the star lines at the time of making the settings, as there may be a great variety in the sharpness of the different lines. The measurer is quite unconscious of the effect on the results of this assignment of weights, and in general he does not know anything as to the accordance of his results until the computation is well advanced.

As an illustration of the reduction I select a plate of *α Arietis*, as the velocity of this star has been accordantly determined by several observers. This is not better than an average plate, although the character of the spectrum permits of a good degree of accuracy of measurement.

The letter B in the sixth column denotes that the wave-length given is the weighted mean for two or more close solar lines. The three titanium lines whose wave-lengths are in heavy face type in the second column were the standards used in the derivation of the formula.

Other published determinations of the velocity of this star are: Campbell, —14.1; Vogel and Scheiner, —14.7; Lord, —14.0 km.

Although it is not the object of this paper to communicate the results of our measures of velocities, which will be published in due time, mention should be made of the case of the star *ε Leonis*, as illustrating the contrast between the old and new spectrographs.

The measures of the plates obtained by Mr. Adams in February, March and April, 1900, gave a range of 23 kilometers in

PLATE B 233 — α *ARIETIS*.

Taken 1901 November 15, G. M. T. 14^h 36^m, by E. B. F.
Measured by W. S. Adams.

Means of settings S = Star lines *Ti* = Titanium lines	Wave-length by formula	Correction to wave-length	Correction to star lines	Corrected wave-length of star line	Wave-length in Sun	Displacement	Velocity km. per sec.
S 28.0279	4442.418		+0.015	4442.433	4442.510	−0.077	−5.20
S 28.1850	4443.880		+0.015	4443.895	4443.976	−0.081	5.47
Ti 28.1937	4443.961	+0.015					
S 28.6055	4447.808		+0.004	4447.812	4447.892	−0.080	5.39
Ti 28.7659	4449.313	0.000					
S 29.0187	4451.691		−0.001	4451.690	4451.752	−0.062	4.18
Ti 29.6436	4457.604	−0.004					
S 30.7931	4468.614		−0.004	4468.610	4468.663	−0.053	3.56
Ti 30.7987	4468.667	−0.004					
S 31.5695	4476.148		+0.003	4476.151	4476.214B	−0.063	4.22
Ti 32.1089	4481.431	+0.007					
S 32.1972	4482.300		+0.006	4482.306	4482.376B	−0.070	4.68
Ti 32.8241	4488.497	−0.004					
S 33.4467	4494.705		−0.016	4494.689	4494.738	−0.049	3.27
Ti 33.6095	4496.337	−0.019					
S 34.1050	4501.329		−0.003	4501.326	4501.431B	−0.102	6.79
Ti 34.1168	4501.448	−0.003					
S 35.2379	4512.873		−0.014	4512.859	4512.906	−0.047	3.12
Ti 35.2425	4512.920	−0.014					
S 35.4846	4515.411		−0.012	4515.399	4515.484B	−0.085	5.64
Ti 36.6468	4527.490	0.000					
S 36.7676	4528.757		−0.006	4528.751	4528.798	−0.047	3.11
S 37.3481	4534.877		−0.033	4534.844	4534.953	−0.109	7.21
Ti 37.3585	4534.986	−0.033					
Ti 38.2862	4544.974	−0.010					
S 38.3976	4546.071		−0.009	4546.062	4546.129	−0.067	4.42
Ti 39.0060	4552.639	−0.007					
S 39.1432	4554.128		−0.013	4554.115	4554.211	−0.096	6.32
Ti 39.2861	4555.682	−0.020					
S 40.0346	4563.876		−0.006	4563.870	4563.939	−0.069	4.53
Ti 40.0409	4563.945	−0.006					
S 40.6947	4571.177		−0.002	4571.175	4571.275	−0.100	6.56
S 40.7752	4572.072		−0.002	4572.070	4572.156	−0.086	5.64
Ti 40.7829	4572.158	−0.002					
S 41.2421	4577.286		−0.007	4577.279	4577.356	−0.077	5.04
S 42.3755	4590.093		−0.020	4590.073	4590.126	−0.053	3.46
Ti 42.3801	4590.146	−0.020					
S 44.6303	4616.228		−0.001	4616.227	4616.305	−0.078	5.07
S 44.7282	4617.383		±0.000	4617.383	4617.452	−0.069	4.48
Ti 44.7340	4617.452	0.000					

Mean .... −4.88

For Hour-Angle 1^h 40^m E. Correction for diurnal velocity .. +0.13

Correction for orbital velocity .. −8.91

Reduction to Sun .... −8.78

Star's velocity, km per sec............ −13.66

the velocity, and the star was supposed by us to be a spectroscopic binary. This was not confirmed by the set of eight values obtained with the Mills spectrograph, subsequently published by Mr. Wright, which showed a range of but 2.7 km, and a mean velocity of + 5.1 km. Three plates have now been obtained with the Bruce spectrograph and measured by Mr. Adams, with the following results :

1900	October 3, $21^{h}.9$ G. M. T.	+ 4.4 km per sec.
	October 23, 22.6	+ 3.5
	October 31, 22.7	+ 4.0

The variability was therefore in the spectrograph instead of the star. The case is instructive as showing the possibilities of a derangement of the instrument which does not affect the definition of either the star lines or comparison lines. Previous determinations of planetary velocities had also yielded results of a very fair degree of accuracy.

FUNDAMENTAL VELOCITY-STARS.

In the present state of line of sight work, which is in progress in six or seven observatories, a certain amount of duplication of work by different observers would not appear wasteful, especially if the conditions are varied. With a good spectrograph in accurate adjustment, the principal uncertainty (aside from subjective effects in the measurements) lies in the assumptions that we know the actual wave-lengths of the lines (1) in our comparison spectra, and (2) in the stellar spectra.

The first presumes that the tabular values taken to apply to the comparison spectra were obtained under conditions similar to those in the source of the comparison spectrum. Shifts in the wave-lengths of lines in the comparison spectrum of the sort reported by Haschek[1] in spark spectra would most seriously modify the stellar velocities deduced from lines so affected. Inasmuch as the arc is employed at Potsdam as the source of the comparison spectrum, and the iron spark (and to some extent the hydrogen tube) at the Lick Observatory, it would seem

[1] ASTROPHYSICAL JOURNAL, 14, 181, 1901.

desirable that the conditions should be further varied with the Bruce spectrograph, and I should prefer to employ the vacuum tube, which may be expected to be less liable to fluctuations and idiosyncracies than the spark or arc. A helium tube that we have lately been using happened to give an excellent compound line spectrum of hydrogen, which contains a large number of fine lines quite well distributed over the region covered by our plates. If we are able to get tubes giving this spectrum uniformly, it may be regularly employed here after the wave-lengths of its lines have been accurately determined.

The plates of the Moon and planets taken at frequent intervals by all observers to control the general adjustment of the spectrograph tend of course to disclose any peculiarities of the individual lines of the comparison spectrum, so that such effects could ultimately be largely eliminated. But measures of planetary plates do not give us a check on the second of the assumptions mentioned above, namely, that the wave-lengths of the star lines are exactly the same as for corresponding lines in the Sun, or in laboratory spectra. The conditions of pressure (and temperature) in the stellar atmospheres may well be such as to modify the wave-lengths quite appreciably, and differently for lines due to different elements. These as yet uncertain effects can probably be best studied by the comparison of the results of different observers using entirely different stellar lines in different parts of the spectrum. In general, the greater the increase in the accuracy of the measurements the more likely are we to find systematic differences in the results with different instruments and different observers, as is of course always the case with fundamental determinations. The time should therefore soon come, if it has not already arrived, when a short list of suitable stars, not spectroscopic binaries, should be made out and observed regularly as fundamental stars by the different spectrographs engaged in line of sight work. Faint as well as bright stars should be included, and representatives of some of the different types of spectra; and the stars should be properly distributed for the different seasons. The comparison of the

stellar velocities thus obtained should be of great value both as establishing certain fundamental values, and as indicating the systematic errors of the different instruments and observers. This Observatory will be glad to take part in any such scheme of coöperation, which it is hoped may be arranged.

YERKES OBSERVATORY,
December 1901.

ON THE ORIGIN OF DOUBLE LINES IN THE SPECTRUM OF THE CHROMOSPHERE, DUE TO ANOMALOUS DISPERSION OF THE LIGHT FROM THE PHOTOSPHERE.[1]

By W. H. JULIUS.

A PECULIARITY, appearing in the photographs which Professor A. A. Nyland obtained with the prismatic camera during the total eclipse of May 18, 1901,[2] caused me to investigate more closely, in the line of my former paper on solar phenomena, what characteristics the chromospheric lines must show, when they really derive their light from the photosphere.[3]

At the meeting of February 24, 1900, I developed considerations which lead us to expect that the light of the chromosphere is to a large extent composed of photospheric light, which has undergone anomalous dispersion in the absorbing vapors of the Sun. The wave-length of the bright lines in the spectrum of the prominences, chromosphere and flash cannot, according to this hypothesis, be exactly the same as the wave-length of the corresponding absorption lines of the ordinary solar spectrum. For every bright line belonging to an absorption line of wave-length λ was supposed to be produced by two groups of radiations, whose wave-lengths are respectively all smaller and all larger than λ. The light on the red side of the absorption lines

[1] Communicated by the author, from *Proceedings of the Royal Academy of Sciences, Amsterdam.* Meeting of October 26, 1901.

[2] With the consent of Messrs. Nyland and Wilterdink (the members of our expedition who were most concerned with the spectrographic researches) only this special feature of the photographs will be shortly referred to in this paper. The report, containing a full account of the various observations made by our party, will be published later.

[3] I shall frequently make use of the terms photosphere and chromosphere, but I wish to state emphatically that I mean by them only the white disk of the Sun and the more or less colored edge or light ring, as they appear to our eyes. I do not imply the idea of a sharply limited ball, emitting white light and surrounded by a translucent shell, which itself emits colored light.

will perhaps in most cases be a little more intense than that on the violet side, because, however variable as to place and time the density relations of the solar gases may be, it is always a little more probable that the average density of the layers which are penetrated by the light that reaches us, increases towards the Sun's center, than otherwise.[1] Where powerful "Schlieren" occur, however, the wave groups on the violet side may be the stronger ones.

Further it is clear that from each group, those rays whose wave-lengths differ much from λ, can in general only be seen close to the Sun's edge, for there only a small abnormality in the refractive index is necessary to deflect photospheric rays to our eyes. Light whose wave-length differs less from λ can reach us from a broader strip of the chromosphere; and far from the Sun's edge, as a rule, we may expect to see only rays whose wave-lengths differ very little from λ.[2]

To this rule, too, exceptions may be found at places, where mighty prominences show us the presence of great irregularities in the density distribution of the Sun's gases.

Let us now consider what under ordinary circumstances the light distribution in a chromospheric line would be, if we were only concerned with refracted photospheric light, unmixed with any appreciable radiation emitted by the absorbing gas.

In Fig. 1 is given a representation of the form which the dispersion curve of the absorbing gas will assume in the neighborhood of one of the absorption lines. Let the line XX' be the axis of wave-lengths with the value λ at the point O, and let an ordinate zero represent that the refractive index is equal to unity. If no absorption line existed in this part of the spectrum, the dispersion curve would be a nearly straight line NN' at a small distance above XX' and almost parallel to it. But if rays of wave-length λ are strongly absorbed, then the curve consists of two branches of the form represented.

[1] W. H. Julius, *Proc. Royal Acad. Amsterdam*, **2**, 581, 585, *Astron. Nachr.*, **153**, 439.

[2] *Proc. Royal Acad. Amsterdam*, **2**, 581.

Light with a wave-length λ cannot now occur in the chromospheric spectrum. Rays $\lambda \pm \delta$, in the normal spectrum belonging to positions a and a', will reach us from a chromospheric ring of relatively great width, but naturally with greater intensity from the inner than from the outer parts of the ring. Rays $\lambda \pm 2\,\delta$, belonging to places b and b', come only from a smaller chromospheric ring, etc. All these rings have the photosphere for their inner limit. The breadth of the rings from which we can receive light of wave-lengths $\lambda \pm \delta$, $\lambda \pm 2\,\delta$, etc., will depend upon the ordinates of the dispersion curve at the points given by a, a', b, b', etc. We can, as a first approximation, put these widths proportional to the quantities $a_1\ a_2$, $a_1\ a'_2$, $b_1\ b_2$, $b'_1\ b'_2$, etc., by which these ordinates differ from the ordinates of the normal dispersion curve $N\,N'$.

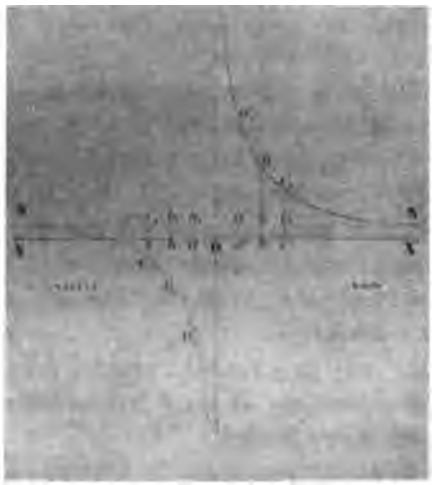


FIG. 1.

In recent eclipse work both the slit spectrograph and the prismatic camera (or the objective grating) have been used; up to this time most results have been obtained by the latter. We shall, therefore, investigate the character of a chromospheric line as it must show itself in ordinary circumstances in the prismatic camera.

The prismatic camera gives for every monochromatic radiation, coming from the chromosphere, an image of the crescent, ranging these images according to the wave-lengths. The light distribution in such an image shows us the intensity with which the corresponding radiation comes out of the various parts of the crescent. Consequently a pure monochromatic image will, as a

rule, possess the greater intensity on the concave side, where it is limited by the Moon's edge, and will gradually fade away on the convex side.

The images due to neighboring rays will, however, partially overlap. This will be especially noticed with the two-ray groups which together form a chromospheric line ; in this combination of arc images we may expect a quite different distribution of the light than would be found in an image, formed either by monochromatic light or by one simple ray group, such as a more or less rarefied gas would show us in its emission lines.

Let Z (Fig. 2) be a portion of the Moon's edge at the instant of the second or third contact of a total eclipse. We may now consider the compound light, arising from a small column $Z\,a$ of the chromosphere, dispersed into a horizontal spectrum parallel to the line $P\,P'$. In order to obtain more easily an idea of the share which the various rays contribute to the light distribution in the band, we separate the various rays from one another and represent on distinct lines $P\,P'$, $Q\,Q'$, $R\,R'$ . . . . those parts of the spectrum, where chromospheric light is found of wave-lengths equal respectively to λ, $\lambda \pm \delta$, $\lambda \pm 2\,\delta$, etc.

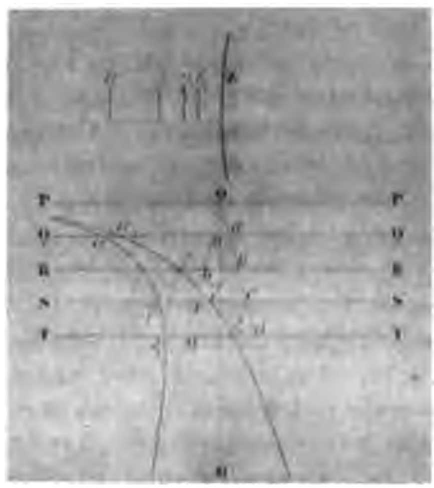


FIG. 2.

The point O may indicate the place where the Moon's edge would be seen if absolutely monochromatic light of wave-length λ appeared on its left.

The rays of wave-length λ are, however, completely absorbed, so that nothing need be represented on the line $P\,P'$.

On the line $Q\,Q'$ we find first the light of wave-length $\lambda - \delta$,

which projects from the sharp edge of the Moon at a and reaches (with decreasing intensity) from there to α, and secondly the light of wave-length $\lambda + \delta$, which reaches from a' to α'.

In the same way we find on $R R'$ the rays $\lambda - 2\delta$ and $\lambda + 2\delta$, corresponding respectively to the sections $b\beta$ and $b' \beta'$; on $S S'$ the rays $\lambda - 3\delta$ and $\lambda + 3\delta$ at the sections $c\gamma$ and $c' \gamma'$, etc.

Because the sections $a\alpha$, $a' \alpha'$, $b\beta$, $b' \beta'$ . . . represent the width of the chromospheric rings corresponding to the various sorts of rays, we have considered them proportional to the lengths a_1 a_2, a'_1 a'_2, b_1 b_2, b'_1 b'_2 of Fig. 1. Hence the extremities α, β, . . . and α' β', . . . etc., lie on two curves, whose shape is closely related to that of the dispersion curve. The share which all intermediate waves bear in the light distribution is thus shown, if we only notice that for each kind of light the intensity decreases from right to left. This is represented by shading in the upper part of Fig. 3. Finally, to obtain the light distribution in the chromospheric line, we only need suppose that the figure is compressed in the vertical direction and that thus the light intensities are added together. The resulting intensity is then found to be approximately distributed as is shown by the shading in the spectrum given below. Hence a *double line* is produced, each of the components of which shades off gradually on each side, so that there is still light of a somewhat considerable intensity in the intervening space.

If the rays whose wave-lengths are less than λ are on the average of the same intensity as those with wave-lengths greater than λ (this case is shown in the figure), the "center of gravity" of the chromospheric line is shifted a little to the convex side of the image with respect to the place belonging to the absorption line of wave-length λ. If, on the contrary, we consider the inner limit of the crescent, it appears that the line has shifted to the other side. This must involve us in difficulties when trying to find the exact wave-length of a chromospheric line.

Moreover, all kinds of variations may be expected in the intensity distribution. The ray groups whose wave-lengths are

greater than λ, may be intenser or *vice versa.* In such a case the displacements of the chromospheric line, both with regard to limit and to position of center of gravity, may assume quite other values. Such displacements of variable character are actually often observed (by Campbell, Frost, Lord, and others).

The figure represents a case where on the convex side of the crescent the intensity of the system decreases faster than on the concave side (just otherwise than we should expect from a cursory examination; indeed the chromospheric crescent, observed without a spectroscope, is sharply limited on the concave side). This peculiarity too has been often seen in the chromospheric spectrum (cf. Frost, ASTROPHYSICAL JOURNAL, **12**, 315, Dec. 1900). In general, many of the irregularities in the form of the lines of the chromosphere and the flash, as given by Mascari,[1] Campbell,[2] Brown,[3] Lord,[4] Frost,[5] and also the principal features of the chromospheric spectrum, recently once more discussed by Sir Norman Lockyer,[6] can be easily explained if we suppose the lines to be produced by anomalous dispersion.

FIG. 3.

A convincing argument for the correctness of our explanation would be obtained, if it appeared that all chromospheric lines were really double lines of the above described character.

Hence I have repeatedly sought for dark cores in the chromospheric arcs on photographs taken during former eclipses, and

[1] MASCARI, *Mem. Spettr. Ital.*, **27**, 83–89; *Ref. Naturw. Rundsch.* **13**, 618.

[2] CAMPBELL, ASTROPHYSICAL JOURNAL, **11**, 226–233.

[3] BROWN, *ibid.*, **12**, 61–63.

[4] LORD, **12**, *ibid.* 66–67. [5] FROST, *ibid.*, **12**, 307–351.

[6] LOCKYER, *Recent and Coming Eclipses*, chaps. X and XVIII (London, 1900).

have indeed found several indications of them; but a plate where this peculiarity was the rule, where all the chromospheric lines were double, has certainly never before been obtained, for if so, the phenomenon could not have escaped notice.

The Dutch expedition had the fortune to get the first plates which quite clearly show all the chromospheric and flash lines, visible on them, to be *double lines*.

This important result is in the first place due to the great care with which the whole plan of observation with the beautiful prismatic camera of Cooke was designed and elaborated by Professor Nyland, and not less to the extraordinary exactness, with which both before and during the eclipse he has performed all necessary manipulations. But besides, it is not impossible that the result was favorably influenced by the cloudiness of the sky, so very unfortunate in other respects. For if the light had not been considerably weakened, the chromospheric lines would have been found on the plate both broader and in greater number, and the doubling would have been perhaps as little marked as on the plates obtained on former occasions.

Shortly after the second contact five exposures were made on one plate, each of them during about three-fourths of a second. They show each only nine lines, all double. On the four plates, prepared for the corona spectrum, some of the stronger chromospheric lines are represented by arcs often interrupted. The light of these evidently comes from prominences which project rather far beyond the photosphere. Here it appears not so easy to distinguish the duplication, just as we might expect by our theory; but still it is visible at many places.

On the sixth plate another set of five exposures, of three-fourths of a second each, were taken a little after the third contact. In the first of the spectra thus obtained (reaching from λ 3880 to λ 5000) 150 double chromospheric lines can be counted between λ 3889 and λ 4600, these being also visible in the other four spectra, as far as the increasing scattered light permits.[1]

[1] On the original negatives the duplication can only be distinguished with a magnifying glass. Enlargements (which were shown in the meeting) will soon be reproduced and published.

A little below the continuous spectrum, due to the just appearing edge of the Sun, the double lines are most conspicuous. We find there, parallel to the spectrum, a bright narrow streak which appears broader in the following exposures and which is probably owing to a small depression in the Moon's edge or to a projecting part of the apparent Sun's edge. In the fifth exposure, below the light band so produced there appears a similar streak. These bands give, so to speak, repeated spectra of the flash (a fortunate circumstance, for the totality was over sooner than was calculated and the exposures were thus a little later than was intended) so that we obtain at one and the same exposure both the pure flash spectrum and the continuous spectrum of the Sun's limb.

Professor Nyland and I have discussed together the possibility of ascribing the origin of double lines to disturbing circumstances, such as irregular motion of the siderostat, vibrations of the prismatic camera, light reflections, etc.,[1] but we were not able to find any disturbance which could account for the observed phenomena and we must conclude that here we really have a property of the chromospheric lines.

The Fraunhofer lines in the continuous spectrum are weak. This may in part be due to the diffusion of light by the clouds. For the edge of the photosphere which had just appeared, which plays the same part with the prismatic camera as the illuminated slit with an ordinary spectroscope, was not darkly limited, but surrounded by a marked aureole (this can be seen in some of our corona photographs). The clouds, however, cannot have been the only cause of the faintness of the absorption lines in the first stage after totality, this phenomenon having been also observed in a clear sky.[2] There must therefore be another reason for the partial absence of the lines. Our theory gives such a reason immediately. For the chromospheric spectrum will at the end of totality become more and more like a continuous spectrum, because more bright lines will continually appear, each of which,

[1] The mounting of the instruments will be fully discussed in the report of the expedition.

[2] CAMPBELL, ASTROPHYSICAL JOURNAL, 11, 228, April 1900.

according to our hypothesis, forms a double band in which the absence of the absorbed waves is not easily perceived. But as soon as a portion of the photosphere appears, the already existing, apparently continuous spectrum will be dominated by a more really continuous spectrum, the source of light of which is limited by two nearly sharp edges (those of the photosphere and of the Moon).

In this spectrum the absence of absorbed rays must show in the usual way as Fraunhofer lines. The light of the chromospheric arcs will, of course, partially overlap those lines, but compared with the direct photospheric light it is weak enough for the dark lines to be visible. Thus, not considering the presence of clouds, the absorption lines must yet, during the transition from the flash spectrum to the Fraunhofer spectrum, at first show very faint and with abnormal relative intensities, then grow stronger, intensities appearing normal.

Because the double lines are not sharply defined objects, it is difficult to give the width of these systems. But we can make settings on the brightest parts of the components and measure their distance with a reading microscope. It differs for the different double lines, still it generally lies between 0.7 and 1.3 Ångström units. Wider and narrower systems follow each other in irregular succession, but on an average the distance of the components appears to decrease as we proceed from the green to the violet. Perhaps this fact may be important for dispersion theories.

With some lines the stronger component is that which has the greater, in others that which has the smaller wave-length. It happens that even in the same line (*e. g.*, in the arcs of $H\gamma$ and $H\delta$ on our plate) the two cases occur close by one another, which means that in neighboring places of the Sun's atmosphere the density distribution of the absorbing gas is different in this, that at one place the average density along the path of the ray increases, at another decreases toward the Sun's center.

Campbell states[1] that in some cases where dark and bright

[1] CAMPBELL, ASTROPHYSICAL JOURNAL, 11, 229.

lines are to be found together, they are separated from one another by a distance of from 0.4 to 0.5 Ångström units. This is about the half of the distance between the components of our double lines. We may here reasonably suppose that Campbell was concerned with cases where one of the components was strongly marked. A similar case is found on our photograph in $H\beta$, where the component with the greater wave-length is stronger over nearly its whole length than that with the smaller wave-length, and such is the case not only at the third contact but also during the second and even on the four plates, prepared for the corona spectrum, which were exposed for 5, 20, 190 and 60 seconds respectively.

I have not found until now in any chromospheric line a peculiarity in the distribution of the light, which would make it necessary to ascribe even a part of this light to radiations emitted by incandescent chromospheric gases. Now we can hardly assume that these gases really do not emit any light; the question is only, in what cases and how far the intensity of the true chromospheric emission is comparable with the intensity of the abnormally refracted photospheric light.

Perhaps the photographs obtained by our expedition are accidentally so extremely well fitted to show the part played by anomalous dispersion in causing chromospheric light, that they induce one to overestimate the importance of the new principle.

It would therefore be very interesting if the plates of other expeditions were also studied from this point of view.

CONTRIBUTIONS TO THE SOLAR THEORY.[1]

By R. EMDEN.

IT has been shown by Helmholtz[2] that layers of air of different densities, flowing with different velocities, can exist side by side with a well-defined surface of discontinuity between them ; we then have conditions similar to those given when the wind passes over a water surface, and the boundary surface is forced into a huge parallel formation in the direction of the more swiftly moving layer, in front of the advancing train of waves. These, which are generally invisible, can be observed by means of the parallel cloud strata (formed at the wave crests) which often cover a large part of the sky; by means also of the violent rain storms which, broken by periods of fair weather, often return at equal intervals during the day; and moreover by the movement which these waves impart to an air balloon accidentally caught by them. A fortunate opportunity allowed me to ascertain, by the flight of a balloon, the length of these waves, as well as the condition of both the contiguous layers of air; and to establish the agreement between the measured wave-length and that required by Helmholtz's theory.[3]

In a series of papers Helmholtz has explained the significance of this wave formation for the general circulation of the atmosphere. The amount of heat which the atmosphere receives in the equatorial regions and carries in vast streams in its upper layers toward the poles must also be brought back to the surface of the Earth in middle latitudes. A simple descent of these upper layers is out of the question, for, retaining their angular momentum, they would cause in lower latitudes at regular intervals

[1] Translated from the *Sitzungsberichten der mathem.-phys. Classe der kgl. bayer. Academie der Wissenschaften*, Bd. XXXI, 1901, Heft III.

[2] HELMHOLTZ, *Gesammelte Abhandlungen*, Bd. I, u. III.

[3] R. EMDEN, "Eine Beobachtung über Luftwogen," *Wied. Ann.*, LXII, p. 62, 1897.

storms surpassing in violence any that have been observed even in exceptional cases. The coefficient of heat conduction is much too small to cause an equalization of heat energy by conduction; the coefficient of friction is too small to equalize the angular momentum by internal friction. Even more truly will the air masses, supplied with energy at the equator and streaming toward the poles, continually separate themselves by surfaces of discontinuity from the lower air masses which are less rich in heat energy, and are moving back to the equator. These waves, continually gaining power, will advance with ever-steepening wave front, and will finally, like water waves, overhang and break; from every train of waves there is formed a vast horizontal whirl in which eventually both layers of air are intermingled. Since, by the formation of surfaces of discontinuity, the head of the wave is subjected to unstable equilibrium, the breaking up of these surfaces produces a constant transfer of energy with reference to angular momentum and heat; which transfer, without this power, would have been impossible, owing to the smallness of the coefficients of heat conduction and friction.

Similar conditions must exist in the Sun, which is conceived of as fluid within, and as rotating and radiating heat. The purpose of the following discussion is to pursue this line of thought in detail.[1]

For greater convenience let us regard the Sun as a rotating sphere. The conclusions given may be directly applied to a rotating ellipsoid. To avoid too rapid a cooling of the outermost layers, inasmuch as by heat conduction alone too small an amount of heat would be transferred to the surface, we are forced to regard the Sun, at least to considerable depths, as a liquid mass which, by the loss of heat, is becoming more dense, so that by radiation and by convection currents, also by the mixing of these currents, a more or less uniform interchange of heat is produced. It makes no difference whether the liquid be

[1] I notice that M. BRILLOUIN, in a short remark in the French translation of the treatise of W. THOMSON, *On the Sun's Heat*, has already referred to the formation of layers, which is to be described in the following discussion.

compressible or incompressible. Let us take the first case as the more general one. Inasmuch as we do not know the equation of condition of a gas at so high a temperature and under such pressure, we will make as a fundamental hypothesis the assumption that the entire part of the Sun which we are considering obeys the equation $pv = HT$, where p = pressure, v = mass of unit volume, H = the gas constant, T = the absolute temperature.

We will further assume that the entire mass of the Sun obeys the laws of gases, and that the process of cooling extends throughout it by means of convection streams. If the Sun had a solid core it would make no difference in the following discussion; the formation of layers which takes place would then extend only to the surface of this solid core. This sphere of gas must in the first place be in adiabatic (indifferent) equilibrium; that is, the density, pressure, and temperature throughout the whole mass must vary so that any given particle of the Sun, under an arbitrary displacement and protected from exchange of heat, shall be in constant equilibrium as regards density, pressure, and temperature with the particle which momentarily presses against it. In a sphere which is not rotating this condition must always be brought about by the mingling of ascending and descending currents.

Frictional forces should operate only in case of finite differences of velocity.

Let us view the Sun from its north pole, and consider a motion in the direction of its actual rotation as plus.

The masses on the surface of the Sun lose heat, become denser, and must sink. If the Sun did not rotate then, at any assumed position of equilibrium, these masses would sink to the center and an equal mass of matter would be forced up to fill the unoccupied space on the surface. But the rotation of the Sun will completely change this form of convection current.

From considerations of symmetry the surfaces of equal pressure are surfaces of rotation, and lines of pressure cut the Sun's axis; also, the masses which have become more dense

through cooling and are sinking inwards, must retain their angular momentum. As they approach the axis of the Sun their linear velocity continually increases, and their tendency to fly off radially (centrifugal force) diminishes by reason of the increase of their angular velocity. The ascending masses continually quicken their speed in the reverse direction, retaining their smaller angular momentum; and their centripetal force diminishes. We then obtain different rapidly rotating masses of gas of unequal densities which can glide by each other in a clearly marked surface of discontinuity. We have then, surfaces of discontinuity which can occur at any position we please in the inner regions of the Sun. We have no *à priori* knowledge of their form except that, from grounds of symmetry, it coincides with rotation surfaces; but in most cases we shall be concerned only with more or less extended portions of such. At these surfaces of discontinuity are given the conditions requisite for the formation of vast waves.

On the axis of the Sun waves or trains of waves which are not indiscriminately placed, are always formed in huge proportions; rushing forward they overhang and from every wave, by its breaking, is formed a vast whirl, in which a complete equalization of the angular momentum and heat condition of both layers takes place. Only in this manner can a uniform cooling process in the rotating Sun occur, for the difference of the angular momentum hinders the formation of any considerable convection currents in a radial direction; because of the smallness of the coefficient of friction, the internal friction cannot in a short time equalize the angular momentum; nor, in like manner, can the heat conduction equalize the different amounts of heat.

This intermingling process just described should be investigated more fully. We have in the first place to determine the form and position of these surfaces of discontinuity, and hence the position of the layers of the Sun which are separated by them.

Let us designate by R the distance of a particle from the center of the Sun, by r its distance from the Sun's axis; and let

the Sun's diameter be D. The angular momentum of unit mass which rotates about the Sun with angular velocity ω is:

$$\Omega = \omega r^2 . \tag{1}$$

If p and ρ signify pressure and density, X, Y, Z, u, v, w acceleration and velocity in the direction of the axes x, y, z, then we have the hydrodynamical equations:

$$\begin{aligned} X - \frac{1}{\rho}\frac{dp}{dx} - \frac{du}{dt} - u\frac{du}{dx} - v\frac{du}{dy} - w\frac{du}{dz} &= 0 . \\ Y - \frac{1}{\rho}\frac{dp}{dy} - \frac{dv}{dt} - u\frac{dv}{dx} - v\frac{dv}{dy} - w\frac{dv}{dz} &= 0 . \\ Z - \frac{1}{\rho}\frac{dp}{dz} - \frac{dw}{dt} - u\frac{dw}{dx} - v\frac{dw}{dy} - w\frac{dw}{dz} &= 0 . \end{aligned} \tag{2}$$

$$\frac{d\rho}{dt} + d\frac{(\rho u)}{dx} + d\frac{(\rho v)}{dy} + d\frac{(\rho w)}{dz} = 0 . \tag{2a}$$

The origin of coördinates lies at the center of the Sun; the x-axis is coincident with the axis of the Sun; the y-axis lies so that it can be made to coincide with the z-axis by a positive rotation. X, Y, Z are the accelerations which the mass of the Sun imparts to a unit mass situated within it. If this be at a distance R from the Sun's center, the potential of the total mass of the Sun upon it will be:

$$V = -4\pi \left\{ \frac{1}{R}\int_0^R \rho R^2 dR + \int_R^{\frac{D}{2}} \rho R dR \right\} .$$

Therefore it matters not whether we regard the Sun as having a solid core or of a gaseous constitution throughout. (Throughout the gaseous part ρ is known as a function of R, as soon as the adiabatic which represents its indifferent equilibrium and the nature of the gas are given.) If we consider the Sun not as a sphere, but as an ellipsoid, then in the following discussion V would have to be considered as the potential of this ellipsoid. The following equations are always true:

$$X = -\frac{dV}{dx} , \quad Y = -\frac{dV}{dy} , \quad Z = -\frac{dV}{dz} .$$

We consider only a rotating movement about the Sun's axis. Therefore:

$$u = 0 \,,$$
$$v = -\omega z = -\frac{\Omega}{r^2} z \,,$$
$$w = \omega y = \frac{\Omega}{r^2} y \,.$$

The three equations (2) may be reduced if the motion become uniform, to:

$$\begin{aligned} \frac{dV}{dX} + \frac{1}{\rho}\frac{dp}{dX} &= 0 \\ \frac{dV}{dr} + \frac{1}{\rho}\frac{dp}{dr} &= \frac{\Omega^2}{r^2} \end{aligned} \tag{3}$$

($2a$) is identically satisfied.

The expression $\frac{1}{\rho}\frac{dp}{ds}$, where s signifies any chosen direction, permits of an important transformation when the process is adiabatic. Define the condition of the gaseous mass at any definite moment by the values p_0 and ρ_0. If we treat the mass as in adiabatic condition, then all the values of p and ρ which the mass passes through are related to p_0 and ρ_0 by the equation:

$$\frac{p}{\rho^k} = \frac{p_0}{\rho_0^k} \tag{4}$$

(if k is the ratio of the specific heats) and at any time

$$\frac{p}{\rho} = HT \tag{4a}$$

must be true.

The amount of heat of a gaseous mass is measured by its potential temperature. This is usually defined as that temperature which the gas reaches if it is brought adiabatically to a more accurately determined normal pressure. Inasmuch as, in contrast to such an arbitrary normal pressure, unit density is a magnitude immediately and clearly determined by the absolute system of measure, the following definition of potential temperature should be more suitable, as well as because it simplifies the formula whenever the potential temperature enters:

The potential temperature is that temperature which a gas reaches when it is adiabatically brought to unit density. We will call this temperature Θ.

By this arrangement, without further explanation, the potential pressure may be defined as that pressure which a gas exercises when it is brought to unit density. We will call this Π. Π and Θ are constant for an adiabatic change. If the Sun is in adiabatic equilibrium, Π and Θ have a constant value throughout the whole solar mass. If a particle of the Sun radiates heat into space, Π and Θ fall in value.

Therefore we have from $(4a)$:

$$\Pi = H\Theta \ .$$

If we choose in equation (4) for ρ_0 and p_0 the values $\rho_0 = 1$ and $p_0 = \Pi$, then the equation of the adiabatic becomes:

$$p = \rho^k H\Theta \ .$$

Using these relations, we may write:

$$\frac{1}{p}\frac{dp}{ds} = (H\Theta)^{\frac{1}{k}} p^{-\frac{1}{k}} \frac{dp}{ds} = \frac{k}{k-1}(H\Theta)^{\frac{1}{k}} \frac{dp^{\frac{k-1}{k}}}{ds} \ .$$

Placing

$$\theta = \frac{k}{k-1}(H\Theta)^{\frac{1}{k}} \ , \quad \pi = p^{\frac{k-1}{k}} \ ,$$

we have:

$$\frac{1}{\rho}\frac{dp}{ds} = \theta \frac{d\pi}{ds} \ , \quad \theta = \text{constant} \ .$$

As $k > 1$, θ varies directly with Θ, and can therefore also serve as the measure of the heat condition of a gaseous mass. In like manner π varies directly with p. Instead of two variables, ρ and p, we have but one, π, because θ remains constant for adiabatic processes. In the case of adiabatic equilibrium, θ has a constant value throughout the whole solar mass.

By reason of the above mentioned radiating and convective processes, layers may form in the Sun within which the heat condition and the angular momentum possess constant values, while both quantities change abruptly in passing from one layer to another. Such a layer, in which θ and Ω are constant, is called a *homogeneous* layer.

By means of the foregoing expressions equation (3) may be written:

$$\frac{dV}{dx}+\theta\frac{d\pi}{dx}=0\ .$$

$$\frac{dV}{dr}+\theta\frac{d\pi}{dx}=\frac{\Omega^2}{r^3}\ .$$

Within a homogeneous layer, therefore, the relation

$$V+\theta\pi=-\frac{1}{2}\frac{\Omega^2}{r^2}+C \text{ holds.} \qquad \text{(I)}$$

Let us now consider two contiguous layers, 1 and 2, and distinguish them by

$$\theta_1\ ,\ \ \Omega_1\ ,\ \ C_1\ ,\ \ \text{and}\ \ \theta_2\ ,\ \ \Omega_2\ ,\ \ C_2\ .$$

In order that a surface of discontinuity may exist, the pressure and also π must have the same values on both sides of it. On every portion of the boundary there must also hold the following:

$$\pi_1-\pi_2=0\ ,$$

where along this boundary π_1 and π_2 vary and, on the surface of the gaseous sphere, π_1 and π_2 assume the value $\pi_1=\pi_2=0$.

According to this we have as the equation of the meridian curve of the surface of discontinuity (the surface of contact of two homogeneous layers), expressed as a function of r and R:

$$V\left(\frac{1}{\theta_2}-\frac{1}{\theta_1}\right)=\frac{1}{2}\frac{1}{r^2}\left(\frac{\Omega_1^2}{\theta_1}-\frac{\Omega_2^2}{\theta_2}\right)-\frac{C_1}{\theta_1}+\frac{C_2}{\theta_2}\ . \qquad \text{(II)}$$

The direction of the tangent to this meridian curve is given by differentiation with respect to r and R:

$$\frac{dV}{dR}dR=\frac{dr}{r^3}\left(\frac{\Omega_1^2\theta_2-\Omega_2^2\theta_1}{\theta_2-\theta_1}\right),$$

or:

$$\frac{dr}{dR}=r^3\frac{dV}{dR}\left(\frac{\theta_2-\theta_1}{\Omega_1^2\theta_2-\Omega_2^2\theta_1}\right)\ . \qquad \text{(III)}$$

Therefore, the differential quotient always has the same sign as

$$\frac{\theta_2-\theta_1}{\Omega_1^2\theta_2-\Omega_2^2\theta_1}\ .$$

If this expression vanishes. which [illegible] when $\theta_2=\theta_1$.

$\Omega_1 = \Omega_2$, then the meridian curve becomes a parallel to the Sun's axis.

The surfaces of separation between layers which are characterized by different angular momenta but possess the same amount of heat are, in this special case, circular cylinders, parallel to the axis of the Sun and concentric with it.[1]

In order to obtain in the more general case a further insight into the form of these surfaces, and the position of the layers 1 and 2, we will employ the process used by Helmholtz in his treatment of the surfaces of discontinuity in the atmosphere.

The equation of the surface of separation reads $\pi_1 - \pi_2 = 0$ and therefore for every direction, s, within the surface of separation,

$$\frac{d(\pi_1 - \pi_2)}{ds} = 0 .$$

If we subject the surface at a certain point to a small deformation, π_1 and π_2 will change and thus $\pi_1 - \pi_2$, in case the equilibrium of the surface is not, by chance, indifferent. If we mark off, on the normal to the surface, a small portion dn, then the quotient $\frac{d(\pi_1 - \pi_2)}{dn}$ can be positive or negative, and, for a continuous pressure-distribution on each side of the surface, the same sign is also possessed by the quotient $\frac{d(\pi_1 - \pi_2)}{dh}$, where h is drawn in any direction we please. If the differential quotient be positive, the deformation will cause on that side an excess-pressure which will force the surface back again; the equilibrium of the surface is then stable. If the differential quotient be negative, then the ensuing difference of pressure will increase the deformation and the equilibrium becomes unstable. To determine the kind of equilibrium it is sufficient to form the differential quotient in the two directions, dr and dR, and see into which layer dr or dR projects for the case of stable equilibrium.

We first perform the operation $\frac{d(\pi_1 - \pi_2)}{dR}$ with constant r; that

[1] Compare E. J. WILCZYNSKI, *Hydrodynamische Untersuchungen mit Anwendungen auf die Theorie der Sonnenrotation. Inauguraldissertation.* Berlin, 1897. P. 8.

is, we pass outward parallel to the Sun's axis. Equation I gives

$$\frac{d(\pi_1 - \pi_2)}{dR} = \frac{dV}{dR}\left(\frac{1}{\theta_2} - \frac{1}{\theta_1}\right). \tag{7}$$

The differential quotient is + if $\theta_1 > \theta_2$; therefore, if the warmer layer lie higher in a direction toward the Sun's pole, the equilibrium of the surface is stable.

There still remain two possibilities. If we approach the boundary from within we can either approach or recede from the Sun's axis. In the first case the hotter layer lies on the side of the surface away from the axis; in the second the point itself may be on the side turned toward the Sun's axis. In order to separate these we form from equation I, $\frac{d(\pi_1 - \pi_2)}{dr}$, keeping R constant, and obtain:

$$\frac{d(\pi_1 - \pi_2)}{dr} = \frac{1}{r^3}\left(\frac{\Omega_1^2}{\theta_1} - \frac{\Omega_2^2}{\theta_2}\right). \tag{8}$$

The differential quotient is positive if $\frac{\Omega_1^2}{\theta_1} > \frac{\Omega_2^2}{\theta_2}$, that is, if the gas having the greater angular momentum, Ω_1, is the cooler or at least as cool as the other. In the Sun, when prevented from radiating, θ has everywhere the same value, and Ω increases from the axis to the equator. With radiation taking place θ diminishes uniformly over the entire surface, so that $\frac{\Omega^2}{\theta}$ increases from pole to equator and from the Sun's axis perpendicularly outward. Moreover, if cooling occurs, θ decreases for the masses which lie at the surface and are sinking, therefore Ω increases; while the rising masses are characterized by greater θ and smaller Ω. On this account in the layers of the Sun to a greater Ω always belongs a lesser θ.

If we move along the surface of a sphere surrounding the center of the Sun, then at stable equilibrium the layer which is hotter and has the smaller angular momentum lies on that side of the surface of separation which is turned toward the axis of the Sun.

The surfaces of separation of the layers which must be formed through radiation in the rotating Sun occupy therefore a position such that, in our motion on the same, we would recede from the Sun's axis if we move outward. Thus the layers which are warmer and have the smaller angular momentum lie on that side of the surface of separation, which is turned toward the Sun's axis.

In agreement with this (III) shows that $\frac{dr}{dR}$ is positive.

If we pass outward, parallel to the Sun's axis, we continually come upon warmer layers, in the same way as if we approached the Sun's axis from the equatorial plane along the surface of a sphere. On this account by no two paths can we twice cut the same surface of separation. Therefore it follows that:

The boundary surfaces are not closed, but are surfaces of revolution which cut the surface of the Sun.

The angle of intersection is given by the value of $\frac{dr}{dR}$ at the Sun's surface.

Of the form of this surface we can, in general, say little; it is determined by $\frac{dr}{dR}$. From (III) it follows that:

$$\frac{dr}{dR} = r^3 \frac{dV}{dR} \frac{\theta_2 - \theta_1}{\Omega_1^2\theta_2 - \Omega_2^2\theta_1} = r^3 f(R)\, \phi(\Omega\,,\ \theta)\,.$$

If the gas constant and the ratio of the specific heats of the Sun's masses are known, $f(R)\frac{dV}{dR}$ can be calculated with sufficient accuracy for the state of adiabatic equilibrium.[1]

From a value of 0 at the center it increases, then, after crossing a maximum, the position of which on the radius is given by k, it decreases to the value, $-g$, on the surface. Of the value of the function ϕ (Ω, θ) we can say nothing without a knowledge of the magnitudes of Ω and θ, except that it is $+$ and increases with increasing difference of the amount of heat

[1] A. RITTER, "Untersuchungen über die Höhe der Atmosphäre und die Konstitution gasförmiger Weltkörper," *Wied. Ann.*, XI, p. 332, 1880.

in the two layers. If we pass a plane through the x (Sun's) and y axes then we may also write for the equation $\frac{dr}{dR}$:

$$\frac{dx}{dy}=\frac{1}{x}\left(\frac{R}{r^3f(R)\phi(\Omega\,,\ \theta)}-y\right);$$

and we see from this that the surfaces of separation cut the equatorial plane perpendicularly and that, at the same place, within the Sun, the tangents to the meridian curve are so much the steeper with reference to the equatorial plane, the smaller the value of ϕ $(\Omega,\ \theta)$ is.

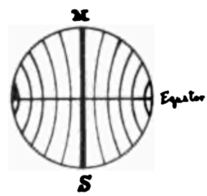


FIG. 1.

The curvature of the bounding surfaces increases with increasing difference in the heat condition and angular momentum of the two adjacent layers. If the angular momentum alone, and not also the potential temperature, of the two layers were different, the surfaces of separation would be cylindrical surfaces parallel to the axis of the Sun.

The form of the surfaces of separation is shown in the accompanying figure.

A division of the rotating Sun into an indefinitely great number of such homogeneous layers would give a condition of stable equilibrium, provided we neglected the friction between the bounding surfaces, so that, upon motion outward along the equatorial plane, layers of greater Ω and smaller θ would continually be met with.

Each one of these rotating layers shows an entirely different relation from that exhibited by the Sun conceived as rotating as a whole. While the latter possesses the same potential temperature throughout its whole mass, in the former it has a constant value only within one layer, and this changes its value abruptly at the surface of discontinuity. In every layer the moment of rotation is always constant; the smallest impulse is sufficient to cause a part of the mass to pierce a layer in any direction. In each layer there exists a velocity potential, while the rotation of the Sun causes a whirling motion. Within each layer the

ngular velocity increases inversely as the square of the radius of a rotation, the linear velocity inversely as the first power, and the centrifugal force inversely as the third power of the same. The difference of the linear velocity at the surface of contact of two layers is therefore not constant but increases in proportion as the boundary surface approaches the axis. The lower down the surface is within the Sun the greater will be the difference of its tangential velocity, and, on this account, the greater will be the effect of friction along the separating surface.

The formation of these layers, and the form of the separating surfaces, is wholly independent of the presence of a solid core in the Sun. In this latter case the formation of layers would be restricted to the surface ; and the solid core of the Sun, with the stratified gaseous envelope reaching to the photosphere, would correspond fully to the Earth with its stratified enveloping atmosphere. The sole difference is this—that the position of the layers and their bounding surfaces, corresponding to those in the Sun, can only occur in an exceptional and locally-bounded manner in the atmosphere of the Earth, where, in general, the tangents to the meridian curve of the surface of separation cut the celestial sphere between the horizon and the pole. The reason for this lies in the fact that at the equator the atmosphere almost always possesses the greatest angular momentum and also the greatest amount of heat, so that by reason of the heating the quotient $\frac{\Omega}{\theta}$ diminishes ; while in the case of the contiguous layers in the Sun greater angular momentum is accompanied by a less amount of heat, and through cooling $\frac{\Omega}{\theta}$ increases.

Let us suppose that there occurs between the two layers at their boundary a mixture of masses m_1 and m_2 of the two layers characterized by $\Omega_1\,\theta_1$ and $\Omega_2\,\theta_2$; we may compute the resulting angular momentum Ω of the mixture (because only inner forces are operative) and the potential temperature, θ, according to the law of the center of gravity.

$$(m_1 + m_2)\,\Omega = m_1\Omega_1 + m_2\Omega_2$$
$$(m_1 + m_2)\,\theta = m_1\theta_1 + m_2\theta_2$$

Equation III was:

$$\frac{dV}{dR}\frac{dR}{dr}=\frac{1}{r^3}\left(\frac{\Omega_1^2\theta_2-\Omega_2^2\theta_1}{\theta_2-\theta_1}\right).$$

Subscript 1 applies to the warmer layer. In order to find the position of the bounding surface of the mixture on the side toward layer 1, which may be denoted by $\frac{dr_1}{dR_1}$, we must replace Ω_2 and θ_2 in this equation by Ω and θ and we find:

$$\frac{dV}{dr}\left(\frac{dR_1}{dr_1}-\frac{dR}{dr}\right)=\frac{m_1\theta_1}{m_1+m_2}\left\{\frac{(\Omega_1-\Omega_2)^2}{\theta_2-\theta_1}\right\}$$

and, as $\theta_1 > \theta_2$, we have:

$$\frac{dr_1}{dR_1}>\frac{dr}{dR}.$$

The new bounding surface on the side toward layer 1 is therefore more strongly inclined to the equatorial plane than the original. In like manner we obtain for $\frac{dR_2}{dr_2}$, which gives the position of the bounding surface of the mixture on the side toward layer 2:

$$\frac{dV}{dr}\left(\frac{dR_2}{dr_2}-\frac{dR}{dr}\right)=\frac{m_2\theta_2}{m_1+m_2}\left\{\frac{(\Omega_1-\Omega_2)^2}{\theta_1-\theta_2}\right\},$$

and therefore:

$$\frac{dr_2}{dR_2}<\frac{dR}{dr}.$$

The new bounding surface on the side toward layer 2 therefore stands more steeply inclined to the equatorial plane than the original. From the point chosen at which the mixture took place, two new bounding surfaces are formed which seek to penetrate layers 1 and 2 and bound a roof-like empty space open in the direction of the equatorial plane. On this account the intermingled parts must move along the original bounding surface toward the equator (the normal relation in the atmosphere of the Earth being toward the pole). In proportion as more and more masses enter into the mixture the mixed layer will also move outward along the bounding surface into the empty space, and form a new layer of mean angular momentum

and mean heat condition, which lies between the two original layers and in contact with them.

Now it is quite certain that the Sun, or at least the gaseous part of it, is not completely split up into a more or less great number of such homogeneous layers. We have rather to conceive of the processes actually entering in in such a way that more or less extended portions of these surfaces of discontinuity are formed during the cooling of the rotating Sun from without. The difference of the linear velocity on the two sides of the surface of separation causes this to form waves which become continually larger and finally overhang and break and thus change into a vast whirl, the inner part of which is filled with a mixture of extensive portions of the masses of both layers. In between, in other places, new bounding surfaces have been produced by newly formed layers in which the same mixing process is repeated. Solely by a mechanism such as the one described, a mechanism which has nothing hypothetical in it, and which must of necessity occur in a fluid rotating mass radiating heat, can a uniform cooling of the Sun's mass take place, and a much too rapid cooling of the outer layers be prevented. For heat conduction and internal friction of the gases are too small to account for the equalization of heat condition and angular momentum. Only by the above portrayed formation of surfaces of discontinuity, and their rolling up through mingling of the gases involved, can the different angular momenta and potential temperatures be equalized.

We have already shown that in a homogeneous layer the angular velocity diminishes as the square of the distance from the rotation (Sun's) axis. It follows, therefore that it is impossible to speak of an angular velocity of the rotating Sun. If the Sun were by chance to rotate at any time with a constant angular velocity everywhere, this constancy would be destroyed by the formation and intermingling of layers which would occur. The angular velocity must be variable, not only throughout the whole Sun's mass, but also in the course of time at one and the same point. It is not necessary that throughout the

mass it should vary uniformly at any one moment, but it may change at a surface of discontinuity in an abrupt manner. If a section be made across a surface of discontinuity in the Sun's upper surface (photosphere), parts will be obtained which will move by each other with unequal angular velocities. This consideration is of value with reference to the potential temperature. If k for the solar mass, and the function $\phi(\Omega, \theta)$ be known, then we can calculate approximately the mean distribution of velocity (angular momentum) and potential temperature. By reason of the absence of such data we must use the following general reasoning:

If the Sun were stationary and were cooling at its surface, then the process of cooling would be uniform over its entire surface, because the intermingling produced by the convection streams would extend to equal depths of the Sun. If the Sun rotate, then those streams which at the poles pass along the Sun's axis are in no way disturbed. But the nearer we approach to the equator, the less deep can the streams penetrate, and the nearer to the surface will they be confined by the formation of surfaces of discontinuity; and the exchange of heat energy can take place only through the rolling up of these and the formation of other surfaces much more slowly in the depths of the Sun. The loss of heat in the equatorial parts will, on this account, take place more slowly than in polar regions; the potential temperature of the latter must therefore remain, in general, higher. But, as under equal pressure the actually observed gas temperature increases with the potential temperature, we may frame the following law:

(*a*) The surface of the Sun must possess in the polar regions a higher temperature than at the equator. Whether this difference is sufficiently great to be determined by radiation measurements must be settled by experiment.

We may arrange this whole discussion with reference to the exchange of angular momentum (velocity) in polar and equatorial regions. The surface of the Sun contracts as a result of cooling; its angular velocity increases, and the exterior must

advance toward the core. In the polar regions the convection currents which are undisturbed, even at the greatest depths, account for the exchange of angular velocity. The nearer we advance toward the equator the sooner are the currents restrained by surfaces of discontinuity, and the more slowly do they, by the progressive rolling up and formation anew, impart to the lower portions an increasing velocity from the outside. There follows thus the well-known law:

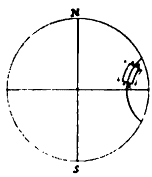


FIG. 2.

(*b*) The surface of the Sun must possess in its equatorial regions a greater angular velocity than in its polar regions.

Laws (*a*) and (*b*) are parallel laws which are based on the same ultimate cause.

As to these surfaces of discontinuity, the process of their rolling up is marked in still a different way than by the retardation of a uniform angular velocity of the rotating Sun.

The difference of linear velocity produces surfaces and waves which finally overhang and break. In the place of every train of waves there is formed a powerful whirl which revolves in the direction of the Sun's rotation, and does not lie greatly inclined toward the Sun's axis. The difference of linear velocity on the two sides of this surface of separation increases with proximity to the solar axis (p. 49). Therefore the location of the maximum wave- and whirl-formation will be found within the Sun and not on its surface. In Fig. 2 we see such a whirl schematically. The theory of the vortex shows that the pressure decreases along its axis. Therefore the whirl sucks a mass in along its axis only to eject it again at other points. This cyclonic, sucking action of ouratmosphere is well known ; every vertical whirl in a river asserts itself in a depression of the surface. If the whirl which forms by the rolling of the surface of discontinuity is not too far from the Sun's surface it will show itself like the whirl upon the surface of water. If we grant the validity of Wilson's theory of the constitution of Sun-spots as depressions in the Sun's surface, then we need only to seek their

cause in the Sun whirls in order to obtain a satisfactory explanation of most of the phenomena which we observe in the case of Sun-spots and their accompanying manifestations.

We cannot in the space allotted this treatise discuss the whole vast material of observation dealing with Sun-spots with a view to elucidating the whole question in this way. It is sufficient here to show that the characteristic phenomena which Sun-spots offer can be predicted *à priori* if we connect them with these whirls in the interior of the Sun.

If a surface of discontinuity not too far distant from the surface of the Sun roll itself up, then the resultant whirl will gradually be shown upon it in the manner just mentioned. Disturbance of the surface and multiplied formation of faculæ are signs of Sun-spot formation, and, according to our point of view, a proof that the mill within the Sun is already in action. The sucking action of the whirls will soon lay hold upon the masses which lie on the surface of the photosphere. At one or more points the masses are beginning to sink. There is being formed a highly irregular crater; the streaming becomes gradually stationary, and in the same degree the crater assumes a regular cross-section. The photospheric masses are rushing into this crater in radial streams, the appearance of the absorption lines in the spectrum shows the violent motion within this whirlpool. "The darker parts as well as the rest of the core are clearly the openings of tube-like cavities which extend into unknown depths" (Dawes).

The masses which are sucked in must be replaced by masses from the interior of the Sun and the Sun-spots therefore will be surrounded by a region rich in faculæ and protuberances. "A spot is, as a matter of fact, generally surrounded by a ring of eruptions, and has an appearance as if the eruptive masses were flowing together into one and the same cavity, as if the masses were actually being sucked down, as if a spot exhibited a sucking effect which is strong enough to draw into the interior of the spot the broken masses surrounding it" (Young).

When the whirl has gradually exhausted itself by friction in

the Sun's interior parts, the sucking ceases, the crater in the Sun's surface fills up, and only after the lapse of time does the increased activity of faculæ at this point show that in the interior of the Sun forces are still active which gradually die away. If, while the whirl is still in existence, a current does not flow in the direction of the axis because of the asymmetry which enters, then the crater on the Sun's surface may disappear to appear again after the renewal of the symmetrical flow. In this way Sun-spots often disappear and again break out at the same point. If the waves and whirl arise at too great a depth, their arrival at the surface will be attended by increased formation of faculæ, not in the formation of craters. In this manner the "veiled spot" mentioned by Trouvelot is explained (*cf.* YOUNG, *The Sun*, p. 129).

If the whirl originates near the Sun's surface its direction of rotation (in the direction of the solar rotation) must become apparent in the motion of the spots in a similar direction, such as is observed sometimes. Generally the whirl arises at a considerable distance from the Sun's surface, so that the direction of motion of the spots is chiefly conditional upon the unsymmetrical side-currents of the masses which are sucked in. The diverting force of the Sun's rotation, in the case of slow angular velocity, has little influence upon these currents (at the same latitude and with equal angular velocity about twenty-five times as small as upon the Earth), especially in the lower latitudes in which most of the spots are found. Hence it may also readily happen that in one and the same spot a difference of direction of rotation may be found owing to the lack of symmetry of the currents.

The origin, appearance, and disappearance of spots are fully explained, according to Wilson, by the rolling up of the surfaces of discontinuity. Quite as clearly in this manner the distribution of spots over the Sun's surface can be explained. Fig. 1 shows the manner of the formation of layers and a study of it indicates that a zone of minimum spot-activity must often be present about the equator. Only as an exceptional

case—perhaps unsymmetrically formed—can a surface of separation by an unsymmetrical rolling up cause a spot in this region. Moreover, in higher latitudes, surfaces of discontinuity are seldom formed, and then such that with them only at great depths does there enter a sufficient difference of linear velocity on each side, and as a result a wave- and whirl-formation. In higher latitudes we often find faculæ, also veiled spots, but no full-fledged spots. In middle latitudes lie the positions of maximum spot-activity; these latitudes are marked by the maximum formation of layers and are indicated even on the surface by the great difference continually existing in their angular velocity. If the function ϕ (Ω, θ) were known then the position of maximum spot-activity could be calculated. As long as this is unknown we must rather inversely conclude, from the frequency of spots, as to the positions of maximum layer formation. Therefore, most surfaces of separation must be formed in middle latitudes where the direction of the tangent to the parts lying nearer to the Sun's surface cuts the latter at angles varying from 10° to 40°, because between these boundaries (with few exceptions) the spot-zone is enclosed. This position of the maximum, and most active, layer formation has no element of improbability about it, so that we may explain on the basis of our hypothesis the distribution of spots in a purely mechanical and unconstrained manner.

Sun-spots frequently occur at the same latitude arranged in series. Our hypothesis anticipates this. For the surface of discontinuity does not often form a wave; but several waves follow one another. To every train of waves there corresponds, upon its dissolution, a whirl, and to every whirl there may correspond a Sun-spot. Thus spots originate which usually occur at equal latitudes at approximately the same time. (A series of Sun-spots and a system of parallel-formed cirrus layers in our atmosphere are produced by similar mechanisms.)

After a period of minimum Sun-spot activity, spots, entering again in great numbers, begin to form in higher latitudes, and the formation of spots then advances into lower latitudes. Our hypothesis anticipates this. If the Sun's mass be at the period

of greatest quiet, then the masses cooling at the surface can cool comparatively strongly before they sink. The surfaces of discontinuity begin to form in greater depths and higher latitudes, and similarly the Sun-spots. In proportion as the Sun is active will the unstable equilibrium of the cooling masses at the surface be more easily destroyed; the masses must sink earlier and in a less cooled condition, and corresponding layers and spots are formed in lower and lower latitudes.

By the loss of the unstable equilibrium planets may possibly be influenced by Sun-spot phenomena.

If the Sun-spots are caused by whirls, they must also possess a proper motion. A simple vortex filament in an infinite, extended, motionless fluid possesses no proper motion. But if it lie adjacent to a solid wall or to a parallel fluid surface, then it will move parallel to this surface in the same direction as in consequence of its rotatory movement the liquid between the whirl and the wall flows along, and with a velocity equal to one fourth of that with which the fluid flows at the base of the perpendicular let fall upon the solid wall. The whirl in the inner part of the Sun does not lie parallel to the Sun's surface; but if we resolve it into two component whirls, perpendicular and parallel to the Sun's surface, then the latter will have a large value for the whirls in lower latitudes. In the lower latitudes the whirls, particularly if they lie not too deep, must possess a proper motion and must certainly move in the direction of the angular movement of the Sun. We thus throw light upon the law of Dunér, that from the observation of Sun-spots we obtain a greater angular velocity for the Sun than from spectrum observations based on Doppler's principle. The whirls which are not perpendicular to each other exert a mutual influence on their proper motion, by which fact we can explain the complicated proper motion of Sun-spots mentioned by Faye. The phenomenon of a whirl (Sun-spot) splitting up into several whirls can be frequently observed in the corresponding case of a whirlpool in a liquid.

Inasmuch as according to this explanation Sun-spots are the

consequences of a mixing process on the rotating Sun, at the time of their maximum frequency the loss of heat at the Sun's surface will be equalized most completely by the mixing with lower-lying hotter masses. The times of maximum Sun-spot activity will, therefore, coincide with the periods of increased solar radiation (fluctuations of climate).

Young has stated (*The Sun*, p. 173) the possibility "that spots found in the region of eruptions are depressions of the photosphere which are developed—as a consequence of eruptions—not directly by the pressure of the upper layers on the lower, but by the diminution of the pressure of the lower upon the upper." On this he sought to base a rather ingenious theory of Sun-spots.

The formation of a whirl by the rolling up of the surfaces of discontinuity gives directly the diminished pressure in the interior of the Sun sought by Young. Faye's vortex theory possesses, on account of its manifold advantages, the possibility of great extensions despite the fact that, contrary to experience, the interior Sun-spots must rotate as a whole in the same direction as the Sun and that the mechanical explanation of this state of affairs in the whirl is not free from objections. The theory outlined here possesses all the advantages which are exhibited by Faye's theory, and none of its disadvantages.

Inasmuch as over the spots, in case they arise from the sucking of the whirl at work within the Sun, there must enter a descending stream of the gas surrounding the photosphere such as Oppolzer postulates as the basis of his theory of Sun-spots, here, then, the various advantages of Oppolzer's theory are developed to good effect. The descending streams from which Oppolzer started here find their explanation.

The formation of layers, velocity of rotation variable in space and time, and forms analogous to the Sun-spot, are the necessary consequences of the cooling process — produced by radiation — in a rotating celestial body composed wholly, or only in its outer layers, of a fluid mass.

Minor Contributions and Notes

AN EXPERIMENTAL INVESTIGATION OF THE PRESSURE OF LIGHT.[1]

KEPLER was probably the first (in 1619) to attribute the form of comets' tails to a repulsive force from the Sun, and to explain this repulsion as the result of the pressure exerted by the Sun's rays on the matter in the tail. The same phenomenon also led Euler, in 1746, to ascribe a pressure to the solar radiation, and in 1754 de Mairan made the first attempts to test these ideas experimentally, but he did not obtain a positive result.

A dual theoretical basis for the pressure of radiation was independently and almost simultaneously given by Maxwell in 1873, as a consequence of the magnetic theory of light, and by Bartoli in 1876, as a consequence of the second law of thermodynamics.

If a beam of parallel rays is normally incident on a plane surface, the amount of the Maxwell-Bartoli light-pressure p is determined if we know the quantity of energy E received per second, the reflecting power ρ of the surface, and the velocity v of light. For then

$$p = \frac{E}{v}(1 + \rho) ,$$

where ρ varies between 0 for an absolutely black surface and 1 for a perfectly reflecting surface.

This pressure is very small: for solar rays falling normally at the Earth's distance upon a square meter, the pressure exerted is 0.4 mg for a black surface and 0.8 mg for a mirror.

In the experimental investigation of the pressure of light, two large sources of disturbance arise, the one due to the well known radiometric forces discovered by Crookes, and the other to convection in the residual gas. It is possible, however, to diminish these disturbing forces and to make their effect on the observations harmless, if the experiments are made with very thin metal vanes and the

[1] Translated from an abstract communicated by the author of his extended paper in *Annalen der Physik*, **6**, 433-458, 1901.

exhaustion is carried as far as possible by using a mercury pump and condensing the mercury vapor by freezing mixtures.

Without entering into the details of the experimental arrangements, the principle of the method employed may be briefly explained as follows: A torsion thread hangs in a highly exhausted bell jar and carries a vertical glass rod. Thin disks of 5 mm diameter, of the metal to be investigated, are attached to this rod at a distance of 10 mm from its axis. If the radiation from an arc lamp is concentrated on one of the disks the incident radiation will exert a pressure upon it, and it will retire until the pressure due to radiation is balanced by the torsion of the glass thread; the angle of torsion is measured by a mirror and scale as for a galvanometer. This observation permits the determination of the *absolute* magnitude of the pressure (in dynes) if the directing force of the torsion thread is measured in absolute units by one of the well known methods.

In order to compare the observed pressures with those computed according to Maxwell and Bartoli from the amount of energy incident and the reflecting power of the vane used, the same beam of light was directed upon a circular aperture of exactly the same size as the vane, and the rays passing through were caught by a calorimeter. If we divide the quantity of energy incident per second, as measured by the calorimeter, by the velocity of light, we obtain the amount of pressure in dynes exerted by the light upon a perfectly absorbing body, according to Maxwell and Bartoli.

The measurements, which were made repeatedly and with different apparatus, yielded accordantly the following results:

Vane Used	Radiation Pressure	
	Observed	Computed
Black, platinum plated	1.1	1.0
Bright platinum	1.8	1.6
Bright aluminium...............	2.0	1.8
Bright nickel..................	1.4	1.6

For such complicated and difficult measurements a better agreement between observation and computation cannot be expected. A discussion of the possible errors of observation shows that they are con-

siderable, but that within the limits of error the existence of the light pressure of Maxwell and Bartoli are *quantitatively* confirmed.

This result is of importance to astrophysics as furnishing a much simpler explanation of the repulsive force of the Sun than the hypothetical ones of electrical charges. A firm basis, amenable to computation and assured by experiment, is thus given to the view expressed by Kepler.

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November, 1901.

PRESSURE DUE TO LIGHT AND HEAT RADIATION.[1]

ALTHOUGH the idea of a pressure due to radiation is at least three centuries old, and definite experiments directed toward its measurement were made a century and a half ago, yet Maxwell[2] was the first to accurately state the theory of a radiation pressure, in the form in which it is now held. In astrophysics the problem is interesting, not alone for its bearing upon the repulsion of comets' tails by the Sun[3] but also for its possible effect upon the Solar Corona and the Aurora Borealis.[4]

The present experiments were of two kinds: (1) The determination of the light pressure by observing the deflection, either static or ballistic, of a torsion balance when one vane of the balance was exposed to light, and (2) the determination, in ergs per second, of the intensity of the light falling upon the vanes. The image of an aperture, upon which the rays from an arc lamp were concentrated by two condensing lenses, was focused in the plane of the glass vanes placed symmetrically with regard to a rotation axis held by a quartz fiber of known torsion coefficient. The torsion balance was covered by a bell jar connected to pressure gauges and a mercury pump. To eliminate the disturbing action due to the residual gas in the receiver, the following devices were used: (1) The vanes were silvered and highly polished,

[1] An abstract of a paper read in Denver, August 29, 1901, before a joint session of the American Physical Society and Section B of the American Association for the Advancement of Science.

[2] J. C. MAXWELL, *Electricity and Magnetism*, 1st ed., Oxford, 1873, **2**, 391.

[3] P. LEBEDEW, *Wied. Ann.*, **45**, 292, 1892.

[4] S. ARRHENIUS, *Konig. Vetenskaps-Akademiens Förhandlingar*, 1900; also ASTROPHYSICAL JOURNAL, **13**, 344, 1901.

thus making the absorption small and the reflecting percentage large. (2) The silver surface of one vane was turned towards, and of the other, from the light, thus making the effect of the gas action and light pressure in the same direction on one vane, and in opposite directions on the other. (3) The pressure of the air in the bell jar was varied, and pressures were chosen in the vicinity of that pressure at which the gas action was very small. (4) The length of exposure of light on the vane in most of the observations was only six seconds. The gas action, which begins at zero and increases with the length of exposure, was thus reduced in comparison with the instantaneous action of the radiation pressure. By means of an inclined glass plate placed in front of the aperture, a portion of the incident light was thrown on a thermopile. The deflection of a galvanometer connected with the latter gave the *relative* light intensities.

Two methods of determining radiation pressure were used:

(1) The vane was exposed continuously to the light until the turning points of the vibration of the balance showed that static conditions had been reached. The other vane was then exposed. Finally the whole suspended system was turned through 180°, and the vanes were exposed in turn. The mean of the angles of deflection, multiplied by the torsion coefficient of the fiber and divided by the lever arm, gave the force in dynes acting on the vane. (2) The vanes were exposed in the same order as before, but only for a quarter of the period of the suspended system. The period, damping coefficient, torsion coefficient, and lever arm being known, the value of the radiation pressure could be found. The two methods gave practically the same result except for the air pressures for which the gas action was large.

Measurements of the radiation pressure were made with eight different gas pressures in the bell jar. A comparison of the static with the ballistic deflections, within this range, showed that the pressure due to gas action varied from $\frac{1}{10}$ of, to 6 or 7 times, the radiation pressure. The radiation pressures observed by the ballistic method and reduced to constant radiant intensity, are given in the accompanying table:

Gas pressures in millimeters of mercury	Radiation pressures in 10^{-4} dynes
96.3	0.9
67.7	1.0
37.9	1.0
36.5	1.0
33.4	1.0
1.2	1.0
0.13	1.3
0.06	1.1
	Mean 1.04×10^{-4} dynes.

The energy falling upon the vanes was measured by means of a bolometer consisting of a thin disk of platinum, about the size of the vanes, covered with platinum black. The bolometer, occupying exactly the position which the glass vane had previously occupied, was made one of the arms of a Wheatstone bridge. The bridge was balanced, the bolometer exposed to the light, and the throw of the bridge galvanometer read. Later the disk was heated by an electric current which entered and left at two equipotential points on the bridge current, and the galvanometer throw was read again. The current strength and the resistance of the disk being known, the energy developed in the disk is $i^2 R \times 10^7$ ergs per second. The reading of the thermopile before mentioned made it possible to reduce all observations to a constant light intensity. If $E =$ energy per second falling upon a surface, $a =$ the percentage of the radiation reflected from the silvered vane, $v =$ the velocity of light, then, theoretically, the value of the radiation pressure is $\frac{(i + a)E}{v}$.

In the experiment, $R = 0.278$ ohm; the current causing the same resistance changes in the platinum disk as the light source, was $i = 0.75$ amp.; $a = 0.92$, and $v = 3 \times 10^{10}$. We have thus

$$\frac{1.92 \times 0.278 \times 0.75 \times 10^7}{3 \times 10^{10}} = 1.34 \times 10^{-4} \text{ dynes}.$$

The observed value of the radiation pressure is thus seen to be about 80 per cent. of the theoretical value as computed from the heat measurements. Unfortunately there were certain systematic errors in the energy measurements due to the construction of the bolometer and which could not be eliminated. The writers have therefore

greater confidence in the accuracy of the observed radiation pressures than in the theoretical values computed from the heat measurements.

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SPECTROSCOPIC BINARIES: A SUGGESTION.

THE spectroscopic binaries are naturally divisible into two chief classes, which may be conveniently distinguished by the Roman numerals I and II. Class I comprises those systems in which one component only yields a perceptible spectrum; in Class II the spectra of both components are visibly superposed. Stars of the first class — so far as our present knowledge extends — are distinctly more numerous than those of the second. But the latter are in general more interesting, since the study of such systems must lead frequently to conclusions of special value to the astrophysicist.

As an illustration, I may point out that it will be possible to deduce the *average* mass[1] for stars of Class II; to compare the mean masses for stars of different spectral types, or those comprised in different orders of magnitude, etc. By combining such results with photometric, parallactic, and other data, we may further investigate the relation between mass and intrinsic brightness, which is one of the fundamental problems of stellar physics.

The preponderance in numbers of Class I, as noted above, may prove to be illusory. In other words, a careful examination, by special methods of these star-spectra would doubtless often reveal the supposed absent secondary spectrum. Such is my suggestion, and it now remains to point out the nature of the special methods referred to.

In general, a marked disparity in brightness between the components of a binary star is accompanied by dissimilarity of the spectra. Thus, if the primary spectrum be of the solar type (or some closely-related sub-type), the secondary spectrum is almost invariably of the type of Sirius.

[1] If m_0 denote the required mean mass (the Sun's mass being taken as unity), I find

$$m_0 = [\bar{7}.24630]\frac{1}{n}\Sigma UV^3 ,$$

where n is the number of binaries, U the period of revolution in *days*, and V cosec, i the relative mean orbital velocity in kilometers per second. It is here assumed that n is very large, and that the orbit-poles are distributed equally in all directions.

Now the contrast in brightness of such spectra should be much reduced (or even reversed) in the *extreme ultra-violet region*, whose radiations are largely absorbed by object-glasses and prisms.[1] Hence a direct concave grating spectroscope[2] should be utilized for the purpose suggested; or, in lieu of this, a reflecting telescope, with grating or quartz prisms.

On the other hand, isochromatic plates might prove to be a valuable auxiliary. For in some cases — perhaps not infrequently — the secondary lines or bands might appear to best advantage in the more luminous part of the spectrum.[3] The plates used should be of various grades, with the region of maximum sensitiveness at different wave-lengths.[4]

These methods should be supplemented by certain devices of a purely photographic nature. By varying the exposures[5] between wide limits, and by resorting to known processes of development and intensification, it will be possible to greatly emphasize the relatively weak spectra, and, in a corresponding degree, to increase the chances of their detection.

Polaris with its relatively close companion, would form an admirable subject for preliminary experiments in the direction indicated. The star has a distinct yellow tinge, its spectrum being closely allied to the solar type. A photograph of this spectrum, obtained under the conditions noted, might well reveal the extreme ultra-violet bands of

[1] Sir William Huggins has called attention to a certain falling off in brightness in the ultra-violet spectra of white stars. On the other hand we have ample evidence that the spectra of Sirian stars extend considerably further into the ultra-violet than do those of solar stars of equal brightness.

[2] See ASTROPHYSICAL JOURNAL, March 1898, p. 157, and June 1899, p. 29.

[3] Other factors than relative brightness must of course be considered. A very favorable case would be that of the projection (or partial projection) of a *well-defined* line in the secondary spectrum upon a strong dark line or band of the primary spectrum.

[4] In accordance with Sir William Abney's recent researches, much may depend here upon the "gradation" of the plates employed. For rays of any assigned wave-length, the "steepness" of the gradation-curve varies according to the region of maximum sensitiveness.—(*Phil. Trans.*, March 2, 1901; *Observatory*, November 1901, p. 427.)

[5] In the extreme ultra-violet the spectra would be under-exposed, which tends to heighten contrast. The method of copying faint or delicately-graded images on "slow" plates, with (if necessary) subsequent intensification, is also available here.

the hydrogen series, due (according to our probable view) to the Sirian-type companion. The spectrum of the *outer* companion is probably much too faint to be thus recorded.

It is of course quite possible that the hydrogen bands in the spectrum of *Polaris* itself might also appear in the far ultra-violet, with the prolonged exposure which may be necessary to photograph the companion-spectrum. If so, the methods of increasing contrast, as already noted, may suffice to differentiate the two spectra, provided the bands are sufficiently well-defined. Otherwise, we must rely upon measurements of the *mean* position of the bands, or (with better prospects of success) upon a careful study of narrower bands or lines in the composite spectrum.

If the primary spectrum be of the first type (*e. g.*, *Spica*, *Algol*, *δ Orionis*, etc.), we should look for a secondary spectrum of the same general type. But the relative intensity of the two spectra will probably vary considerably at different wave-lengths. Our search for faint superposed lines, etc., should here extend from the far ultra-violet to the infra-red, special attention being given to the various methods for heightening contrast in the image.

The case of *Algol* is exceptionally interesting. At minimum, the light of the secondary body is relatively strengthened. At the opposite phase (when this body is *behind Algol*), its rays are wholly cut off. Hence a careful comparison of photographs taken at these phases would add much to our chances — otherwise favorable — of detecting the companion spectrum.[1]

It is worthy of remark that the masses of such secondary bodies will in general be notably smaller than those of their primaries, and hence the orbital velocities of these bodies will be correspondingly large. Thus, many interesting systems, which would otherwise escape observation, may yet be brought to light by a systematic search for faint secondary spectra.

J. MILLER BARR.

ST. CATHARINES, ONTARIO,
Canada, November 22, 1901.

[1] Possible changes in the star's spectrum, due to its axial rotation, must be taken into account. (See the *Observatory*, September 1887, p. 320; November 1887, p. 388; and June 1900, p. 254.) The foregoing remarks will apply (with some slight modifications) to other variables of the *Algol* type.

THE EXPLOSION HYPOTHESIS IN THE LIGHT OF THE RECENT PHENOMENA OF *NOVA PERSEI.*

IN the September number of the ***Observatory*** Miss Clerke criticises the explanation of the *Nova* spectrum published in the ASTROPHYSICAL JOURNAL for May of this year (p. 277). She states first that in a few hours the expanding gases would be too cool to give out any light of their own, and consequently would be unable to produce an absorption band. Secondly, "the initial velocities of the expelled gases, even if sufficient to carry them finally away from the parent-body, should be steadily diminished by its attraction." From this she argues that the residual velocities would be inadequate to produce the observed separation between the bright and dark lines.

These remarks would be true enough if applied to the instantaneous explosion of a powder magazine, but in the case of a *Nova* it is evident that the outrush of the gases lasts for weeks, and that even if the first gases emitted did cool down as she supposes, and cease to absorb light, there are constantly fresh supplies of new gases appearing, which would maintain the effect observed. The recent photographs obtained at the Yerkes and Lick Observatories, however, seem to dispose of these two objections so completely that little else needs to be said. A luminous mass of gas is at the present time found to be receding from the *Nova* in all directions with an enormous velocity.

The former separation of the bright and dark lines in the spectrum according to Vogel corresponded to a relative velocity of about 450 miles per second, or 225 miles relatively to the star. This velocity is nearly equal to the maximum velocities observed in the solar explosions, which, according to Young, sometimes reach 250 miles per second.

That the velocities at present found in the nebula surrounding the *Nova* are much higher than this now seems certain. Assuming a parallax of $0''.1$, they must be about 100 times greater. This is a result which could probably only be obtained from a continuously acting force. However these high velocities lately observed may be explained, it now seems almost certain that the *Nova* did at first give out continuously large volumes of luminous gas in all directions, and if this is the case, it seems difficult to see how it could have avoided having a spectrum of bright and dark lines such as was actually observed, and

such as it was attempted to explain by an explosion in the article above mentioned.

WILLIAM H. PICKERING.

HARVARD COLLEGE OBSERVATORY,
November 25, 1901.

ERRATA

THE following corrections to M. Rogovsky's article in the November number of the ASTROPHYSICAL JOURNAL have been sent by the author.

Page 239, line 7, *for* has had *read* has.

Page 242, line 4 from foot, *dele* the words "placed in column 7 of Table II and."

Page 244, Table I, column 3, *for* −635 *read* −63.5.

Page 244, end of footnote, *for* the oblateness of *Uranus*, *read* their oblateness.

Page 246, line 2 from end of footnote, *for* 8, *read* 7.

Page 250, line 6, *for* 1500, *read* 1505.

Page 253, line 8, *for* from the given celestial body, *read* from the surface of the given celestial body.

Page 254, line 9 from foot, *for* λ5000.5 *read* λ5005.

Page 254, line 7 from foot, *for* 5002$\mu\mu$, *read* 5002.

Page 255, line 4, *for* 1.6, *read* 16.

Page 256, line 3, from end of footnote, *for* Leipzig 1874, read *Sitzb. d. preuss. Akad. d. Wiss. zu Berlin*, 1895, 1–25.

Page 256, line 2, from end of footnote, *for Ibid.*, read H. C. Vogel, *Untersuchungen über die Spectra der Planeten*. Leipzig, 1874.

Page 258, line 1 from end of footnote, *for* Overmann's *read* Wildermann's.

Reviews

Theorie und Geschichte des photographischen Objectivs. Von MORITZ VON ROHR. Verlag Julius Springer, Berlin, 1899.

DR. VON ROHR in his book on the theory and history of the photographic objective has aimed to gather together from the original sources an account of the improvements that have been made in photographic lens-construction. The task seems by no means an easy one when we consider the number of journals and individual publications that have been consulted by him in the preparation of the work. An author-bibliography covering thirty-seven pages is given at the end of the book and is a most valuable guide to the technical literature of the subject when the original sources must be consulted. However, Dr. von Rohr has given within a small compass a thorough review of the works bearing on the *history* of the development of the photographic lens. His work is entirely historical. The eighty pages of theory consist chiefly of statements of facts concerning lenses and are intended, it seems, to give definitions and concise explanations of the errors which occur in actual lenses.

In the theoretical part of his book, Dr. von Rohr gives first the Gauss theory and applies it immediately to lenses of small angular aperture. He then discusses the "pin-hole" diaphragm and the effects of diaphragms of finite aperture. The errors which enter in actual practice are next defined. Seidel's five spherical errors — aberration for a point on the axis, non-fulfillment of the sine-condition, astigmatism with possible coma, curvature of field, and distortion of field are explained with a method of illustrating the amounts of some of these errors by graphical constructions. Chromatic aberration for thin and thick lenses are the subjects of following sections, and here, too, the question of the diminution of the secondary spectrum is raised. In connection with the account of chromatic difference of spherical aberration a method is given to show the amount for each zone of the lens and each line of the spectrum. A short chapter contains some remarks on the reflections which occur at the surfaces of a photographic

objective and on the loss of light by absorption during transmission.

The purely historical part of Dr. von Rohr's book gives according to a well arranged plan the different developments in the photographic lens and introduces the subject with a short discussion of the condition of camera-obscura optics before the invention of the photographic process. Attention is called to William Wollaston's lens and to the theoretical work of Airy "On the spherical aberration of the eyepieces of telescopes." Mention is made of William R. Hamilton's "Characteristic Function" which years later was used so cleverly by Maxwell in his investigations of the influence of refractions on the character of thin pencils of light.

The history of the real photographic lens is discussed in full. The progress of the subject is traced in a national way by a threefold division of the book — French-Italian Optics, English-American Optics, and German-Austrian Optics. The forms of construction devised by the French-Italian opticians are considered first, with the theoretical advances made by the same school. Under this division accounts are found, among others, of the lenses of Chevalier, of the investigations of Foucault and Fizeau on the actinic power of lights of various colors, of the work of Breton (de Champ) on astigmatism, and of the contributions of Adolphe Martin and Wallon to the theory of the photographic lens.

Under the title of English-American Optics are included the designs of such men as Andrew Ross, Dallmeyer, Dennis Taylor, and Alvan G. Clark, with the theoretical writings of Thomas Grubb and Robert Bow. Short biographical sketches of many of the persons mentioned in the book are given, while in small type are detailed accounts of special investigations.

Dr. von Rohr is a scientific associate of the great Zeiss firm at Jena and one is not disappointed with the thoroughness of his treatment of German-Austrian optics. Here is found a discussion of the analytical methods of Petzval, Seidel, and Zinke (gen. Sommer), and extended accounts are given of the magnificent results of Voigtländer, Steinheil and Rudolph. The histories of the aplanatic and anastigmat objectives are given at some length and are clear and interesting. Some chapters are devoted to scientific glass making. Its improvements are traced from Fraunhofer's time down to the present, and special remark is made on the experiments of Abbe and Schott, and Mantois.

At the end of the volume are drawings of eleven portrait lenses,

twenty-two "universal" objectives, nine wide angle objectives and six landscape lenses with curves showing their errors of spherical aberration and astigmatism. The curves are drawn from the results of trigonometrical calculations made by Dr. von Rohr, the data for the computations being given in the body of the book where the various lens types are discussed.

Altogether the work is a thoroughly satisfactory history and is valuable in giving an accurate outline of what has been done up to the present in the line of improving the photographic objective.

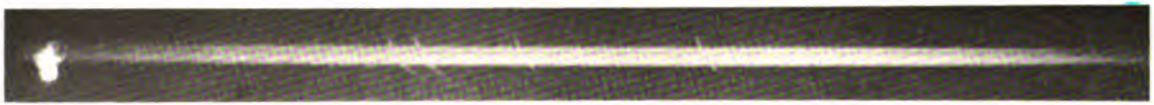
STANLEY C. REESE.

PLATE IV.

UNITED STATES NAVAL OBSERVATORY

SUMATRA ECLIPSE EXPEDITION

May 18, 1901.



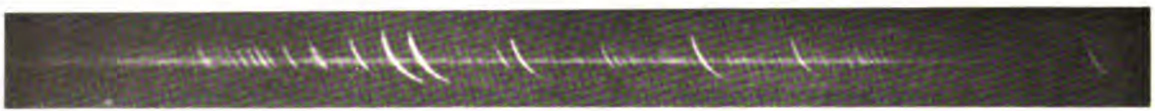


FIG. 1.—SPECTRUM 5 SEC. AFTER THIRD CONTACT.

FIG. 2.—FLASH AT THIRD CONTACT.

Photographed with an objective plane grating by S. A. MITCHELL.

Nine-tenths natural size.

THE

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MEASURES OF ABSOLUTE WAVE-LENGTHS IN THE SOLAR SPECTRUM AND IN THE SPECTRUM OF IRON.[1]

By C. FABRY and A. PEROT.

I. INTRODUCTION.

AT the present time spectroscopic measures are ordinarily made by referring each line to neighboring lines whose relative wave-lengths have been determined by Rowland. The measures are thus based upon a series of standards, and every accidental error in one of these standards produces systematic errors in the measures of a certain part of the spectrum.

Interference methods permit the ratio of the wave-lengths of any two bright lines to be measured directly. In the form employed by us, they render possible the solution of the same problem either for two solar lines or for a solar line and a bright line. Contrary to the present custom, we therefore select a single standard in the spectrum, and compare all wave-lengths with the wave-length of this standard. This should be a rigorously defined radiation, easily reproduced without change of character.

[1] Communicated by the authors, as advance proof of an article to be published also in the *Annales de Chimie et de Physique*.

The choice of one of the two D lines should certainly be avoided, since these lines are too wide, both in the solar spectrum and in flames, where they undergo remarkable variations in appearance. In the solar spectrum hundreds of lines might serve much better as standards (Peirce). But it is not sufficient that the line selected be narrow: its wave-length must also be constant. The wave-length of solar lines varies very appreciably, depending upon the point of the Sun observed, on account of its axial rotation; the motion of the Earth in an elliptic orbit and its axial rotation may also produce changes which, though very small, may not be entirely negligible in a fundamental standard; finally, it is not certain that variations in the solar activity may not cause slight changes in wave-length, through change of pressure. Moreover, laboratory methods render possible the production of bright lines whose wave-lengths are much better defined than those of the sharpest solar lines, and which offer every guarantee of invariability.

The problem has been studied and completely solved by Professor Michelson, in connection with another research (choice of a metrological standard). But the conditions of the two problems are precisely the same, and the best metrological standard is also the best standard from the spectroscopic point of view. We shall therefore choose as a standard the red line of cadmium produced by the electric illumination of the vapor at low pressure.

This standard having been chosen, it is desirable to designate its wave-length. From a purely spectroscopic point of view, the choice of this number is only of conventional importance: wave-lengths are used in spectroscopy only as ratios, and the designation of the wave-length of the standard by this or that value is a matter of perfect indifference. It nevertheless seems rational to refer wave-lengths to the unit of the metric system, by attributing to the wave-length of the red cadmium line the value $643.84722 \mu\mu$ in air at $15°$ and 760 mm, the value found by MM. Michelson and Benoit. If future investigations yield a more exact value of this wave-length, it

will only be necessary to multiply all the other numbers by the same coefficient.

It should be added that, as a matter of fact, the greater part of our determinations have been made by comparison with the green line, which is brighter and more conveniently situated in the spectrum. It is true that this line is not single; but in the conditions of the experiments (Michelson tube excited by an alternating current and not by a coil), the principal component so strongly predominates that this complexity can lead to no error. Furthermore, we have frequently had occasion to determine that the ratio of the wave-lengths of the red and green lines of cadmium given by MM. Michelson and Benoit is accurate to a ten millionth (at least when only the principal component of the green line is considered). Under these conditions it comes to exactly the same thing to take the green line as the standard, giving it the wave-length 508.58240 $\mu\mu$.

To avoid reducing to vacuum, which requires a very precise knowledge of the optical properties of air, the wave-lengths are given in air. These wave-lengths vary with the temperature and pressure; but it is easy to show that the relative wave-lengths remain practically unchanged, at least under ordinary atmospheric conditions.

Thus for a given condition of the atmosphere let D be the density of air, λ, λ' the wave-lengths of the two lines, n, n' the corresponding indices of air. If the temperature and pressure vary, we have

$$\frac{\lambda}{\lambda'} : \frac{n}{n'} = \text{const.}, \quad \text{whence} \quad \frac{d\frac{\lambda}{\lambda'}}{\frac{\lambda}{\lambda'}} = \frac{dn'}{n'} - \frac{dn}{n} ;$$

but

$$\frac{n-1}{D} = \text{const.}, \quad \text{whence} \quad \frac{dn}{n-1} = \frac{dD}{D} ,$$

$$dn = (n-1)\frac{dD}{D} \quad \text{and} \quad dn' = (n'-1)\frac{dD}{D} ,$$

finally

$$\frac{d\frac{\lambda}{\lambda'}}{\frac{\lambda}{\lambda'}} = \frac{dD}{D}\,\frac{n'-n}{nn'} .$$

Under the conditions of our measurements, which extend from λ 435 $\mu\mu$ to λ 650 $\mu\mu$, the error resulting from a change of pressure of 20 mm of mercury and a temperature change of 15° (never realized in practice) is less than three ten millionths. As the precision attained is one millionth, no attention need be paid to this point.

The values given may thus be considered to have the following significance: If a line is defined by the value λ, the ratio $\frac{\lambda}{643.84722}$ is the ratio of the wave-length of the given line to that of the red cadmium line, both being observed in air under the same conditions. Further, if we consider with MM. Michelson and Benoit that the wave-length of the red cadmium line in air at 15° and 760 mm of mercury is 643.84722, our values represent absolute wave-lengths for the same conditions.

We may remark, in consequence of what has been said, that it is unnecessary to record the temperature and pressure in wave-length comparisons, provided that these quantities do not change during the measures.

Precision of wave-length determinations.— Even when we except the compound lines observed by us and previously recognized by Professor Michelson, a spectral line never appears as an infinitely narrow line, such as would correspond to a simple pendular motion, either in the case of a bright line or a dark absorption line. Without the necessity of investigating the physical cause of this phenomenon, it is evident that this appearance must render difficult the determination of the center of gravity of the line, and by this fact in itself prevent the possibility of giving the wave-length with more than a certain number of significant figures; it is just as though we were dealing with a group of lines and not with a single one in the mathematical sense of the word.

It is desirable that the methods of comparison should be sufficiently perfect to permit this limit to be reached; in other words, that they should be free from systematic errors and be affected by only such accidental errors as are involved in the

definition of the quantity measured. Finally, the best defined lines obtainable should be selected as standards.

II. METHOD OF COMPARING WAVE-LENGTHS.

1. *Interpolation methods.*—The purpose of all spectroscopes is to arrange luminous radiations according to a continuous function of their wave-lengths. Of two given radiations, any spectroscope whatsoever will render it possible to determine which has the greater wave-length. If a sufficiently large number of standards of known wave-lengths are available, it will always be possible to find by interpolation the wave-length of any line. This is the method currently employed in both terrestrial and astronomical spectroscopy. It is equally adapted in principle for prism and for grating spectroscopes, though the interpolation is easier in the latter case, as the formula is sensibly linear.

The interpolation method is not a true method of wave-length determination; it assumes that a certain number of lines forming a wave-length scale have already been compared, and the numbers of this scale must be obtained by another method.

2. *Direct comparison methods.*— Every direct measurement of the ratio of two wave-lengths is based on the observation of interference fringes, which are indispensable for the determination of the periodicity of luminous phenomena.

Consider a radiation of wave-length λ, employed for the production of an interference phenomenon. To any point A in space there corresponds a difference of path δ and an *order of interference* p; we have

$$\delta = p\lambda . \qquad (1)$$

For a second radiation of wave-length λ', at a point A' (which may coincide with A), we have

$$\delta' = p' \lambda' ; \qquad (2)$$

whence

$$\frac{\lambda'}{\lambda} = \frac{\delta' p}{\delta p'} . \qquad (3)$$

To obtain the desired ratio, it is necessary to measure the ratio of two lengths and two numbers, p, p'. This principle may be applied in various ways:

A. GRATINGS.—The orders of interference are in this case very small (never greater than ten); the integral part of these small numbers is known at once.

Deviation method.—p and p' are equal *integers*. Then

$$\frac{\lambda'}{\lambda} = \frac{\delta'}{\delta},$$

and this ratio of lengths is derived from angular measures.

Method of coincidences. (Rowland and his pupils.)—The points A and A' coincide; $\delta = \delta'$, p and p' are no longer integers, though nearly so; they are small but different (up to 7 in Rowland's measures). The small fractional part of these numbers is determined by direct measurement.

Thus with gratings interference phenomena of low orders are employed, and consequently the numbers p and p' must be determined with great precision. This is possible, thanks to the special properties of gratings, which are based on the fact that there are not merely two interfering waves, but a great number (as many as there are rulings).

B. INTERFERENCE METHODS.—In this case the orders of interference p and p' are, on the contrary, very high, and may attain tens or hundreds of thousands.

Suppose that the points A and A' coincide. As there are no dispersive media, $\delta = \delta'$ (within very small limits) and consequently

$$\frac{\lambda'}{\lambda} = \frac{p}{p'} .$$

λ being the standard, known by hypothesis, we have

$$\frac{d\lambda'}{\lambda'} = \frac{dp}{p} - \frac{dp'}{p'} .$$

The relative error of λ varies as the relative error of the order of interference. If infinitely narrow lines are employed dp is certainly less than 0.1; the precision may thus be increased indefinitely by increasing the order of interference. In fact, the limit is imposed by the limit of definition of the lines.

It is evident from what precedes that there is no difference in principle between the grating method and interference

methods for the comparison of wave-lengths. The difference consists simply in the very different order of magnitude of the parameters. In the case of gratings, the order of interference does not exceed a few units; it may attain hundreds of thousands in interference methods. Conversely, the order of interference is defined in the case of gratings with very great absolute precision; with ordinary interference apparatus, where there are but two interfering waves, the absolute precision of p can hardly surpass 0.05. The difference between the two methods is still further diminished if it is noticed that in the case of gratings, if N is the number of lines, the difference of path between the extreme waves is $p \times N$. In both cases great precision cannot be obtained without the intervention of waves having great difference of path; only in the case of gratings there are also present a great number of intermediate waves.

From this point of view our interference methods may be regarded as a combination of the two preceding methods; the value of p is as high as in ordinary interference methods and is limited solely by the fineness of the lines; as in the case of gratings, the number of supplementary waves may be great, though always much less than with gratings and not all of the same intensity. It follows from this that the value of p is determined with a precision at least ten times as great as in ordinary interference methods.

Professor Michelson's echelon spectroscope enjoys the same properties, but in it the value of p is determined for each line, while our interferometer enables us to give this the greatest value admissible.

Finally, the properties of our apparatus resemble those of gratings in that it directly separates the various radiations that enter in a confused mixture, which a two-wave system cannot do; we can thus attack problems which seem beyond the reach of the old interference methods (measurement of wave-lengths in the solar spectrum, for example).

The measurement of wave-lengths in absolute value (*i. e.*, by comparison with a unit of length of another kind) has always

been effected by means of interference phenomena. The equation $\delta = p\lambda$ gives λ if δ is known.

The earliest measures of this character must be attributed to Newton, and they have all the precision that could be expected from a spectrum without standards of comparison. Measures which were absolute as well as relative were not taken up again until after the invention of the grating. The method of deviations is the only one available for absolute measures: the wide range of the results obtained is sufficient to indicate the extreme difficulty of the method. Relative measures, as carried out by Rowland by the method of coincidences, attain a precision considered to be one part in a million; it actually is of this order when neighboring lines are compared, but it is notably less when widely separated lines are concerned (see § V). It follows that the grating is an excellent instrument for measurement by interpolation, only fairly good for relative measures of lines widely separated in the spectrum, and bad for absolute measures.

A return was made to interference methods for the purpose of effecting absolute measures. A very interesting first attempt was that of M. Macé de Lépinay in 1886;[1] the grating was employed only as a dispersing instrument. The problem was attacked and solved in a much more direct manner, first by MM. Michelson and Morley and later by MM. Michelson and Benoit. These experiments led to the determination of the absolute wave-lengths of four lines, of which at least two (the red and the green) are perfectly defined. The superiority of interference methods for absolute measures was then demonstrated. Their precision for relative measures is the result of this work. Unfortunately the apparatus was complicated, and the experiments long and delicate. A dozen new lines were compared by M. Hamy with the lines measured by Professor Michelson, by means of much simpler apparatus. Finally, we have used our fringes from silvered plates for the same purpose. Our first measures, made by the method of coincidences, involved a rather extended investigation; since that time we have devised

[1] *Annales de Chim. et de Phys.*, (6), 10, 166, 1887.

a much simpler method, which is convenient and reliable in practice; this is described in the present paper.

Summing up, we may say that the grating is likely to remain the principal spectroscopic apparatus; although its resolving power can be exceeded by the use of interference methods, particularly by our own, it nevertheless remains without a rival because of its convenience; by interpolation it is capable of

FIG. 1.

giving wave-lengths with a precision attaining a millionth. But this interpolation is possible only with standard lines, and interference methods only can give the relative wave-lengths of these standards to a millionth.

III. PHENOMENA AND APPARATUS EMPLOYED.

The phenomena which we utilize are always those of interference from silvered films, the theory of which we have given previously. As we are concerned with interference at great difference of path, fringes at infinity are employed, produced by

two plane and parallel surfaces. These fringes consist of concentric circles, whose order decreases as their diameter increases.

The interferometer is employed when we wish to be able to change the order of interference. When this is not necessary, the two plane surfaces are permanently fastened together, parallel to each other, and form what we call a standard of thickness. We have previously described one form of standard;[1] a new and more compact form is shown in Fig. 1; the silvered plates are held against pieces of polished steel, with rounded ends, by springs whose pressure can be regulated by means of the butting screws V. By varying this pressure, the deformation of the steel and the glass at the point of contact is very slightly changed, which permits the parallelism of the silvered surfaces to be adjusted, after having been previously made almost perfect by systematically grinding the pieces of steel. The adjustment of these standards is perfectly stable, and their thickness remains almost absolutely constant after successive dismountings and and remountings. We have employed three standards, of 2.5, 5, and 10 mm thickness, respectively.

The method is always as follows: as the two radiations λ and λ' produce at the same point in the field the orders of interference p and p', we have

$$\frac{\lambda'}{\lambda} = \frac{p}{p'} \ .$$

The radiation λ is always the principal component of the green line of cadmium, produced by the illumination of a Michelson tube through the use of a high tension alternating current, obtained from a small transformer and a converter supplied by a storage battery. Under these conditions the principal component of the green line is predominant, while the others are very faint. Moreover, the tubes last much longer than when excited by a coil; finally, the apparatus is noiseless and the interrupter of the coil, always a source of trouble and uncertainty, is gotten rid of.

[1] *Annales de Chim. et de Phys.*, April 1901; also ASTROPHYSICAL JOURNAL, **9**, 87, 1899.

In brief, everything depends on the determination of the two numbers, p and p'. Each is composed of an integral part, P, P', and a fractional part ϵ, ϵ';

$$p = P + \epsilon \ . \qquad p' = P' + \epsilon'.$$

A. METHOD OF COINCIDENCES.—In our earlier measures, instead of determining directly the fractions ϵ and ϵ', we sought a coincidence of the two systems of fringes, the apparatus being illuminated by the two sources simultaneously. This plan required that a large number of fringes should pass through the field, and when once a coincidence had been found, it became necessary to make observations in order to find the orders p and p' (whole numbers). These observations rendered the work very long. By this method we determined the wave-lengths of the mercury lines, the experiments involving a difference of path of 6.3 cm.

Subsequently we simplified the operations, by always working in the neighborhood of some given thickness. For this purpose we set the interferometer for a known thickness, by means of superposition fringes obtained with the aid of an auxiliary standard. In this manner we measured the lines of various metals produced by our vacuum interrupter.[1]

Even with this improvement the method had several serious disadvantages:

1. The necessity of illuminating the apparatus by two sources simultaneously involved important losses of light.

2. The coincidences are easily observed only when the two ring systems are comparable in brightness, and this condition is not always easy to realize.

3. The search for a coincidence always involves uncertainty, and one can never be sure (when the period is short) of finding one that is exact. For these reasons we no longer recommend the method of coincidences. It may be said, however, that for bright and easily isolated lines it is apparently no less precise than the method we are about to describe, which nevertheless has the

[1] *Comptes Rendus*, **130**, 406 and 492, 1900; *Journal de Physique* (3), **9**, 369, 1900.

advantage of being simpler and more uniform in application, while it is also applicable to much fainter lines.

B. METHOD OF DIAMETERS.—The order of interference at the center of a system of rings is determined directly for the two radiations in succession. It is no longer necessary to illuminate by the two sources simultaneously; the distance between the two silvered surfaces, instead of being variable, should, on the contrary, remain fixed during the measurement; the interference is therefore produced by means of a standard of thickness Consider at first the radiation of cadmium, λ. It gives a system of rings. Let p be the order of one of these rings, *e.g.*, the first from the center; this easily determined whole number is supposed known. The order of interference at the center will be $p=P+\epsilon$ It is necessary to determine this number ϵ, which ordinarily lies between zero and one. The diameter of the ring considered increases with ϵ; the measurement of this diameter should thus permit ϵ to be measured, as we shall now show.

Let e be the thickness of the layer of air. The order of interference at the center is $p=\frac{2e}{\lambda}$. In a direction making an angle i with the normal, the order of interference is

$$\frac{2e}{\lambda}\cos i = p\cos i \ .$$

If x is the angular diameter of the ring P, the observing telescope being focused for infinity, we have

$$p\cos\frac{x}{2} = P \ ,$$

or since x is small,

$$p = P\left(1+\frac{x^2}{8}\right) .$$

Now

$$\epsilon = p - P, \quad \text{whence} \quad \epsilon = P\frac{x^2}{8} \ . \tag{1}$$

Treating in the same way the radiation to be measured, λ', we have

$$\epsilon' = P'\frac{x'^2}{8} \ , \tag{1'}$$

and if the integer P' is known, we may derive from it λ' as a function of λ; thus

$$\lambda(P+\epsilon)=\lambda'(P'+\epsilon'), \text{ whence } \lambda'=\lambda\frac{P+\epsilon}{P'+\epsilon'},$$

or, substituting for ϵ and ϵ' their values, and making permissible approximations,

$$\lambda'=\lambda\frac{P}{P'}\left(1+\frac{x^2}{8}-\frac{x'^2}{8}\right). \tag{2}$$

Thus the ratio $\frac{\lambda'}{\lambda}$ is derived from angular diameters x and x' and whole numbers P and P'.

Arrangement, adjustment, and measurement of the standard.—It is indispensable that the standard be rigorously adjusted, *i. e.*, that the plane silvered surfaces be exactly parallel. A slight maladjustment may produce important systematic errors. If the two radiations do not illuminate exactly the same part of the standard, and if the surfaces are not precisely parallel, the result will be the same as though there were a difference in path when substituting one radiation for the other.

The standard is carried on a support provided with rack and pinion which permits it to be moved horizontally or vertically in its own plane. In front, carried by an independent support, is a screen pierced by a small opening (2 or 3mm in diameter) upon which is projected the image of a monochromatic source. A very small part of the standard is thus used, and the region employed may be changed by moving the standard. If the adjustment is perfect, these displacements will have no effect on the appearance of the rings; if not, the rings will expand or contract. The least defect in adjustment and the precise change necessary to produce perfect parallelism is thus indicated to the observer. The adjustment is effected by varying the pressure of the springs on the steel buttons. The precision of this adjustment is limited only by the curvature of the glass plates, which is always very slight. A position is reached for which the thickness is a maximum or a minimum in the central region; over a limited area it may be regarded as absolutely constant; this area only is used, the standard remaining henceforth fixed with reference to the

perforated screen, upon which are projected the images of the sources employed. The systematic error referred to above is thus certainly avoided, since the part of the standard used is small and perfectly defined, and also since the parallelism of the silvered surfaces is perfect in this region.

The standard, after being adjusted, is measured in terms of the cadmium wave-lengths by the methods already described; it is necessary to use the interferometer for this purpose. The thickness is subject to variation only as the result of expansion; the 1cm standard expands about 0.1μ per degree, or 4 fringes for 10°. The coincidences of the red and green lines of cadmium enumerate the green fringes by fives. A glance into the telescope indicates the numbers of the various fringes; they remain the same from day to day. Thus P is known at once.

Although temperature changes have no influence on the integral part, they nevertheless produce troublesome variations in the fractional part of the order of interference (for the 1 cm standard, $0\overset{\circ}{.}01$ produces a change of 0.004 fringe, which exceeds the limit of precision). As a complete measure lasts only a few minutes, these variations are easily avoided; they can also be eliminated by making the measures in the proper order.[1]

When sources giving numerous monochromatic radiations are employed, a suitable dispersive system must be used to separate them. Sometimes absorbing tanks are sufficient. In the case of cadmium we observe directly, each radiation gives its own system of rings, and as there are but few of these, settings can easily be made on any ring unless the ring observed corresponds to a coincidence, when an absorbing tank is necessary.

Measurement of the diameters—The essential part of the measurement consists in the determination of the angular diameter of

[1] It will be advantageous to avoid these displacements by employing the metal *invar* in the construction of standards. One advantage would be lost, however: with steel standards, the fractional part is quite variable from day to day; if a measure is repeated, it is made again in quite different conditions, which gives a chance control on its exactness. With the metal *invar* great temperature changes would be necessary to appreciably affect the diameter of the rings, and each measure would always be repeated under the same conditions, especially in the case of standards of small thickness.

a ring for each of the radiations to be compared. For this purpose we employ a telescope focused for parallel rays, and provided with an eyepiece micrometer; the objective has a focal length of 19 cm. The magnification should not be too great; we ordinarily use an eyepiece of 3 cm focal length, giving a magnification of only five diameters. The micrometer has a single movable thread; there is also a fixed thread parallel to the movable one. The telescope, carried by a massive cast-iron column, is provided with all motions of adjustment; in particular, it is movable about a horizontal axis by means of a screw.

A measure of the diameter of a ring is made by setting the fixed (horizontal) thread on the upper part of the ring and the movable wire on the lower part, each bisecting the ring. The micrometer has been calibrated with the aid of a large circle divided by Brunner—belonging to the University of Marseilles. The viewing telescope and the telescope attached to the circle having been directed toward each other, with the objectives face to face, the movable micrometer thread was set on the thread of the goniometer for a series of angular positions 5 minutes apart. The resulting measures are well represented by the formula

$$\alpha = (l - 3.93) \times 9'.1060 .$$

α being the angular separation of the two threads, expressed in minutes, and l the position of the movable thread, expressed in revolutions of the screw.

In the measurement of the diameter of a ring made as indicated above let l be the reading of the micrometer in revolutions; the diameter of the ring in revolutions is

$$\delta = l - 3.93 .$$

In minutes this diameter is

$$\alpha = 9'.1060 \times \delta ,$$

and in radians

$$x = 0.002649\,\delta .$$

The fractional part of the order of interference at the center is

$$\epsilon = P\delta^2 \times 0.87728 \times 10^{-6} = BP\delta^2 ,$$

where

$$B = 0.87728 \times 10^{-6} .$$

Similar formulæ will be obtained for the second radiation, λ'. Thus formula (2), which gives λ', becomes

$$\lambda' = \frac{P\lambda}{P'}(1 + B\delta^2 - B\delta'^2) .$$

Precision of the measures.—The maximum accidental error of ϵ, for a single measure of the diameter, is less than 0.01, when the radiation which produces the fringes is very nearly monochromatic. An example of the measures follows:

2.5 mm standard: green line of cadmium; number of the ring measured, $P = 9612$. The formula then gives

$$\epsilon = 0.8433 \times 10^{-2}\delta^2 .$$

Observer	δ	ϵ	Mean
Fabry	11.45	1.105	1.104
	11.44	1.103	
	11.45	1.105	1.106
	11.46	1.107	
Perot	11.47	1.109	1.105
	11.44	1.103	
	11.44	1.103	
	11.48	1.110	1.110
	11.47	1.109	

The horizontal lines indicate a few minutes' rest. The temperature was rising very slowly, which explains the slight increase in ϵ (about 0.006). It is evident that in a single determination ϵ may be considered reliable to a few thousandths. δ is determined in this case to one division on the head, *i. e.*, the angular diameter of the ring is measured within about 6″.

This example suffices to show that the errors possible in the determination of these fractional parts are much smaller than in the case of ordinary interference fringes. This results from the appearance of the fringes: each ring appears as a sharply defined circle, and the thickness of the line hardly exceeds a tenth of the distance between the two fringes. Assuming the fractional part of the order of interference at the center to be

0.9, which indicates that the center is barely luminous, the diameter of the first ring for the 2.5 mm standard and λ 500 is $1^\circ 32'$, and its width is only $2'.5$.

Repetition of the measures reveals only the accidental errors; but systematic errors may also affect the results. The fractional part of the order of interference at the center may be deduced from the measurement of any one of the rings. It is advantageous to employ the smaller ones, for a given angular error affects the order of interference at the center by an amount which decreases with the diameter of the ring. Nevertheless, settings on the very small rings involve an error due to the fact that the geometrical middle of the ring and the light maximum which defines the physical middle do not coincide. This arises from the fact that the law of distribution of the fringes is not linear, but is given by a cosine, and may be considered parabolic.

Let N be the order of a ring, y the angular distance from the physical middle point of this ring to the center, $N + \epsilon$ the order of interference at the center; then

$$N = (N + \epsilon) \cos y \,, \quad \text{or} \quad \epsilon = N \frac{y^2}{2} \,.$$

If the edges of the ring are defined by the radii y' and y'', and if its semidiameter expressed in fringes is a, we have

$$\epsilon - a = N \frac{y'^2}{2} \,, \quad \epsilon + a = N \frac{y''^2}{2} \,,$$

whence

$$y'^2 + y''^2 = 2y^2 \,.$$

In practice we measure the radius y_1, defined by

$$y' + y'' = 2y_1 \,,$$

whence we obtain

$$\epsilon_1 = \frac{N y_1^2}{2} \,.$$

The error involved is

$$\epsilon - \epsilon_1 = \frac{N}{2}(y^2 - y_1^2) = \frac{N}{8}[2(y'^2 + y''^2) - (y' + y'')^2] \,,$$

whence

$$\epsilon - \epsilon_1 = \frac{N}{8}(y'' - y')^2 \,,$$

which may be written, after expanding and substituting for y' and y'' their values in terms of ϵ and a,

$$\epsilon - \epsilon_1 = \frac{\epsilon}{2}\left(1 - \sqrt{1 - \frac{a^2}{\epsilon^2}}\right) = \frac{a^2}{4\epsilon} .$$

The measured order of interference will thus be too small, and in greater degree as ϵ grows smaller; in fact, special measures have shown that the values of ϵ derived from settings on the first ring when barely visible and on the second ring have slightly different fractional values, as the following table indicates:

	I	II	III
First ring, ϵ_1	0.237	0.121	0.087
Second ring, ϵ.......	1.244	1.130	1.093
$\epsilon_1-\epsilon$	0.007	0.009	0.006

These differences are clearly in the direction expected, and give for a:

a - - - - - -	0.060	0.057	0.054
Mean - - - - - -		0.057	

If it is desired that the error be less than 0.005, we must have for the silvered surfaces in question

$$\epsilon > \frac{(0.057)^2}{4 \times 0.005} \quad \text{or} \quad \epsilon > 0.4 .$$

We may add that the direct measurement of the width, made by determining the order of interference at the center for a barely visible ring, in the third of the above experiments, gave $a = 0.052$, which is in perfect agreement with the computed value, in spite of the extreme difficulty of the measures, in which the order of interference is determined to about $\frac{1}{2000}$. We have treated this matter in some detail, as the above results indicate the extreme precision of our measures.

It follows from this that results which are somewhat in error may be obtained if the settings are made on too small a ring; if the first ring is of too small diameter ($\epsilon < 0.3$, for example), the next one would be measured. ϵ will then be between 1 and 2;

the formulæ are not changed, P always remaining the number of the ring measured.

Thus we see that ϵ and ϵ' are known with an absolute error which is certainly less than 0.01; the same is true of the orders of interference p and p', which are obtained by adding whole numbers to ϵ and ϵ'. Let us now find the precision of the result λ'.

The equation $\frac{d\lambda'}{\lambda'} = \frac{dp}{p} - \frac{dp'}{p'}$ shows that as dp and dp' are less than 0.01, in order that λ' may be known to $\frac{1}{1{,}000{,}000}$, it will suffice that p and p' be of the order of 10,000, *i. e.*, that the thickness of the standard shall be from 2 to 3 mm.

For a broad line the rings will of course be less sharply defined, the settings less precise, and the result less exact, but this is due to a lack of definiteness in the aspect of the object measured. After a certain limit has been reached nothing is gained by using rings of a higher order; the rings may even become so ill-defined that measurement is wholly impossible.

Finally, in order to compute λ', it is necessary to know the whole numbers P and P', the orders of the rings whose diameter is measured. We have seen that the first, which corresponds to cadmium, is always known without difficulty. P' is deduced from an approximate value of λ'. In the equation

$$\lambda' = \frac{P\lambda}{P'}(1 + B\delta^2 - B\delta'^2)$$

the parenthesis is known, and also λ and P. If we give to λ' an approximate value, an approximate value of P' may be obtained. If the value of λ' is in error by the amount $d\lambda'$, the value of P' will be in error by the amount dP', and we shall have

$$\frac{dP'}{P'} = -\frac{d\lambda'}{\lambda'} .$$

This error, dP', must be so small as to introduce no ambiguity in the whole number to be chosen for P'. In general, this condition will be satisfied if the error dP' has an absolute value less than $\frac{1}{3}$. In this case

$$\frac{d\lambda'}{\lambda'} < \frac{1}{3P'} .$$

For instance, if a standard of 2.5 mm is used, P' is of the order of 10,000, and it will suffice in general that λ' be known to $\frac{1}{30,000}$. For all the radiations that we have measured we already have values of λ' whose precision exceeds this limit; it has not been advantageous to employ a standard of less thickness.

Order in an experiment.—To eliminate the effect of changes of temperature, it is desirable to make the measurement of one of the ring systems between two measurements of the other system. We proceed as follows: an observer makes two or three settings on the radiation of cadmium, then two or three on the radiation to be measured, then two or three on cadmium. The mean of all the settings on cadmium is used to calculate δ; the mean of the settings on the radiation λ' gives δ'. The second observer repeats the same measures; the observations of the two observers are computed separately and give two independent values of λ'.

Influence of change of phase by reflection.—Let p be the order of interference at the center for a radiation of wave-length λ. The quantity $e = p\frac{\lambda}{2}$ is, by definition, the optical thickness of the layer of air lying between the two silvered surfaces. Hitherto we have assumed this thickness to be independent of λ. This is not rigorously true, on account of the change of phase by reflection on the silver, which varies slightly with the wave-length. In other words, it is just as if each kind of radiation underwent reflection at a certain plane, which may be called the optical surface of the silvered glass;[1] this surface varies slightly with the wave-length. It is evident that this phenomenon, which, as we shall see, is extremely small, must modify the results a little. To eliminate it, it is only necessary to make two observations with the same silvered surfaces and very unequal differences of path. The same result may be attained by making a preliminary

[1] The actual surface of silver, which would be extremely difficult to define, has no part in these purely optical questions.

study of the phenomenon in order to derive the corrections which it involves for the wave-lengths computed by the formulæ given above.

Let λ be the green radiation of cadmium, λ' another radiation; for the same layer of air, we shall have the orders of interference p, p', and the optical thicknesses e_λ, $e_{\lambda'}$, differing slightly from each other.

We have

$$2e_\lambda = p\lambda\,, \qquad 2e_{\lambda'} = p'\lambda'\,.$$

whence

$$\lambda' = \frac{e_{\lambda'}}{e_\lambda}\frac{p}{p'}\lambda\ ;$$

instead of which we have calculated

$$\lambda_0' = \frac{p}{p'}\lambda\ .$$

To this value of λ_0' it is therefore necessary to add a correction

$$\gamma = \lambda' - \lambda_0' = \frac{p\lambda}{p'}\frac{e_{\lambda'} - e_\lambda}{e_\lambda} = 2\frac{e_{\lambda'} - e_\lambda}{p'}\ . \qquad (1)$$

To calculate this it is only necessary to know the difference of the optical thicknesses corresponding to the two radiations. This difference is evidently independent of the distance between the silvered surfaces. It is determined by special experiments made by means of interferences at small difference of path, of radiations of known wave-length.

The silvered plates are removed from the standard and attached to the interferometer; they are then brought together until they are nearly in contact, so as to produce a thin film with silvered surfaces. The fringes of this film are observed in parallel light, normal to the surface, by means of a telescope set on the thin film.

The apparatus is illuminated simultaneously by two radiations, λ, λ', and, for clearness, let us assume that $\lambda > \lambda'$. In this double system of fringes an exact coincidence between two fringes is sought. Suppose that the qth coincidence appears exact, *i. e.*, that a certain fringe of order p, due to the radiation λ, coincides exactly with the fringe $(p + q)$ due to λ'. Then

let e_λ, $e_{\lambda'}$, be the optical thicknesses at this fringe of the thin film for the two radiations. We have

$$2e_\lambda = p\lambda \ , \quad 2e_{\lambda'} = (p+q)\lambda' \ ,$$

whence

$$e_{\lambda'} - e_\lambda = \frac{\lambda - \lambda'}{2}\left(q\frac{\lambda'}{\lambda - \lambda'} - p\right) = \frac{\lambda - \lambda'}{2}\rho \ ,$$

in which

$$q\frac{\lambda'}{\lambda - \lambda'} - p = \rho \ .$$

Hence we can calculate $e_{\lambda'} - e_\lambda$, the difference of the optical thickness for the two radiations used in the experiment. (It should be remarked that $\frac{\lambda'}{\lambda - \lambda'}$ is the period of coincidence expressed as a function of λ.) In this experiment care should be taken not to choose two radiations λ and λ' which lie too near together.

The search for an exact coincidence really amounts to the measurement of a fringe of one of the systems with reference to the fringes of the other system; but the observation of a coincidence is more simple, and an exact coincidence can be determined with very great precision, as the least inexactness is betrayed by dissymmetry of color. As a matter of fact an exact coincidence is not always found; two are selected which are slightly inexact, the one in one direction, the other in the other; *i. e.*, two which show dissymmetry in the two senses. These will give two values of ρ, one of which is too large, while the other is too small. The search for such cases is easily made, as the interferometer permits the fringes to be moved steadily through the field.

This method of coincidences is remarkably precise; it determines the relative position of optical surfaces for two radiations to about 0.1 $\mu\mu$ (one ten-thousandth of a micron). The radiations which have been employed are the red and the green of cadmium (644 $\mu\mu$ and 508 $\mu\mu$), and the green and the violet of the mercury arc in vacuo (546 $\mu\mu$ and 436 $\mu\mu$).

An example will render clear the procedure employed: Silvered surfaces were used with the 10 mm standard. (These

surfaces are the thickest which we have employed and give the greatest effects of change of phase.) Thickness of the silver film: $61\mu\mu$.

First observation (Radiations 644 and 508).—The 29th and 33d coincidences are nearly exact, but one is inexact in one direction, the other in the other direction. They give respectively for ρ, 0.037 and 0.077. We adopt 0.065, from the appearance of the coincidences, whence

$$e_{644} - e_{508} = -4.4\,\mu\mu\ .$$

Second observation (Radiations 644 and 546).—The 24th coincidence is exact. It gives $\rho = 0.042$, whence

$$e_{644} - e_{546} = -2.1\,\mu\mu\ .$$

Third observation (Radiations 546 and 436).—The coincidences 40 and 41 are inexact, one in one direction, the other in the other, and give for ρ, 0.140 and 0.094; adopting $\rho = 0.12$, we have

$$e_{546} - e_{436} = +6.6\,\mu\mu\ .$$

Reducing all the results to the line 508, we obtain by differences:

λ	$e_\lambda - e_{508}$
644	$-4.4\,\mu\mu$
546	-2.3
508	0.0
436	$+4.3$

With these four points we can find the curve which gives $e_\lambda - e_{508}$ as a function of λ (Fig. 2, Curve A). The corrections γ are obtained from this by equation (1), which may be written in the following form: For the line 508 the order of interference is 39,500; for the line λ' it will be

$$p' = \frac{39500}{\lambda'} \times 508\ ,$$

whence

$$\gamma = 2\,\frac{e_{\lambda'} - e_{508}}{39500}\,\frac{508}{\lambda'}\ .$$

The correction curve is deduced from the curve for $e_\lambda - e_{508}$ by multiplying the ordinates by a quantity proportional to the

abscissae (Fig. 2, Curve B). It will be seen that the correction to the wave-lengths does not exceed $3 \times 10^{-4}\mu\mu$, or about half a millionth.

With standards of less thickness the corrections would be greater; but, on the other hand, the silver films which we have studied are the thickest ones employed and are also those which

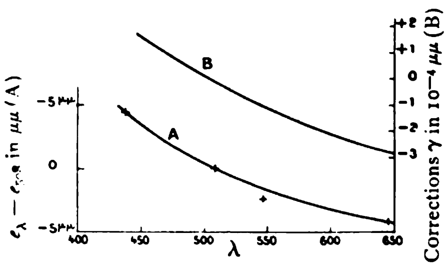

FIG. 2.

have given the largest values for the $e_\lambda - e_{508}$. (The thickness of our silver films varies from 40 to $61\mu\mu$.)

As a matter of fact the phase correction has never exceeded $8 \times 10^{-4}\mu\mu$, or about one and one-half millionths in relative value. If this correction were out as much as 50 parts in a 100, the millionth would not be affected, and it is known to about 0.1. It is evident that this introduces no appreciable error and that, although the change of phase makes necessary a rather delicate investigation, this phenomenon in no degree diminishes the precision of the result; it is even remarkable that it is necessary to take account of such minute quantities.

[*To be concluded.*]

THE FLASH SPECTRUM, MAY 18, 1901.[1]

WAVE-LENGTH DETERMINATIONS AND GENERAL CONCLUSIONS REGARDING THE "REVERSING LAYER."

By S. A. MITCHELL.

UPON the invitation of the former astronomical director of the Naval Observatory, Professor S. J. Brown, the writer became a member of the expedition to view the Sumatra eclipse of May 18, 1901. The party consisted of eleven — six members of the Naval Observatory staff and five invited guests. The former included Professor A. N. Skinner, U. S. N., in charge of the expedition; Professor W. S. Eichelberger, U. S. N.; Professor F. B. Littell, U. S. N.; Mr. G. H. Peters, Mr. W. W. Dinwiddie, and Mr. L. E. Jewell, now of the Johns Hopkins University. The guests of the party were Professor E. E. Barnard, of Yerkes Observatory; Dr. W. J. Humphreys and Mr. H. D. Curtis, of the University of Virginia; Dr. N. E. Gilbert, of Hobart College, and the writer.

A second party, from the Smithsonian Institution, consisting of Professor C. G. Abbot and his assistant, Mr. P. A. Draper, accompanied the Naval Observatory members, the two parties sailing together from San Francisco on February 16. Government steamers carried the expeditions to their destination, the United States army transport "Sheridan" *via* Honolulu to Manila; the United States steamer "General Alava" from Manila to Sumatra, which was reached April 4.

Before arriving in the island it had been decided to occupy two stations for observations on the eclipse; one, Solok, near the central line of totality; the other, Fort de Koch, near the northern edge of the moon's shadow-path.

When we arrived in the East Indies it was soon found that clouds were likely to be exceedingly troublesome, for at no time during the day was the sky perfectly clear. In view of this it was thought best to subdivide the party at Solok, and a

[1] Published in advance of the more complete report by permission of the superintendent of the Naval Observatory.

third station was selected at Sawah Loento, thirty kilometers beyond Solok, at the terminus of the "Staatsspoorweg op Sumatra," the government railroad running inland from Padang. At Sawah Loento were already located Mr. and Mrs. H. F. Newall, of Cambridge, England, and a party from the Massa-

ECLIPSE STATION, SAWAH LOENTO, SUMATRA.

chusetts Institute of Technology under the direction of Professor Burton.

A location was selected near the camp of the latter expedition in the Loento Valley, a mile and a quarter south of the railroad station; and in this connection I wish to express my thanks for the generous assistance of Professor Burton and his party.

The eclipse station was situated 380 meters above sea level, the latitude and longitude being:

$0° 41' 52''$ South,
$100° 46' 40''$ or $6^h 43^m 6^s.7$ East.

The duration of totality was calculated at 5 minutes 41 seconds.

Two instruments were taken to Sawah Loento, a visual telescope of 15.2 cm aperture, but stopped down to 11.4 cm, and of 264 cm focus, and a spectroscope. The former, used to photograph the corona with orthochromatic plates, was in the hands of Mr. René Granger, of Cartersville, Ga., who rendered me very valuable assistance.

The spectroscope consisted of a Rowland flat grating of of 15,000 lines per inch, having a ruled surface of 3½ × 5 inches, and a quartz lens of 8.53 cm aperture and 183 cm focal length, made by Brashear, the whole apparatus belonging to the Naval Observatory. Light from the Sun, reflected by the coelostat mirror in a horizontal direction, fell on the grating, where it was diffracted and was brought to a focus on the photographic plate by means of the quartz lens. Grating, lens, and plate were enclosed in a box.

The coelostat, driving-clock, and box were mounted on brick piers, in the erection of which some very interesting observations were made on methods of labor in the East — observations not always calculated to increase one's peace of mind when regarding the slowness at which the work progressed.

The grating was employed in the manner which gives a normal spectrum, which is the case when the diffracted ray leaves the grating perpendicularly, or the angle of diffraction is zero. The attempt was made to photograph the first order spectrum from λ 3000 to λ 6000, hence with λ 4500 at the center of the plates.

According to investigations of Mr. Jewell, the focal lengths of the quartz lens for different wave-lengths were:

λ	Focal length	λ	Focal length	λ	Focal length
2500	1771.17 mm	4000	1819.00 mm	5500	1833.00 mm
3000	1798.98	4500	1825.78	6000	1835.84
3500	1809.98	5000	1829.66	6500	1837.97

It was thus possible to plot the curve on which the spectrum is brought to a focus. On doing this it was found impossible to

procure glass plates which would bend to such a curvature, and it was, therefore, thought advisable to use films. Through the kindness of the M. A. Seed Dry Plate Co. heavy films were coated with their "Gilt Edge" and "Orthochromatic" emulsions. The films used were 38 mm × 305 mm.

The coelostat was one of those built for the 1874 transit of *Venus*, modified this year by the use of a conical pedulum. Even with this improvement, great difficulty was experienced with the coelostat, and it seemed practically impossible to secure perfectly uniform motion of the mirror. The spectrum was focused a few days before the eclipse by Mr. Jewell, by means of a collimator designed by him.

Valuable assistance was rendered on the day of the eclipse by Naval Cadet W. V. Tomb, who counted time, and by Corporal C. W. Keeter, who carefully attended to changing the plates for me, both having come up from the U. S. S. "General Alava" for the purpose.

The day of the eclipse dawned clear, and our hopes were that these favorable conditions would remain till after totality, which occurred shortly after noon. First contact was observed in a perfectly cloudless sky, but soon after this clouds began to gather, and a quarter of an hour before second contact the sky was completely overcast.

The disappearing crescent of the Sun was watched by a binocular, before one-half of which was arranged a small plane grating in such a way that with one eye the spectrum could be seen, with the other the Sun itself. With this, shortly before the time of second contact, bright lines were seen for a few seconds at F and H and in several places in the green and yellow, but these disappeared almost immediately—the Sun being completely hidden by clouds—and the first flash passed without our being able to see it.

Towards the middle of totality conditions became a trifle better, so that it was possible to see, through clouds, the corona extending for about half a diameter from the Sun, and with the small spectroscope the "coronium" line could be seen quite

distinctly. During no time of the 5 minutes and 41 seconds of totality was an unclouded view of the corona obtained, but nevertheless, the second flash was seen beautifully. An hour after the total phase the clouds cleared away and a perfect sky remained for the rest of the day.

Altogether eight plates—or rather films—were exposed, one before and one just after totality for the cusp spectrum, one at first and one at second flash, and four during the total phase with exposures of 12^s, 120^s, 90^s, and 45^s respectively.

DESCRIPTION OF PLATES.

The first plate was taken 10 seconds before the computed time of the second contact, and was exposed for one-half second. It shows the cusp spectrum and about 60 lines between F and H.

As noted above, the clouds thickened at an inopportune time, with the result that nothing appears on the plate exposed for the flash. The four plates exposed during totality show continuous spectrum of a width equal to the diameter of the Sun, and extending from about λ 4900 to λ 3400, also bright lines of hydrogen from $H\beta$ to $H\eta$ and the helium line λ 4471.6, undoubtedly due to the upper chromosphere.

The second flash seemed fully exposed in spite of the clouds.

An exposure was made as soon after totality as possible—probably 5 seconds after third contact—for the cusp spectrum. An exposure of one-half second was given, the plate closely resembling that made before second contact.

THE SECOND FLASH.

For some reason the spectra were not all of them in perfect focus. As absorption lines suffer from this defect more than bright lines, it was found practically impossible to measure the cusp spectra. For wave-lengths, we are therefore confined to one plate—or rather film,—that of the second flash. This was exposed for three seconds, the exposure being stopped at the first reappearance of the Sun.

The peculiarities of this photograph of the flash are twofold: (1) The normal spectrum, and (2) the great dispersion.

On the plate the distance from F to H is 95.4 mm, and as the spectrum is normal, 1 mm therefore corresponds to a difference of wave-length of 9.37 tenth-meters, or 1 tenth-meter corresponds to a dispersion of about 0.1 mm. This dispersion is about one-fifth of that obtained with the ordinary Rowland mounting with a grating of 20,000 lines and radius of 21½ feet (6.55 meters.)

The plate was measured by one of the Repsold machines belonging to the Columbia University Observatory, by comparing the position of the spectrum lines directly with a millimeter scale. Measures with this machine can be made directly to 0.005 mm, and by estimation to 0.0005 mm, *i. e.*, to 0.005 λ; the sharpness of the lines, however, did not permit them to be carried to quite this degree of exactness.

Although the spectrum was not in the most perfect focus, in view of the great dispersion measures could be made with a high degree of accuracy. Wave-lengths were determined by taking well-defined standards properly distributed, whose wave-lengths were taken from Rowland's table of standard wave-lengths. Three independent measures of the plate were made.

Second and third contacts were not 180° apart, and the instrument was set up to make a compromise between the two positions. This is the reason for the inclination of the lines of the flash.

COMPARISONS WITH THE SOLAR SPECTRUM.

Those who have attempted to identify the bright lines with Rowland's map know the difficulty of this undertaking. Great care was exercised in the determinations of the wave-lengths, and in the comparisons with Rowland's tables. For the flash an arbitrary scale of intensities was taken, where 0 means a line seen with certainty, 10 the strongest line, 00 denotes a line seen with difficulty.

Table I contains the results of the comparisons. The spectrum extends from λ 4924 to λ 3320, but the focus becomes poor at the violet end beyond K, and measures were discontinued at λ 3835, *H*η.

TABLE I.

Flash Intensity	Wave-length	Rowland	Substance	Solar Intensity
Hη 8	3835.2		*Hη*	..
2 *d*	3839.9	3838.435	*Mg-C*	25
		3840.580	*Fe-C*	8
2	3859.9	3860.055	*Fe-C*	20
2 *d*	3878.4	3878.152	*Fe-C*	8
		3878.720	*Fe*	7 *Nd?*
Hζ 8	3889.0		*Hζ*	..
		3894.165	*Cr*	3
0	3894.2	(3894.211)	....	8 *d*
		3894.241	*Co*	5
0	3895.8	3895.803	*Fe*	7
2 *d?*	3900.5	3900.681	*Ti-Fe-Zr*	5
0	3902.3	3902.399	*V*	3
0	3904.2	3903.991	——	2
		(3904.023)	....	8 *d*
		3904.050	*Fe*	5
0	3905.7	3905.660	*Si*	12
00	3907.6	3907.615	*Fe-Sc*	3 *d?*
2 *d?*	3913.3	3913.609	*Ti-Fe*	5 *d?*
0	3917.5		....	..
00	3919.1 group	6 lines,	....	..
00	3922.3	intensities > 2	....	..
K 10	3933.8	3933.825	*Ca*	1000
1	3944.6	3944.160	*Al*	15
0	3948.3 group	8 lines, intensities	....	..
0	3952.1	2 and greater	....	..
		3960.422	*Fe*	4
0 Triple	3961.3	3961.281	*Fe*	3
		3961.674	*Al*	20
H 10	3969.0	3968.625	*Ca*	700
		3970.177	*He*	..
		3981.917	*Ti*	4
1 Triple	3982.5	3982.630	*Ti-Mn*	2
		3982.742	*Y*	3
0 *d?*	3989.0	3989.137	——	3
		3989.232	——	2
0	3991.0	3991.333	*Cr, Zr*	3
1	3995.5 group	6 lines, intensities	....	..
1	3998.3	3 and greater	....	..
1 *d*	4004.9	4003.912	*Ce-Fe-Ti*	3
		4005.408	*Fe*	7
1	4006.0	4005.856	——	3
0 *d*	4009.1	4009.079	*Ti*	3
1	4012.4	4012.541	*Ti*	4
00	4018.1	4018.234	*Mn*	3
		4018.269	*Mn*	4
1	4021.9	4022.018	*Fe*	5
4	4026.0	(4026.342)	*He*	..
0	4028.4	4028.497	*Ti-*	4
1	4030.8	4030.878	*Mn*	4
		(4030.918)	....	..
		4030.947	*Mn*	5

TABLE I.—*Continued.*

Flash Intensity	Wave-length	Rowland	Substance	Solar Intensity
0	4032.0	4031.942	*Mn*	2
1	4033.3	4033.224	*Fe-Mn*	7 *d ?*
1	4034.6	4034.644	*Mn-Fe*	6 *d ?*
0	4035.7	4035.883	*Mn*	4 *d ?*
2	4045.9	4045.975	*Fe*	30
0	4047.8	....	....	..
00	4048.9	4048.910	*Mn-Cr*	5
0 *d?*	4049.7	4049.716	– -	1 *Nd ?*
0	4052.0 } group	Intensities 2 and	....	..
0	4055.7	greater. 10 lines	....	..
00	4058.3	4058.372	*Co-Fe*	4
2	4063.5	4063.759	*Fe*	20
1	4066.6	....	....	..
0	4069.3	4069.221	——	2
2	4072.0	4071.908	*Fe*	15
6	4077.7	4077.885	*Sr*	8
00	4083.1 } group	7 lines. Intensities	....	..
00	4087.2	2 and greater	....	..
Hδ 10	4102.0	4102.000	*Hδ*	..
0	4107.7	4107.649	*Ce-Fe-Zr*	5
0 Triple	4109.5	4108.687	——	2
		4109.215	*Fe*	3
		4109.905	*V*	2
		4109.953	*Fe*	3
0	4118.7	4118.708	*Fe*	5
		4118.934	*Co*	4
0	4121.4	4121.477	*Cr-Co*	6 *d ?*
0	4123.2	4123.384	*La*	12
0	4127.7 } group	16 lines.	....	..
		Intensities	....	..
0	4134.8	2 and greater	....	..
1 *d*	4137.3	4137.156	*Fe*	6
		4137.567	——	2
0 *d*	4140.3	4140.089	*Fe*	6
		4140.558	——	3
0	4142.0	4142.025	*Fe*	4
2 *d?*	4144.0	(4143.919)	*He*	..
		4144.038	*Fe*	15
0	4146.3	4146.225	*Fe*	3
00	4147.9	4147.836	*Fe*	4
1 *d*	4149.3	....	....	..
0	4149.9	....	....	..
00	4152.3	4152.343	*Fe*	3
1	4154.1	4154.071	*Fe*	4
1	4156.2	....	....	..
0	4156.4	....	....	..
0	4157.5	....	....	..
0	4161.7	4161.682	——	4
1	4163.8	4163.818	*Ti, Cr-*	4
1	4167.0 }	4167.438	—	8
1	4167.9			
0	4168.7	....	....	..

TABLE I.—*Continued.*

Flash Intensity	Wave-length	Rowland	Substance	Solar Intensity
		4171.213	*Ti*—	4
2	4171.6	4171.854	*Cr,La,Mn,Ni,Fe*	2
		4172.066	*Ti, Fe*	2
1	4172.4	....	....	..
1	4173.6	4173.624	—	3
		4173.710	——	3
0	4175.3	4175.082	*Fe*	4
0	4175.8	4175.806	*Fe*	5
0	4176.5	4176.739	*Fe-Mn*	5
1	4177.3	4177.698	*Fe*	3
	group	4177.772	——	3
		4178.223	——	2
1	4179.1	4179.025	——	3
0 *d ?*	4180.3	....	....	..
0	4181.8	4181.919	*Fe*	5
0	4184.4	....	....	..
00	4186.2	4186.280	*Ti*	1
0	4187.2	4187.204	*Fe*	6
0	4187.9	4187.943	*Fe*	5
		4188.019	—	3
0	4188.9	4188.894	——	4
0 *d*	4191.7	4191.595	*Fe*	6
		4191.843	*Fe*	3
0	4193.0	....	....	..
0	4194.0	....	....	..
0	4195.3	4195.492	*Fe*	5
1	4198.6	3 lines, 2 *Fe*	....	4, 4, 3
1	4199.2	4199.267	*Zr-Fe*	5
		4200.761	——	1
00 Triple	4200.8	4200.946	*Ti*	1
		4201.089	*Fe*	3
1	4202.2	4202.198	*Fe*	8
00	4204.0	4204.101	*Fe*	3
00	4204.9	4204.916	——	2
0	4205.5	....	....	..
00	4206.1	....	....	..
0	4206.8	4206.862	*Fe*	3
0	4207.7	....	....	..
0	4208.6	4208.766	*Fe*	3
0	4209.1	4209.144	*Zr*	1
0	4209.9	4209.985	*V*	1
0	4210.5	4210.494	*Fe*	4
		4210.561	——	3
0	4211.2	4211.127	——	3*N*
00	4212.0	4212.048	*Zr*—	2
00	4212.8	4212.801	*Cr ?*	3*r*
00	4213.7	4213.812	*Fe*	3
8	4215.7	4215.703	*Sr*	5 *d ?*
0 *d ?*	4217.8	4217.720	*La, Fe-Cr*	5 *d ?*
00	4219.5	4219.516	*Fe*	4
		4219.580	——	3
0 *d ?*	4220.3	4220.509	*Fe*	3

TABLE I.—*Continued.*

Flash Intensity	Wave-length	Rowland	Substance	Solar Intensity
0 *d?*	4222.7	....	....	..
0	4223.4	....	....	..
0	4224.5	4224.337 4224.673	*Fe* *Cr–Fe*	4 3
0 *d?*	4225.6	4225.619	*Fe*	3
g 3	4226.9	4226.904	*Ca*	20 *d?*
0	4231.1	4231.183	*Ni*	4 *N*
3	4233.2	4233.328	*Mn–Fe*	4
0	4235.4	4235.298 4235.450	*Mn* *Mn*	2 3
2	4235.8	4236.112	*Fe*	8
00	4237.2	4237.339	*Fe*	3
1	4238.0	....	....	..
1 *d*	4239.9	4239.890 4240.014	*Mn* *Fe*	3 3
1 *d?*	4243.1	....	....	..
4	4247.0	4246.996	*Sc?*	5
1 *d*	4248.7	....	...	..
2	4250.3	4250.287	*Fe*	8
2	4251.1	4250.945	*Fe*	8
1	4253.1	....	....	..
3	4254.4	4254.505	*Cr*	8
0 *d*	4255.9	....	....	..
0	4256.7	....	....	..
1	4258.4	....	....	..
0 *d?*	4259.5	....	....	..
2	4260.6	4260.640	*Fe*	10
0	4262.2	....	....	..
1 *d*	4263.9	..	....	.
0 *d?*	4266.8	....	....	..
0 *d*	4267.8	....	....	
1	4269.8	....	....	..
1	4271.3	4271.325	*Fe*	6
3	4271.9	4271.934	*Fe*	15
1 *d?*	4273.5	4273.482 4273.643	*Fe* *Zr*	3 *N* 2 *N*
1	4274.3	4274.348	——	2 *N*
3	4274.9	4274.958	*Cr*	7 *d?*
1 *d*	4277.0	4276.836 4277.147	*-Zr* *V-*	2 1 *N*
00	4278.3	4278.390	*Fe–Ti*	3
0	4279.8	4279.874	——	2 *N d?*
0	4281.2	4281.257	*Mn*	2
1	4282.1	4282.127	——	2 *N*
1	4282.6	4282.565	*Fe*	5
1	4286.1	4286.168	*Ti–*	2
1	4287.8	4288.038	*Ti*	2
0	4288.5	....	....	..
2	4289.9	4289.885	*Cr*	5
1	4291.1	4291.114	*Ti*	3
1	4292.1	....	....	..
1 *d?*	4293.3	. .	.	..

TABLE I. – *Continued.*

Flash Intensity	Wave-length	Rowland	Substance	Solar Intensity
2	4294.2	4294.204 4294.301	*Ti* *Fe*	2 5
0	4295.3	4295.383	——	3 *Nd?*
0	4296.1	4295.914	*Cr, Ti*	2
2	4296.7	4296.735	——	3
0	4298.9	4298.828	*Ti*	2
2 *d*	4299.6	4299.410 4299.803	*Ti, Fe* *Ti*	4 2
3	4300.4	4300.211 4300.732	*Ti* *Ti*	3 2
0	4301.3 Group	8 lines. Intensities 2 and greater	....	..
2 *d*	4302.4		....	..
1 *d*	4303.5		....	..
1	4304.1	4304.098	——	4
1	4304.6	....	....	..
2	4305.9	4306.078	*Ti*	4
2	4306.9	....	....	..
G 3	4307.9	4307.907 4308.081	*Ca* *Fe*	3 6
2 *d*	4309.4	....	....	..
2 *d*	4312.3	4312.247 4312.462	—— ——	2 2
3	4312.9	4313.034	*Ti*	3
3	4314.2	4314.248	*Sc*	3
3	4315.2	4315.138 4315.262	*Ti* *Fe*	3 4
1	4317.2	....	....	..
1	4318.9	4318.817	*Ca, Mn?*	4
3	4320.9	4320.907	*Sc*	3
1	4323.9	4324.007	——	3
3	4325.0	4325.152	*Sc*	4
3	4326.0	4325.939	*Fe*	8
0	4427.2	4327.274	*Fe*	3
0	4328.4	....	....	..
0	4329.7	....	....	..
2	4330.9	4330.866	*Ti, Ni*	2
0	4331.8	....	....	..
1	4332.6	....		..
2	4334.2	....	. .	..
0	4335.1	....	....	..
0	4336.1	....	....	..
2 *d*	4337.3	4337.216 4337.725	*Fe* *Cr*	5 3
Hγ 10	4340.6	4340.634	*H*	20 *N*
0	4343.9	4343.861	*Fe*	2
2 *d?*	4344.5	4344.451 4344.670	*Ti-* *Cr*	2 4
00	4347.0	4346.987	*Cr*	1
0 *d*	4347.6	4347.403 4347.705	*Fe* ——	1 1 *N*
3	4351.3	4351.216	*Cr*	3
4	4352.0	4351.930	*Co*	5

TABLE I.—*Continued.*

Flash Intensity	Wave-length	Rowland	Substance	Solar Intensity
2	4353.1	4352.908	*Fe*	4
2	4354.4	....	....	..
2	4355.2	....	....	..
2	4358.7	....	....	..
2	4359.8	4359.874	*Cr*	3
1	4360.6	4360.644	*Ti*	1
1 *d*	4363.2	....	....	..
1	4363.9	....	....	..
0	4364.7	....	....	..
2 *d*	4367.9	{ 4367.749	*Fe*	5
		{ 4367.839	*Ti*	2
1	4369.3	....	....	..
2	4369.9	4369.941	*Fe*	4
1	4371.1	....	....	..
2	4374.7	4374.628	*Sc, Fe ?*	3
2	4375.3	4375.103	*V, Mn*	2
2	4376.2	4376.107	*Fe*	6
1	4377.3	4377.388	——	2 *N*
0	4378.4	4378.419	——	2 *N d ?*
2	4379.4	4379.396	*V*	4
0	4381.2	4381.274	*Cr*	0
4	4383.7	4383.720	*Fe*	15
2	4384.6	4384.873	*V*	3
2	4385.5	4385.548	——	2
1	4387.0	4387.007	*Ti ?*	1
1	4388.5	4388.571	*Fe*	3
0	4389.4	4389.413	*Fe–*	2
0	4390.1	4390.149	*V*	2
1	4391.0	{ 4391.123	*Fe*	2
		{ 4391.192	*Ti*	1
1	4391.9	4391.924	*Cr*	1
5	4395.3	{ 4395.201	*Ti*	3
		{ 4395.413	*V, Zr*	2
1	4397.0	....	....	..
2	4399.8	4399.935	*Ti, Cr*	3
2	4400.6	4400.555	*Sc*	3
0	4401.5	3 lines	*Fe, Fe, Ni*	2, 1, 2
3	4404.8	4404.927	*Fe*	10
1	4407.9	{ 4407.810	*V*	2
		{ 4407.871	*Fe*	4
2	4408.8	{ 4408.364	*V*	2
		{ 4408.582	*Fe*	3
		{ 4408.683	*V*	2
2 *d ?*	4411.0	....	....	..
1	4412.0	....	....	..
3 *d ?*	4415.3	4415.293	*Fe*	8
3	4417.2	....	....	..
3	4418.0	4417.884	*Ti–*	3
0	4422.1	....	....	..
2	4422.9	4422.741	*Fe, Y*	3
1 *d ?*	4424.6	....	....	..
0 *d*	4425.7	4425.608	*Ca*	4

TABLE I.— *Continued.*

Flash Intensity	Wave-length	Rowland	Substance	Solar Intensity
3	4427.7	4427.482	*Fe*	5
1	4430.3	....	....	..
1	4430.7	4430.785	*Fe*	3
2 *d*	4435.4	4435.129	*Ca*	5
		4435.851	*Ca*	4
0	4436.5	4436.516	*Mn*	2
0	4438.0	....	....	..
1	4441.7	4441.881	*V*–	3 *Nd?*
1	4442.7	4442.510	*Fe*	6
5	4444.0	4443.976	*Ti*	5
1	4445.8	....	...	..
1 *d?*	4447.5	4447.302	*Mn, Fe*	2
		4447.892	*Fe*	6
1	4448.3	....	....	..
1 *d?*	4449.6	4449.313	*Ti*	2
3	4450.6	4450.654	*Ti?*	2
1	4452.0	4451.752	*Mn*	3
1 *d*	4453.4	4453.486	*Ti*	2
1	4454.2	....	....	..
3	4455.0	4454.953	*Ca, Zr*	5
00	4456.0	4555.980	*Mn*	2
0 *d*	4457.8	4457.600	*Ti, V, Zr*	2
		4457.712	*Mn*	2
1	4459.1	4459.199	*Ni*	2
0	4460.4	4460.462	*Mn*	1
2	4461.8	4461.818	*Fe*	4
1	4463.3	....	....	..
2	4464.7	4464.617	*Ti?*	2
		4464.844	*Mn*	2
1	4466.7	4466.727	*Fe*	5
3	4468.7	4468.663	*Ti*–	5
8	4471.6	(4471.646)	*He*	..
0	4473.3	....	....	..
1	4476.3	4476.185	*Fe*	4
1	4479.8	....	....	..
0	4482.3	4482.338	–,*Fe*	5
		4482.438	*Fe*	3
1	4483.0	4482.904	*Ti–Fe*	1
0	4484.4	4484.392	*Fe*	4
0	4486.2	....	....	..
1	4489.3	4489.351	——	2
0	4490.2	4490.253	*Mn–Fe*	3 *N*
1 wide	4491.7	4491.570	——	2
		4491.823	*Cr, Mn*	0
1	4494.7	4494.738	*Fe*	6
1	4497.0	4497.023	*Cr*	3
0	4499.1	4499.066	*Mn*	1
3	4501.4	4501.448	*Ti*,–	5
3	4508.5	4508.455	*Fe?*–	4
2	4515.6	4515.508	——	3
1	4517.7	4517.702	*Fe*	3
1	4518.2	4518.198	*Ti*	3

TABLE I.—*Continued.*

Flash Intensity	Wave-length	Rowland	Substance	Solar Intensity
2	4520.3	4520.397	*Fe* ?-	3
3	4522.9	4522.802 4522.974	—— *Ti*	3 2
0	4525.0	4525.314	*Fe*	5
1	4527.5	4527.490	*Ti*	3
1	4529.2	....	....	..
1	4531.4	4531.327	*Fe*	5
5	4534.1	4534.139	*Ti-Co*	6
1 *d*	4535.9	4535.741 4536.094 4536.222	*Ti* *Ti* *Ti*	3 2 2
0	4539.7	....	....	..
1 wide	4541.8	4541.690	*Cr*	2
0	4544.9	4544.864	*Ti*	3
0	4546.1	4546.129	*Fe, Cr*	3
5	4549.9	4549.808	*Ti-Co*	6 *d ?*
5	4554.2	4554.211	*Ba*	8
2	4556.2	4556.306	*Fe-Cr*	4
2	4558.8	4558.827	*Co* ?	3
1	4560.6	....	....	..
5	4563.8	4563.939	*Ti*	4
0	4566.0	4565.842	*Co-Fe*	2
6	4572.2	4572.156	*Ti*-	6
0	4575.0	4574.899	*Fe*	2
1	4576.6	4576.512	——	2
00	4580.2	4580.228	*Cr*	3
4	4584.0	4584.018	*Fe*	4
1	4586.4	4586.408 4586.552	*Cr* *V*	1 1
1	4588.2	4588.381	——	3
2	4590.1	4590.126	——	3
1	4592.7	4592.707 4592.840	*Ni* *Fe*	2 4
0	4593.8	....	....	..
0	4594.3	4594.297	*V*	2 *N*
0	4595.7	....	....	..
0	4596.2	4596.245	*Fe*	2
0	4598.3	4598.303	*Fe*	3
2	4600.2	....	....	..
2	4601.0	4600.932	*Cr*	3
1	4602.1	4602.183	*Fe*	3
1	4603.1	4603.126	*Fe*	6
1	4607.7	4607.831	*Fe*	4
1 *d ?*	4611.3	4611.368 4611.469	*Cr* *Fe*	0 5
2 *d*	4613.5	4613.386 4613.544	*Fe* *Cr, La*	3 3
1	4615.8	....	....	..
1	4616.4	4616.305	*Cr*	4
2	4619.1	4618.971	*Fe*-	4 *d ?*
0	4619.9	4619.711	*Cr*	1
1	4620.6	....	....	..

TABLE I — *Continued.*

Flash Intensity	Wave-length	Rowland	Substance	Solar Intensity
1	4622.0	4622.065 4622.128	*Cr* *Cr*	0 1
1	4622.7	4622.627	*Cr*	1
1	4623.4	4623.279	*Ti*	2
1	4625.2	4625.227	*Fe*	5
2	4626.4	4626.358	*Cr*	5
1	4627.7	....	....	..
3	4629.5	4629.521	*Ti-Co*	6
0	4630.2	4630.306	*Fe*	4
2	4634.2	4634.254	——	2
0	4636.2	....	....	..
2	4637.7	4637.685	*Fe*	5
2 *d*	4639.8	3 *Ti* lines	....	2, 2, 1
2	4646.3	4646.347	*Cr*	5
1	4647.7	4647.617	*Fe*	4
1	4648.9	4648.835	*Ni*	4
2 *d*	4651.9	4651.461 4652.343	*Cr* *Cr*	4 5
2 *d*	4654.7	4654.672 4654.800	*Fe* *Fe*	4 5
1	4656.5	4656.644	*Ti*	3
2	4657.3	4657.380	*Ti ?*	2
1	4662.4	....	....	..
1	4663.8	....	....	..
2	4667.4	4667.626 4667.768	*Fe* *Ti*	4 3
2	4670.3	....	....	..
0	4678.4	....	....	..
1 *d?*	4680.4	....	....	..
2	4682.6	....	....	..
2	4685.3	4685.452	*Ca*	2 *N*
0	4691.6	4691.602	*Fe*	5
0	4698.6	3 lines, 2 *Cr*, 1 *Ti*	....	1, 1, 1
0	4700.3	4700.337	——	4
0	4701.3	....	....	..
1 1	4702.9 4704.8 group	4703.177 4703.994 4705.131	*Mg* *Ni* *Fe*	10 3 4
1 1	4707.4 4710.0 group	6 lines, intensities 2 and greater	....	.. .
3	4713.3	(4713.252)	*He*	..
1	4714.7	4714.599	*Ni*	6
1 wide	4727.9	4727.582 4727.676 4728.732	*Fe* *Mn* *Fe*	3 2 4
2	4731.6	4731.651	*Fe ?*	4
00	4733.6	4733.779	*Fe*	4
2	4737.0	4736.963	*Fe*	6
1 *d*	4740.9	....	....	..
00	4746.0	4745.992	*Fe*	4
0	4748.8	....	....	..
1 1	4761.8 4766.6 group	5 lines. Intensities 3 and greater	4 *Mn*, 1 *Ti*	..

TABLE I — *Continued.*

Flash Intensity	Wave-length	Rowland	Substance	Solar Intensity
1 *d?*	4768.5	{ 4768.519 { 4768.595	—— *Fe*	3 2
1	4771.4	4771.664	——	3
1	4773.0	4773.007	*Fe*	4
00	4776.7	....	....	..
2	4780.2	4780.169	*Co*	2
2	4783.7	4783.613	*Mn*	6
2	4786.8	{ 4786.727 { 4787.003	*Ni* *Fe*	3 2
2	4789.5	4789.528	*Cr*	2
1	4792.6	4792.702	*Ti-Cr*	2
2	4805.4	4805.285	——	3
00	4810.9	4810.724	*Zn*	3
2	4824.0	{ 4823.697 { 4824.325	*Mn* *Fe*	5 3
1 wide	4840.4	4 lines	....	3, 2, 3, 3
00	4843.8	....	....	..
1	4848.4	4848.438	——	2
2	4855.6	....	....	..
Hβ 10	4861.5	4861.527	*H*	30
2	4867.7	....	....	..
2	4871.2	....	....	..
1	4890.7	4890.948	*Fe*	6
1	4891.6	4891.683	*Fe*	8
2	4899.7	....	....	..
0 wide	4903.4	4903.502	*Fe*	5
0 wide	4910.4	4 *Fe* lines	....	2, 3, 2, 2
1	4918.2	....	..	..
1	4920.2	4920.685	*Fe*	10
3	4924.1	4924.107	*Fe*	5

In comparing wave-lengths of the flash spectrum with those of the solar spectrum, it is necessary to bear in mind two points: first, we are dealing with a dispersion of about one-fifth of Rowland's, with the focus not so exact, and therefore it will be impossible to separate in the flash the counterparts of close dark solar lines; second, the emission and absorption lines are formed at different heights above the Sun's surface, and the emission lines will, as a result, be shifted (in this flash) towards the violet.

The focus is best between F and H, and for the present purpose this region only will be considered. Neglecting *H* and *He* lines and those lines identified with groups, 374 lines were measured in the flash between F and H. Ninety-one of these were

unidentified, and 283 were identified with lines in the solar spectrum.

The results of these comparisons are put down in Table II, arranged according to their intensities (scale 0–10.)

TABLE II.

LINES IN THE FLASH BETWEEN F AND H.

Scale 0–10.

Element	∞	0	1	2	3	4	5	6	7	8	Total	Average Intensity
Fe	11	39	37	28	8	2	—	—	—	—	125	1.70
Ti	2	7	22	15	10	—	5	1	—	—	62	1.84
Cr	3	9	10	12	3	1	—		—	—	38	1.24
Mn	4	9	9	4	1	—	—	—	—	—	27	0.74
V	—	5	4	5	—	—	1	—	—	—	15	1.27
Zr	1	3	3	—	1	—	1	—	—	—	9	1.22
Ni	—	1	6	2	—	—	—	—	—	—	9	1.11
Ca	—	1	1	3	3	—	—	—	—	—	8	2.00
Sc	—	—	—	2	3	1	—	—	—	—	6	2.83
Co	2	2	1	1	—	—	—	—	—	—	6	0.50
Y	—	—	1	1	—	—	—	—	—	—	2	1.50
La	—	2	—	1	—	—	—	—	—	—	3	0.67
Sr	—	—	—	—	—	—	—	1	—	1	2	7.00
Ba	—	—	—	—	—	—	1	—	—	—	1	5.00
Mg	—	—	1	—	—	—	—	—	—	—	1	1.00
Zn	1	—	—	—	—			—	—	—	1	0
Ce	1	—	1	—	—	—	—	—	—	—	2	0.50
——	4	16	17	8	1	—	—	—	—	—	46	0.78
Unidentified	3	37	39	11	1	—	—	—	—	—	91	0.70
Total	32	131	152	93	31	4	8	2	0	1	454	

In the column of Elements, — means that these lines are unidentified in Rowland's tables.

(The total includes eighty lines identified with more than one element, either in Rowland's tables or in the comparisons between flash and solar spectra.) The average intensities are given in the last column.

Two points are immediately noticed in comparing the two spectra: first, for each and every element, the brighter the solar line the brighter the flash line corresponding to it; second, the intensities of the solar lines which correspond to a line of given

brightness in the flash differ with different metals. *Fe* and *Ni* lines of intensity 5, *Ti*, *Sc*, and *V* lines of intensity 2 are identified with flash lines of equal strength. These differences for the various elements were so marked that in order to arrive at their significance, and hence draw some conclusions regarding the "reversing layer," comparisons were made between the flash and the solar spectrum.

The 283 lines identified with lines in Rowland's tables are arranged according to their intensities (scale 1–1000), in Table III.

TABLE III.

SOLAR LINES WITH WHICH THE FLASH LINES WERE IDENTIFIED.

Scale 1–1000.

Element	0	1	2	3	4	5	6	7	8	10	12	15	20	30	Total	Average Intensity
Fe	—	3	11	28	33	22	12	2	6	2	..	4	1	1	125	4.02
Ti	—	11	23	13	8	3	4	..	..	..	..	..	..	..	62	2.69
Cr	4	8	3	13	3	4	1	1	1	..	..	..	..	..	38	2.27
Mn	—	3	9	5	5	3	2	..	..	..	..	..	..	..	27	3.08
V	—	5	8	1	1	..	..	..	..	..	..	..	..	..	15	1.87
Zr	—	1	5	1	—	2	..	..	..	..	..	..	..	..	9	2.67
Ni	—	—	4	2	2	—	1	..	..	..	..	..	..	..	9	3.11
Ca	—	—	1	1	3	2	..	..	..	..	..	..	1	..	8	5.88
Sc	—	—	—	4	1	1	..	..	..	..	..	..	..	..	6	3.50
Co	..	..	2	1	2	1	..	..	..	..	..	..	..	..	6	2.07
Y	..	..	..	2	..	..	..	..	..	..	..	..	..	..	2	3.00
La	..	..	..	1	—	1	..	..	..	..	1	..	..	..	3	6.07
Sr	..	..	..	..	..	1	..	..	1	..	..	..	..	..	2	6.50
Ba	..	..	..	..	..	..	..	..	1	..	..	..	..	..	1	8.00
Mg	..	..	..	..	..	..	..	..	..	1	..	..	..	..	1	10.00
Zn	..	..	..	1	..	..	..	..	..	..	..	..	..	..	1	3.00
Ce	..	..	..	1	..	1	..	..	..	..	..	..	..	..	2	4.00
—	..	3	21	17	4	..	..	..	..	..	..	..	..	..	46	2.44
Total	4	34	87	91	62	41	20	3	10	3	1	4	2	1	363	

Table IV gives all the lines in Rowland's tables having an intensity of 2 and greater, arranged according to scale of intensities (1–1000).

There are 874 lines having an intensity of 2 and over (41 of the 915 being identified with more than one element). 657 of these lines have an intensity less than 4.

TABLE IV.

LINES IN ROWLAND'S MAP BETWEEN H AND F. SCALE 1–1000.

Intensities 2 and greater.

Element	2	3	4	5	6	7	8	10	12	15	20	30	Total
Fe	135	108	75	40	15	2	7	3	0	4	1	1	391
Ti	42	24	15	3	4	..	..	..	..	..	..	..	88
Cr	22	19	6	5	1	1	1	..	..	..	..	..	55
Mn	24	14	8	6	3	1	..	..	..	..	..	..	56
V	12	7	2	—	1	..	..	..	..	..	..	..	22
Zr	9	2	—	2	..	..	..	..	..	..	..	..	13
Ni	18	10	3	--	1	..	..	.	..	..	..	..	32
Ca	4	5	9	2	..	..	..	..	..	..	1	..	21
Sc	1	5	1	1	..	..	..	..	..	..	..	..	8
Co	9	4	5	2	1	..	..	..	..	..	..	..	21
Y	—	2	1	..	..	..	..	..	..	..	..	..	3
La	4	1	1	1	..	..	..		1	..	..	..	8
Sr	-	1	—	1	—	—	1	..	..	..	.	..	3
Ba	1	..	..	..	..	.	1	..	..	..	..	..	2
Zn	..	2	.	..	..	..	..	..	..	..	..	..	2
Nd	4	1	..	..	..	..	..	..	..	..	..	..	5
Si	..	..	..	1	..	..	..	..	..	..	..	..	1
C	4	..	..	..	..	..	..	..	..	..	..	..	4
Mg	..	..	..	2	..	..	..	1	..	..	..	..	3
Ag	..	1	..	..	..	..	..	..	..	..	.	..	1
Cd	..	1	..	..	..	..	..	..	..	..	..	..	1
Ce	3	2	—	1	..	..	..	..	..	..	..	..	6
---	120	36	9	1	1	1	1	..	..	..	..	..	169
Total	412	245	135	68	27	5	11	4	1	4	2	1	915

Although we cannot directly compare the intensities of the bright lines of the flash (scale 0–10) with those of the dark lines given in Rowland's tables (scale 1–1000), we arrive at certain theoretical considerations if we compare the ratios of the average intensities of the different elements, that is, $\frac{\text{Flash intensities}}{\text{Solar intensities}}$, and also the ratio of the number of lines of each element identified to the whole number of solar lines for that metal. Forming these ratios and arranging them, we are at once struck with the systematic variations not only in the ratio of intensities, but also in the percentage of lines identified.

The meaning of these systematic differences will be understood if we consider these ratios in combination with the atomic

weights of the various elements, as is done in Table V, where also are put down the number of lines of the flash due to each metal.

TABLE V.

GROUP I. LINES STRONG IN FLASH AND IN SOLAR SPECTRUM.

Element	Atomic Weight	Number of Lines Identified	Intensity Flash / Intensity Solar Lines	Number of Lines Identified / Total Number of Lines
Na	23.0	—	--	-
Mg	24.3	1	0.10	1.00
Al	27.1	—	--	—
Ca	40.0	8	0.34	0.38

GROUP II. LINES STRONG IN FLASH, WEAK IN SOLAR SPECTRUM.

Element	Atomic Weight	Number of Lines Identified	Intensity Flash / Intensity Solar Lines	Number of Lines Identified / Total Number of Lines
Sc	44.1	6	0.81	0.75
Ti	48.1	62	0.68	0.70
V	51.2	15	0.68	0.67
Cr	52.1	38	0.55	0.69
Mn	55.1	27	0.24	0.48
Sr	87.6	2	1.08	0.67
Y	88.7	2	0.50	0.67
Zr	90.6	9	0.46	0.70

GROUP III. LINES WEAK IN FLASH, STRONG IN SOLAR SPECTRUM.

Element	Atomic Weight	Number of Lines Identified	Intensity Flash / Intensity Solar Lines	Number of Lines Identified / Total Number of Lines
Fe	56.0	125	0.20	0.32
Ni	58.7	9	0.36	0.28
Co	59.0	6	0.19	0.29

Looking at the numbers in the last two columns, we see that the lines naturally fall into three groups as given in the above table.

To these may also be added the following lines:

La, atomic weight 138.5, 3 lines at λ 4123.384, λ 4217.720 and λ 4613.544.
Ba, atomic weight 137, 1 line, at λ 4554.211; and the following lines possibly identified:
Si, atomic weight 12, 1 line, at λ 3905.660.
Zn, atomic weight 65, 1 line, at λ 4810.724.
Cr, atomic weight, 92, 2 lines at λ 4003.912 and λ 4107.649.

In Group I would also fall *Al*, if we consider the relative intensities of the two lines λ 3944.160 and λ 3971.674; and undoubtedly *Na*, if our plate took in the D lines.

The grouping of these lines is exactly that adopted by Evershed from his investigations of the Indian eclipse, except that I have put *Zr* with *Sr* and *V* in Group II. *Mn* seems to represent the transition from Group II to Group III.

Sir William and Lady Huggins[1] called attention to the great heights to which *Ca* extends in the Sun's atmosphere, and it is on account of this great extent that H and K are such prominent lines not only in the absorption spectrum, but in the emission spectrum. As Evershed[2] has pointed out, the remarkable variations of the relative intensities in the flash and Fraunhofer spectra are undoubtedly due to the heights to which the vapors of the different metals ascend in the chromosphere. We would naturally expect that these heights vary according to the atomic weights of the metals, those of least atomic weights ascending to the greatest distances; and generally speaking this no doubt is true. But if we have two gases in the Sun's atmosphere, one a gas with an intrinsic brightness 1 and a layer 100 miles in thickness, it would give a photographic line in the flash spectrum just as bright as the other gas of intrinsic brightness 100 and only 1 mile thick, if the Sun and Moon were relatively at rest during the period of the "flash;" but considering the gradual advance of the Moon in covering successive layers of the Sun's atmosphere, we see that in the emission spectrum the photographic brightness of the fainter gas would be many times that of the brighter. The absorption caused by a gas depends on the total number of molecules the solar ray comes in contact with, and will be very nearly equal in the two cases.

In view of these considerations, it would therefore seem that the gases of the metals of Group II extend very high, that they are nowhere very much condensed, and that practically all the gas contributes to the formation of the emission line; and hence

[1] SIR WILLIAM and LADY HUGGINS, "The relative behavior of the H and K lines of the spectrum of calcium," ASTROPHYSICAL JOURNAL, **6**, 77, 1897.

[2] EVERSHED, "Wave-length determinations and general results obtained from a detailed examination of spectra photographed at the solar eclipse of January 22, 1898." *Phil. Trans. Royal Society*, A, **197**, 381–413. 1901.

the flash lines are to be regarded as true reversals of the corresponding solar lines.

The vapors of Groups I and III are somewhat condensed near the Sun's surface (those of Group I, particularly *Ca*, reach far greater heights than those of Group III), but as it is the upper portions that contribute most to the formation of the emission lines, owing to the progressive motion of the Moon, the flash lines are to be regarded as only partial reversals of the Fraunhofer lines, the solar intensities being greater than the flash intensities.

UNKNOWN LINES.

Taking account of lines in the flash identified with groups in the solar spectrum, about half the lines in Table IV have corresponding lines in the flash. From the above considerations we see that it is highly improbable that lines of intensity 2 in the solar spectrum, and belonging to Groups I and III, will have flash lines corresponding to them of sufficient brightness to show in this flash. In fact, by reference to Tables III and IV, we see that although there are 135 *Fe* lines of intensity 2, only 11 of these are found in the flash, and indeed, great numbers of the feebler solar lines are lacking in the flash. But if, on the other hand, we compare the stronger lines, we see that every strong line of the solar spectrum is almost without exception found in the flash spectrum.

And so, remembering the meaning of the differences of intensities, we see no reason for giving up our faith in the existence of the "reversing layer."

We may obtain an approximate estimate of the depths of the layers producing the bright arcs by measuring the angular extent of the arcs. Accordingly the lengths of some of the more conspicuous bright lines have been measured, and thence were deduced the elevations of the luminous layers producing the bright lines of the flash spectrum. In calculating, the semi-diameter of the Sun was taken as 948.″4, and the Moon's augmented semi-diameter 1013.″8. For the purposes of comparison,

the same arcs were taken that Frost has measured. The depths of the luminous layers of the various metallic vapors come out as follows:

TABLE VI.

Spectrum line....	*H* 3970	*He* 4026	*Sr* 4078	*H*δ 4102	*Sr* 4215	*Ca* 4226	*Sc?* 4247	*Fe pair* 4250	*Cr* 4254
Approximate height of layer.	8″	4″	4″	7″	4″	2″.5	2″.5	<½″	2″.5
Spectrum line....	*Cr* 4274	*Sc* 4321	*H*γ 4340	*Ti* 4395	*He* 4471	*Ti* 4501	*Ti* 4549	*Ba* 4554	*H*β 4861
Approximate height of layer.	2″	2″	8″	2″	7″	2″	2″	2″.5	8″

Comparing these heights with the intensities given in my scale (0—10), it is seen that roughly speaking the heights in seconds of arc is 0.8 of the value of the intensity for Group II, and 0.4 for Groups I and III. The arcs of the great majority of the lines are no longer than the *Fe* pair at λ 4250, which correspond to an extent of ½″.

As a result we may safely infer that the average depth of the "reversing layer" is about 1″, although from the above considerations we see that the heights to which the gases extend and their condensations are different for the different elements.

These results differ materially from those of Sir Norman Lockyer given in his *Recent and Coming Eclipses*, p. 111. The numbers of lines photographed at the eclipses of 1893 and 1896 are there given by him as 164 and 464, respectively. As there are 5694 lines in the same region in Rowland's map, he decides that only 3 and 8 per cent. respectively of the solar lines are reversed in the flash at these two eclipses. But as Professor Frost has pointed out,[1] the instruments employed were capable of photographing only a small fraction of the 5694 lines of Rowland's map.

Sir Norman lays great stress on the fact that great numbers of "enhanced" lines, or lines stronger in the spark than in the

[1] ASTROPHYSICAL JOURNAL, 12, 346, 1900.

arc spectrum, are found in the spectrum of the chromosphere. In order to investigate this idea, close comparisons were made between the above flash lines and the latest list of "enhanced" lines given by Lockyer in *Proc. R. S.*, **65**, 452, 1900.

Taking up first titanium, as the "enhanced" lines for this metal are most numerous, it is found that he has given fifty-three lines between λ 3900 and λ 4590. This region is all included in the photographs of the flash. Thirty out of the fifty-three "enhanced" lines certainly appear as bright lines in the flash, eleven do not appear, the other twelve are doubtful from their proximity to strong flash lines, or from their being situated in a group in the flash. Thus 56 per cent. of the "enhanced" lines of *Ti* are found as chromosphere lines; and this would seem to strongly support Lockyer's views. But, on the other hand, every one of these thirty lines, without exception, appears as a strong line in the solar spectrum; and the coincidence between "enhanced" and flash lines does not prove anything definite, for where there are strong Fraunhofer lines we expect strong reversals in the flash. A real test would be the case where there is a strong "enhanced" line, but no strong solar line corresponding to it. Such a line occurs in titanium at λ 4308.60, where the intensity in the spark is 7, and on the same scale in the arc 1–2. There is a line in Rowland's tables at λ 4308.601 with an intensity of 00, but there is certainly no exceptionally strong flash line which we would be led to expect if Lockyer's idea is correct. The appearance of this line may be contrasted with that at λ 4563.94. Both have the same intensity in the spark (*loc. cit.*), but to λ 4563.94 corresponds a strong line of intensity 4 on Rowland's scale, and a strong reversal in the flash of intensity 5 on my scale.

The conclusions from the "enhanced" lines of iron agree with those for titanium, *i. e.*, there are a great number of strong lines in the flash spectrum corresponding to the "enhanced" lines, but they also correspond to strong Fraunhofer lines.

A severe test of the idea that "enhanced" lines are found with great frequency in the spectrum of the chromosphere will

be given by the metal vanadium. Lockyer gives twenty-five lines between λ 3885.05 and λ 4243.10, the majority of which are decidedly stronger in the spark than in the arc. Only four of these lines are with certainty found in the flash, corresponding to Lockyer's lines at λ 4005.85, λ 4035.80, λ 4225.41, and λ 4243.10. The first two of these have corresponding strong solar lines; the second two have not; none of the four lines, however, are identified by Rowland as due to vanadium. Three out of nineteen "enhanced" lines of manganese are found in the flash.

From these comparisons it would seem that there is no close connection between "enhanced" lines and the bright lines of the chromosphere seen in the flash.

Lockyer's latest measures may perhaps serve to find the chemical origin of some of the lines unassigned by Rowland. The following lines, unidentified in Table I, may therefore be assigned to different metals. The numbers in (1) are the intensities given in Rowland's tables.

To *Fe* can be assigned the lines at: λ 4179.025 (3); 4296.735 (3); 4385.548 (2); 4489.351 (2); 4491.570 (2); 4508.455, *Fe ?–* (4); 4515.508 (3); 4520.397, *Fe ?–* (3); 4522.802 (3); 4576.512 (2).

To *Ti*, the lines at λ 4161.682 (4); 4173.710 (3); 4184.472 (2); 4590.126 (3).

To *Cr*, λ 4588.381 (3); λ 4634.254 (2); and to *V* the line λ 4005.856.

COLUMBIA UNIVERSITY,
New York City,
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THE EFFECT OF SODIUM ON THE HYDROCARBON BANDS IN THE SPECTRUM OF THE BUNSEN FLAME.

By PERCIVAL LEWIS.

SCHEINER, in his *Astronomical Spectroscopy* (Frost's translation, p. 217), makes the following statement:

> If sodium vapor be introduced into a Bunsen burner which is giving a fine hydrocarbon spectrum, the latter will not be in the least diminished, but will appear as intense as before, the sodium line simply appearing in addition.

In discussing the causes of the relative changes in intensity of the D lines and the hydrocarbon spectrum of Comet 1882 I, as it approached the Sun, he cites the above fact as evidence of the electrical origin of the luminosity of this and other comets.

It seemed worth while to the writer to put this statement to an accurate photometric test, as it was probably based merely on eye estimation. The first experiment was made by fitting a thin asbestos cylinder in the tube of a Bunsen burner, and measuring the intensity of the green hydrocarbon band before and after the asbestos was moistened with a salt solution. The measurements were made with a Glan spectrophotometer, an incandescent lamp being used as a standard of comparison. The first results seemed to confirm Scheiner's opinion, but it seemed very possible that this was due to the fact that the sodium vapor ascended on the outside of the green cone and did not mix with the radiating gas within the cone. On tipping the burner until the tube was nearly horizontal, so that the sodium vapor must ascend through the cone, a very noticeable effect was perceived, manifesting itself chiefly in a great strengthening of the continuous background which always accompanies the sodium spectrum. The positions occupied by the carbon bands underwent no change noticeable to the eye,

but in properly estimating their real intensity deduction must be made of the intensity of the continuous background, determined as nearly as possibly by taking the average of the intensities on both sides of the band.

Below are several series of measurements, taken at different times. The intensities given are merely relative, corresponding parts of the standard source being taken as 100.

Experiment	+ or − Sodium	Apparent I	Background	Real I
1	−	81	8	73
	+	81	18	63
2	−	81	9	72
	+	76	34	42
3	−	71	9	62
	+	56	19	37
4	−	45	7	38
	+	38	16	22
	−	36	10	26
	+	28	14	14

The fourth set of experiments was made with a burner of the fish-tail type, placed in a vertical position, the edges being moistened with salt solution. In this case, on account of the thinness of the flame, there was an opportunity for the sodium vapor to mix with the gas.

The diminution in the intensity of the green band is very evident, but somewhat irregular, as one might expect from the impossibility of gauging the amount of sodium vaporized. A quantity of sodium merely sufficient to color the flame uniformly will produce no measurable change; the flame must be almost saturated.

Measurements were not made on the other bands, on account of their feeble intensity and the resulting difficulty of measurement.

On account of the relative weakness of the carbon spectrum, the comparison source was also made weak, hence any accurate measurement of the far greater intensity of the D lines was subject to great error; nevertheless one series of such rough estimations was made, with the following result:

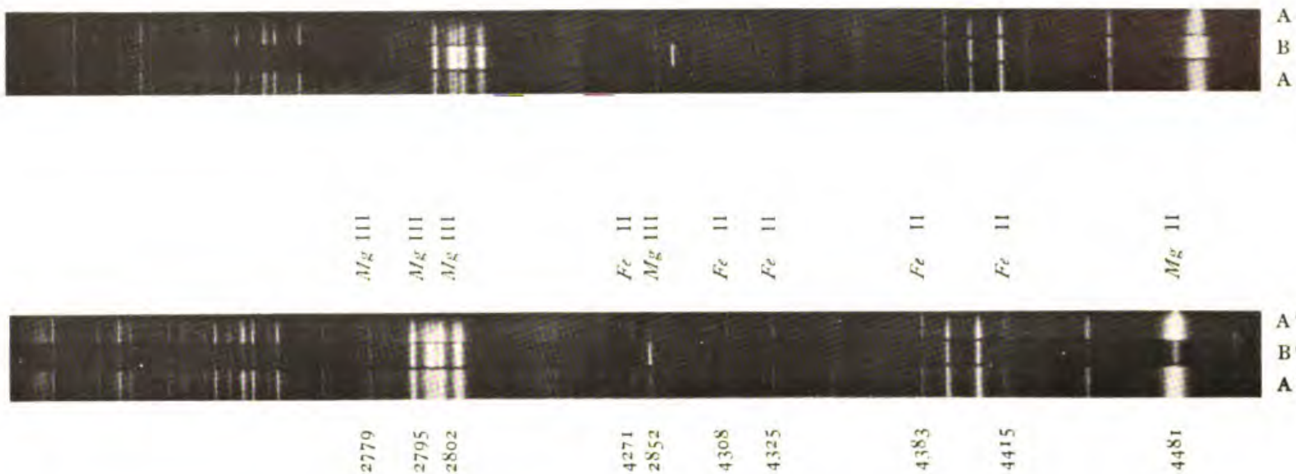
D Lines	Apparent I green band	Background	Real I green band
20	80	8	72
140	80	10	70
600	75	10	64
1200	38	12	26

In this experiment the background was relatively weak and the measurements of it are subject to error.

These facts may not lessen the probability of the electric origin of cometary radiation, but they indicate that there is no apparent difference between the effects produced on the hydrocarbon spectrum by sodium vapor in the arc or vacuum tube and that in the flame, so far as relative intensities are concerned.

UNIVERSITY OF CALIFORNIA,
January 30, 1902.

PLATE V.



DOPPLER EFFECT AND REVERSAL IN SPARK SPECTRA.

Photographed by JOHN FRED MOHLER.

THE DOPPLER EFFECT AND REVERSAL IN SPARK SPECTRA.

By JOHN FRED MOHLER.

SOME time ago I measured the pressure in the electric spark,[1] under certain conditions, using the shift of the lines in the spectra produced as an index. Various causes for this pressure have been advanced, one by Eduard Haschek, who[2] attributes this pressure to the fact that the particles driven off from the electrodes have a very high velocity, and this velocity stopped by collisions with other particles produces pressure. Schuster[3] has given a value to this velocity, having measured it in the spectrum of a spark photographed with a moving film. His values for the zinc lines are from 2,000 to 900 meters per second. Since this work was begun Dr. Schenck[4] has shown that the velocity for some magnesium lines is greater than the values Schuster obtained for zinc. Dr. Schenck used a revolving mirror, and his results show a maximum velocity of 2.5 kilometers per second for the magnesium line at λ 4481.

This work was begun more than a year ago. My plan was to get the spectra of certain metals when the particles were driven away from the slit of the spectroscope, and then get the same lines when the particles were driven toward the slit. The electrodes were of different metals placed in a line perpendicular to the plane of the slit; and after the spectrum was obtained in this position, they were turned 180°, reversing the direction of the particles projected from the electrodes, and this spectrum was photographed alongside of the other. Displacement due to motion in line of sight would thus be doubled and the displacement due to this cause would be in the opposite directions for the lines of the different electrodes.

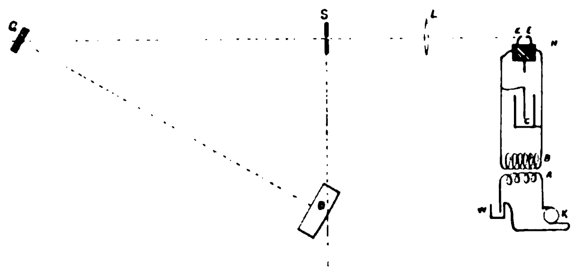
[1] ASTROPHYSICAL JOURNAL, 10, 202, 1899.

[2] *Ibid.*, 14, 181, 1901.

[3] *Nature*, 57, 17, 1897.

[4] ASTROPHYSICAL JOURNAL, 14, 116, 1901.

The diagram of the apparatus will make clear the method. *A* is the primary of an induction coil supplied with current from a 110-volt generator *K*, through a Wehnelt break *W*. The coil thus excited would give a shower of sparks 18 cm long. The secondary *B* was connected to the electrodes *e e* and Leyden jar *C* of 0.0025 microfarads capacity (22.5 meters). Thus arranged, the spark gap was not over 9 mm long, and in most cases the gap was 6 or 7 mm. For part of the work a mercury break was used, but as the Wehnelt break would give good photographs with 5 to 10 minutes exposure and the mercury



break required 30 to 60 minutes, the former was used for most of the work.

The wires used for electrodes were small and arranged as in figure. The block *H* of hard rubber furnished good insulation and permitted the electrodes to be quickly revolved 180°. By slightly inclining the electrodes the wire itself did not obscure the light of the spark. The discharge was oscillatory with this arrangement, and very noisy. *G* is an excellent 4-inch grating of ten feet radius, ruled 14,400 lines to the inch. The work was done in the second, third, and fourth order of spectra. The grating is mounted in a basement on solid masonry pillars and the temperature was kept very constant. The induction coil, break, and resistance for controlling the current, were in the room above the one occupied by the grating. The spectra were photographed in the usual way.

RESULTS.

The most noticeable results were not the displacement of the lines due to the Doppler effect, but, in the case of some of the magnesium lines, the very marked difference in the *character* of the lines. When the spark from the magnesium was going *away* from the slit, the lines at λλ 2795, 2802, and 2852 would be very strongly reversed, similar to the appearance of the line at λ 2852 in the arc, and when the spark was coming toward the slit from the magnesium electrode the lines would be without reversal. This was not due to difference in time of exposure or intensity of the lines, as the photographs themselves plainly show. In much less degree this effect is noticed in the line at λ 2779. When the spark passes in a direction at right angles to the axis of the spectroscope the reversal is still noticed, but is not nearly so marked. Reversing the direction of the current through the primary produced no difference in the effect, for, as stated above, the discharge from the Leyden jar was oscillatory. Motion in the line of sight produces a very small displacement, so small in fact that it is near the limit of measurement with the instruments at my disposal. The displacement of the lines was measured with an eyepiece micrometer made by Bausch & Lomb Optical Co., in connection with a low power objective.

The lines which gave Dr. Schenck his large value of 2.5 kilometers per second as the maximum velocity are far too wide and shaded to be measured accurately. From this maximum the velocity fell off rapidly until a little short of the middle of the spark it was probably only one-tenth as great. If the spark is observed exactly end on, the average of this variable velocity would be measured, but by inclining the spark just a little, light from near the electrode only would illuminate the slit. However, the sparks spread out somewhat in a sheaf so that it was impossible to entirely isolate the one point in the spark near the electrode, and on all plates some of the lines from the other electrode would appear, showing that the light on the slit was from different parts of the spark. What I get, then, is probably

the average velocity of the particles in the spark. The measurements of the plates show that the aluminium lines at $\lambda\lambda$ 3961 and 3944 give an average measured displacement of 0.01 tenth-meters. As this is double the actual displacement from normal position the velocity would be 0.37 km per second. The results for the iron lines at $\lambda\lambda$ 4063, 4071, 4260, 4271, 4308, 4325, 4383, and 4415 give slightly smaller velocity. The magnesium lines at $\lambda\lambda$ 2776, 2778, 2779, and 2781 gave about the same result as the iron lines, and the cadmium lines at $\lambda\lambda$ 4078 and 4800 showed a slightly larger velocity.

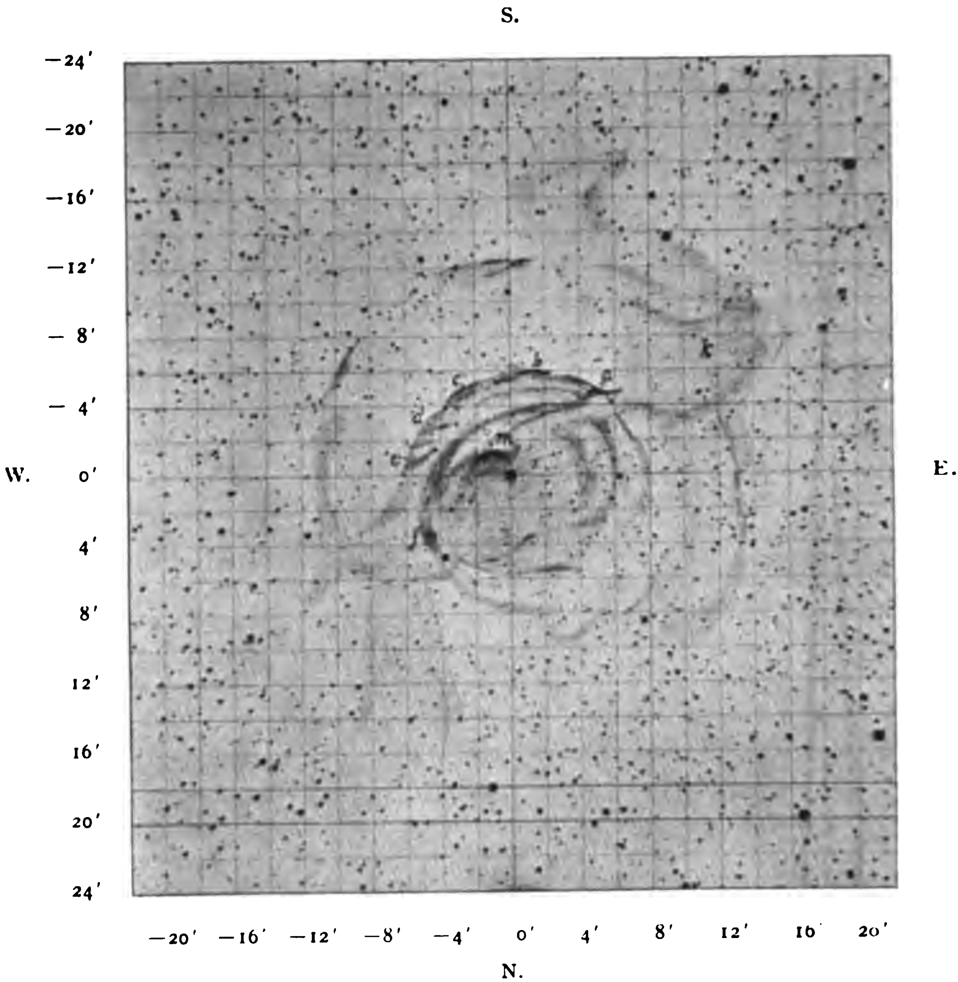
The work is still in progress and I hope to add to the results given above.

EXPLANATION OF THE PLATE.

The spectra *A* were taken when the magnesium was sparking away from the slit and the iron was sparking toward the slit. The spectra *B* were taken with positions reversed. In the second case the *B* spectrum was exposed a shorter time to show that the lack of reversal was not due to long exposure. The Roman numerals refer to the order of spectrum.

DICKINSON COLLEGE,
January 15, 1902.

PLATE VI.



NEBULOSITY ABOUT *NOVA PERSEI*, SEPTEMBER 20, 1901.

Photographed with the Two-Foot Reflector, Yerkes Observatory.

Exposure 3^h 50^m.

NEBULOSITY ABOUT *NOVA PERSEI.*

RECENT PHOTOGRAPHS.

By G. W. Ritchey.

Up to the present time thirteen negatives of the nebula about *Nova Persei* have been obtained with the two-foot reflector, as follows:

Date.	Exposure.	Remarks.
September 20.....	$3^h\ 50^m$	Seeing fine.
November 9.....	1 30	Seeing good. Exposure stopped by clouds.
November 13.....	7 0	Seeing good.
November 20. ..	3 0	Seeing good. Exposure stopped by clouds.
November 30.....	3 0	Seeing poor. Exposure stopped by clouds.
December 4.....	1 45	Seeing poor. Exposure stopped by clouds.
December 14.....	4 30	Seeing poor. Night extremely cold and transparent.
January 2 and 3...	10 0	Seeing poor.
January 7 and 9...	4 30	Seeing fine.
February 8......	2 40	Seeing good.
February 10......	4 30	Seeing poor.
February 25......	1 30	Seeing good. Exposure stopped by clouds.
March 4 and 5....	3 15	Seeing good. Exposure stopped by clouds.

On account of the wide-spread interest in this nebula and in the theories which have been suggested concerning it, it has been thought best to publish now the diagrams which I have prepared from the best five of the negatives, together with a very brief description. These diagrams show the continued changes which have taken place in the principal ring of nebulosity, and include also the more conspicuous of the faint nebulosities outside of the principal ring. These outer nebulosities are in most cases so faint that they would not be recognized with certainty as real nebulosity, in the examination of a single negative; by studying the entire series, however, it is possible to trace these excessively faint outlying masses with considerable certainty, and to follow the apparent changes of position in some of them.

The faint details shown in the diagrams are in nearly every case present in more than one of the negatives. The positions of

these outer nebulosities, and the changes which have occurred in them, are very suggestive in connection with the theories which have been advanced regarding the apparent expansion of the nebula.

Measures and a discussion of the entire series of negatives, including those of March and April (after which time this object will be inaccessible for photographing for several months), will be published soon.

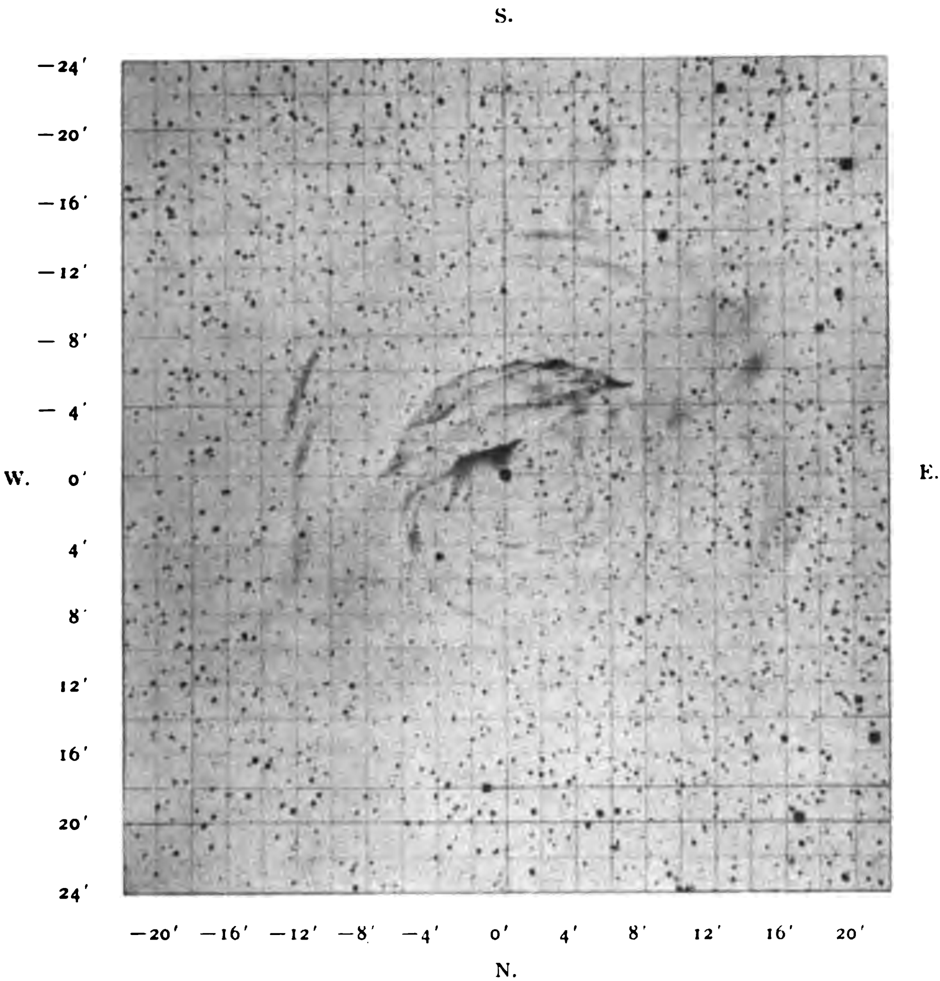
The principal ring of nebulosity, shown in my earlier drawings,[1] has continued to fade rapidly. Only the condensations *a* and *b* remain conspicuous on the later negatives. The parts of the principal ring to the southwest of the *Nova* have become so faint that they can be seen only with the greatest difficulty on the negatives obtained in February and March. The strong condensation *m*, close to the *Nova*, has apparently not changed in brightness since November 9. Its form is changing slowly, however; it is expanding toward the south. This part of the nebula is so distinct that there can be no doubt of this change of form, though it is small.

On the negative of September 20 the outer nebulosities appear as segments of a fairly well-defined ring about 25′ in diameter (Plate VI). On account of favorable atmospheric conditions this negative is very sharp and brilliant, and finer details are shown than in any of the other negatives except that of January 7 and 9. In all of the later negatives parts of this outer ring are shown to be receding from the *Nova;* this is certainly true of those parts to the south and west, and also of the remarkable group of nebulous wisps to the north, which are shown only on the photographs taken after January 1.

The large, diffused mass of nebulosity, *k*, to the southeast of the principal ring, is well shown on all of the negatives which received more than two hours' exposure. No very well-defined details can be seen in it, and no change of form can be detected with certainty. It is a suggestive fact that this large mass lies in the direction from the *Nova* in which the strong condensation

[1] ASTROPHYSICAL JOURNAL, October and November 1901.

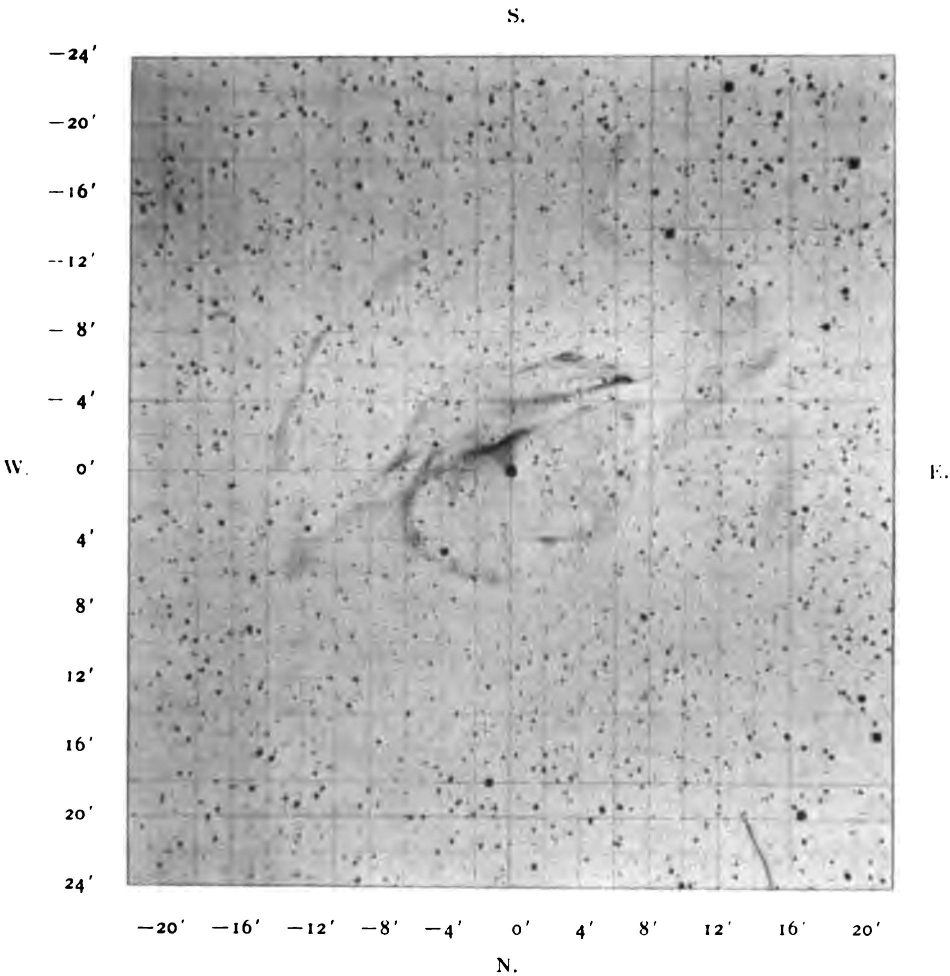
PLATE VII.



NEBULOSITY ABOUT *NOVA PERSEI.* NOVEMBER 13, 1901.

Photographed with the Two-Foot Reflector, Yerkes Observatory.
Exposure 7^h.

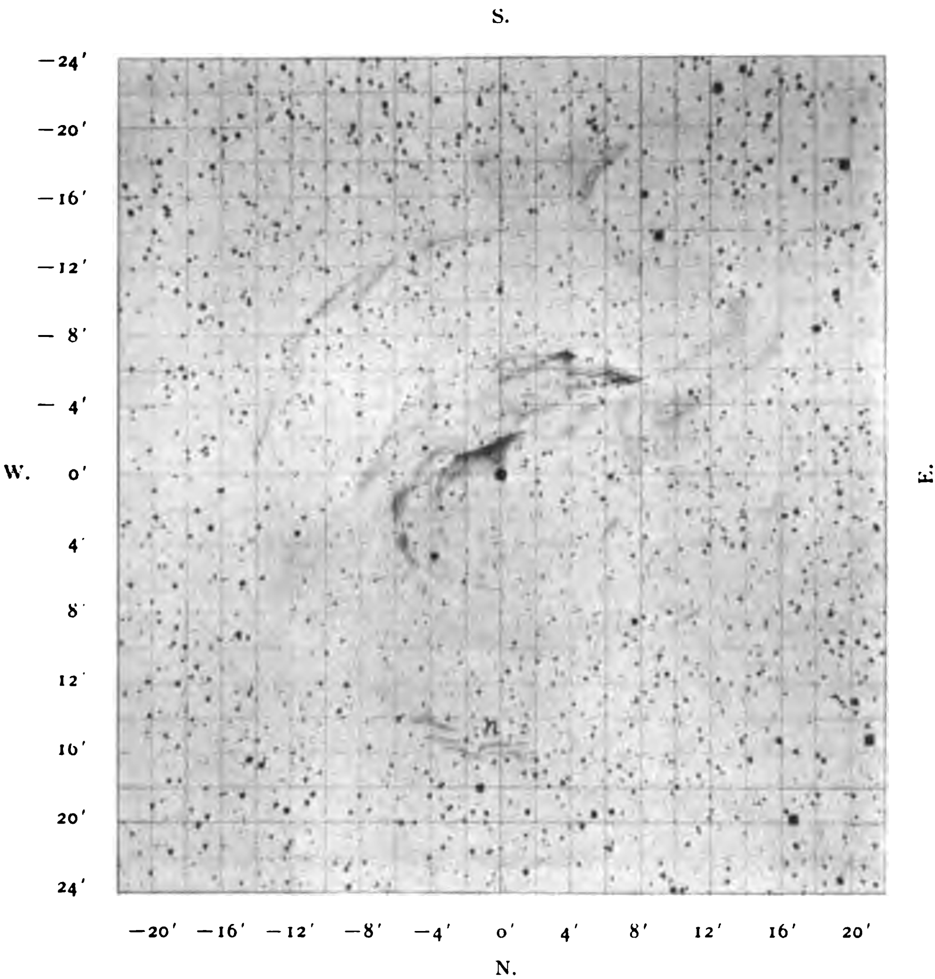
PLATE VIII.



NEBULOSITY ABOUT *NOVA PERSEI*, DECEMBER 14, 1901.

Photographed with the Two-Foot Reflector, Yerkes Observatory.
Exposure 4^h 30^m.

PLATE IX.

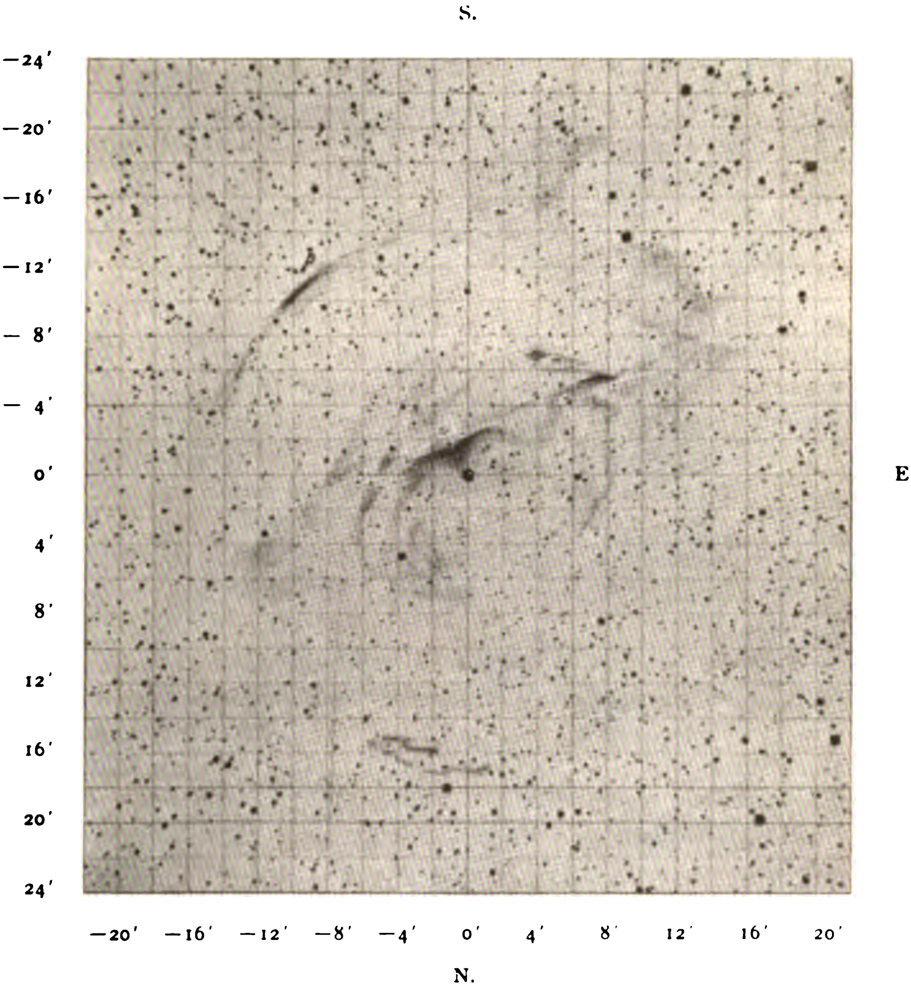


NEBULOSITY ABOUT *NOVA PERSEI*. JANUARY 7 AND 9, 1902.

Photographed with the Two-Foot Reflector, Yerkes Observatory.

Exposure 4^{h} 30^{m}.

PLATE X.



NEBULOSITY ABOUT *NOVA PERSEI*, FEBRUARY 8, 1902.

Photographed with the Two-Foot Reflector, Yerkes Observatory.

Exposure $2^h\ 40^m$.

a of the principal ring is apparently moving. On one negative, that of January 7 and 9, which is one of the sharpest of the series, a faint straight wisp in this mass extends in what is apparently the line of motion of the condensation *a*. (See Plate IX.)

The remarkable group of nebulous wisps, *n*, to the north of the *Nova*, at a distance of about 16′, has already been mentioned. These wisps are invisible on the negatives obtained before January 1, and are strongly shown on the negatives of January 7–9 and February 8, with a well-marked change of position between these dates, as shown in Plates IX and X.

It will be noticed that on the negative of February 8 there is a very strong wisp in the part of the outer ring to the southwest of the *Nova*, at a distance of about 14′ from the latter (*p*, Plate X). On this negative, and on the three which have been secured since February 8, this wisp is the strongest part of the entire nebula, excepting only the condensations *m* and *a*. This wisp is so strong and distinct on the four negatives just mentioned that there cannot be the slightest doubt of its rapid change of position. This apparent movement is outward and nearly radial.

In speaking of motion and of change of form in the above paragraphs I have in all cases meant *apparent* change only, and have not intended to express the opinion that there is actual motion of matter. The rapid fading of parts of the principal ring, and the sudden brightening and rapid outward movement of parts of the outer ring, notably those to the north and to the southwest of the *Nova*, are phenomena which strongly support the theory suggested by Kapteyn and others, that the apparent motion is due to changes of illumination of a stationary nebula.

On account of the illness of the writer, the exposures in the telescope of all of the negatives obtained since December 1 have been made by Mr. Francis G. Pease.

YERKES OBSERVATORY,
March 1902.

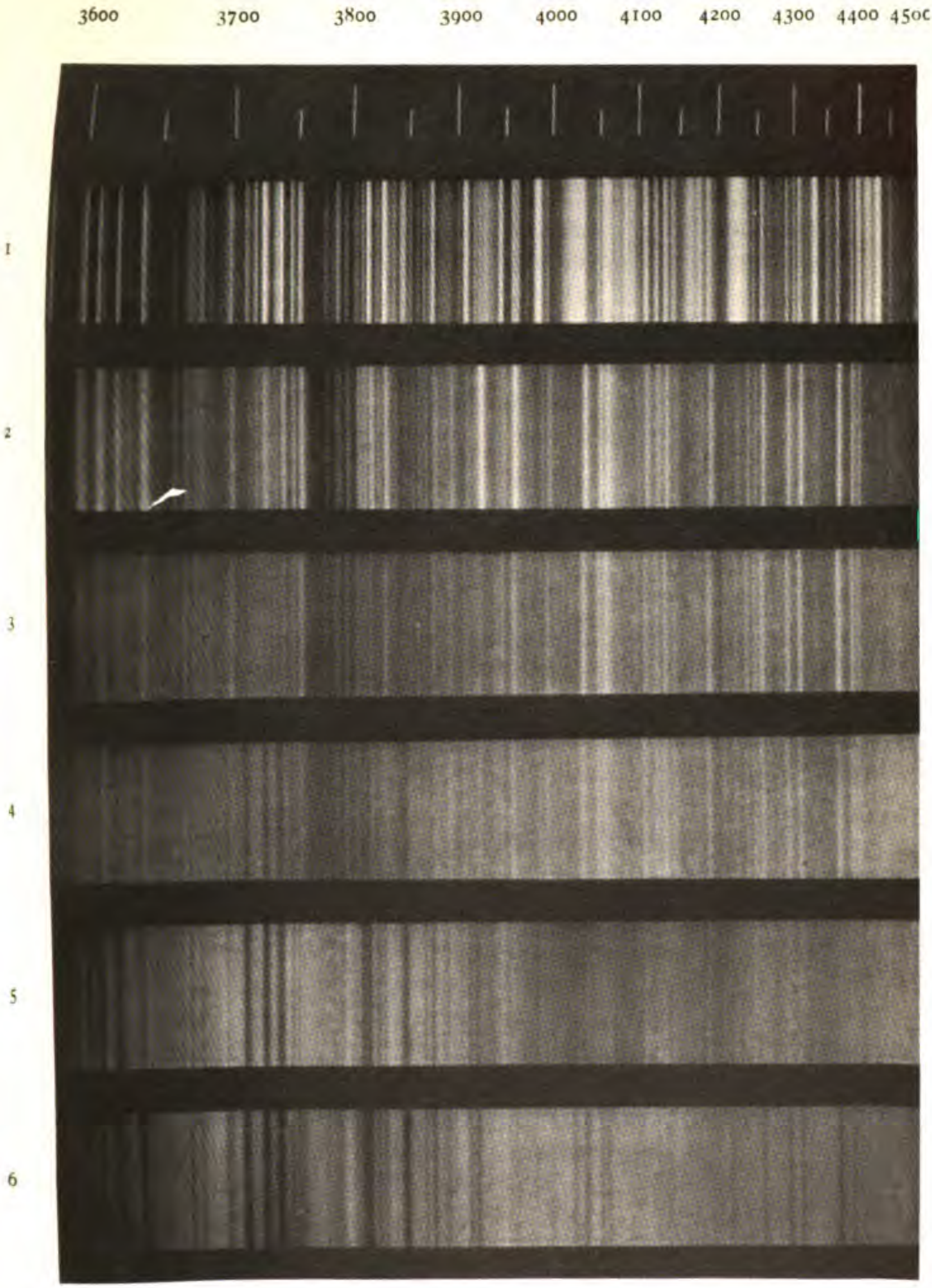
NOTE ON THE SPARK SPECTRUM OF IRON IN LIQUIDS AND IN AIR AT HIGH PRESSURES.

By George E. Hale.

In a study of the spark spectrum of iron and other metals in liquids, undertaken in connection with investigations of the spectra of temporary and red stars, I have encountered certain phenomena of which some preliminary notice seems desirable. It is to be understood that most of the results here presented are derived from a general reconnaissance, and that only a prolonged quantitative investigation, which is now in progress, can be expected to yield data suitable for study in connection with related astronomical and physical phenomena.

The transformer employed is wound to give either 15,000 or 30,000 volts when supplied with an alternating current of 110 volts, but in the present work a resistance of about four ohms was inserted in the circuit of the 3 K. W. alternator (133 cycles), reducing the voltage of the primary current to about 25 and the current to about 20 amperes. Under these conditions, with a condenser of 0.0015 microfarads capacity, electrodes (in water) of Bessemer steel 1 mm in diameter, with flat ends, separated from each other by a distance of about a millimeter; and auxiliary electrodes (in air, connected in series with the electrodes in water and the terminals of the transformer) of Bessemer steel, 1 mm in diameter, with flat ends, separated from each other by a distance of about 12 millimeters: the spark in water gives a spectrum of the type shown in Plate XI, Fig. 2. By changing the diameter or separation of the electrodes, the capacity of the condenser, etc., or by replacing the water by some other liquid, as will be explained below, spectra like those shown in Figs. 3 and 4 can be obtained. With the above transformer this degree of absorption has been surpassed in pure water, but if the spark is taken in a solution of 1 part of common salt in 800 parts of distilled water the effect is intensified,

PLATE XI.



PHOTOGRAPHS OF THE SPARK SPECTRUM OF IRON IN AIR (1), IN WATER (2, 3, 4), AND IN SODIUM CHLORIDE SOLUTIONS (5, 6).

Made with a One-Prism Spectrograph by George E. Hale.

and the spectrum becomes like that shown in Fig. 5. A 4 per cent. solution of common salt gives the absorption phenomena shown in Fig. 6, while an 8 per cent. solution further increases the intensity of the dark lines. A 9.5 per cent. solution of $BaCl_2$ produces the strongest absorption effect hitherto observed. The accompanying illustrations give some of the more marked stages in the process of reversal, but many intermediate steps have been recorded.

Within certain limits, it may be said that the reversals tend to increase in number and intensity (1) with the length of the auxiliary (air) spark; (2) with the diameter of the electrodes at either spark gap; (3) with the capacity of the condenser; (4) with the pressure of the water (other things being equal, spectra photographed with the spark 2.5 cm and 615 cm below the surface of the water respectively show a very distinct difference); (5) in solutions of sodium chloride and other salts, with the strength of the solution. On the other hand, the reversals tend to decrease in a very striking way as the length of the spark in the liquid is increased. Thus with an auxiliary spark gap of 12 mm, a spark in water 0.2 mm long gives a spectrum similar to Fig. 4, while an increase of the length to 1 mm changes the spectrum to the type shown in Fig. 2. In this case the electrodes were of iron wire 2.3 mm in diameter, with flat ends; the auxiliary electrodes of Bessemer steel were of 0.5 mm and 3 mm diameter respectively, with flat ends. It is perhaps well to point out that all of the changes referred to relate to the iron lines of the spark, and not to lines due to substances present in the particular liquid employed.

Other experiments, to be described in detail later, may be briefly referred to here. Spectra containing no dark lines were obtained from the spark which occurs in a Wehnelt interrupter (iron rod in *KOH* solution; direct current, 110 volts); from a rotating arc with iron poles in water (direct current, 110 volts); from the discharge between iron poles in water of an Apps induction coil wound to give a 30 cm spark (alternating current of 11 amperes through primary); and from the discharge

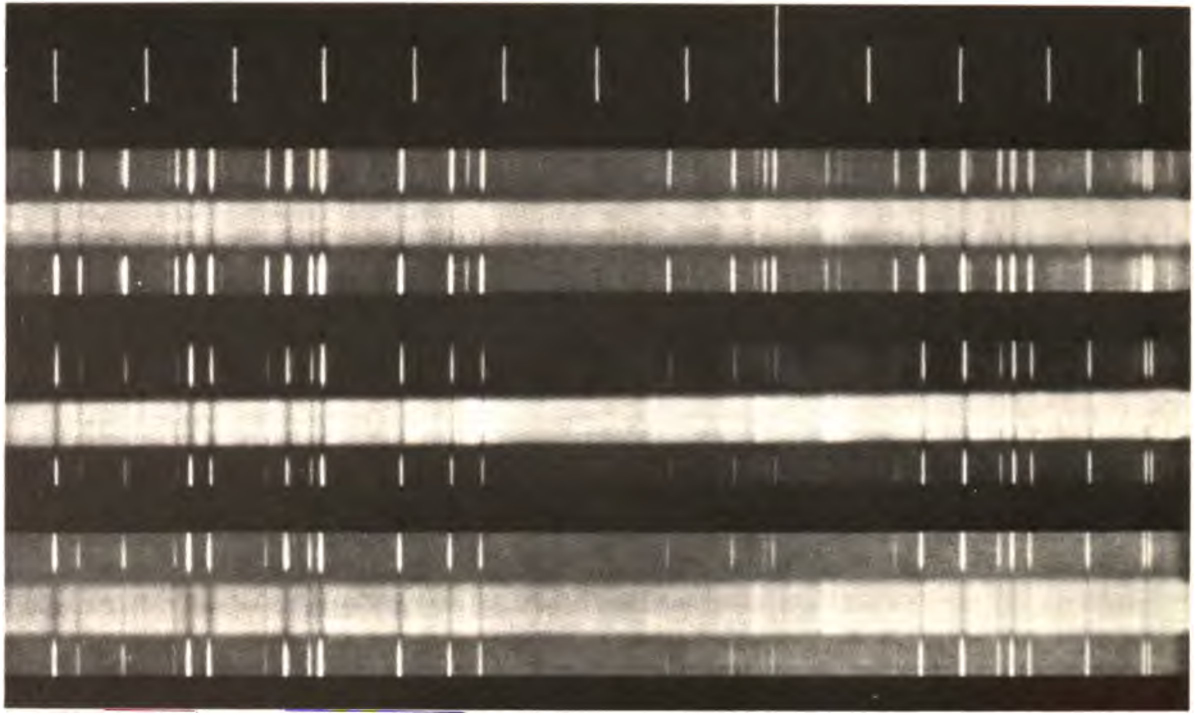
in water of a high frequency coil whose primary was connected with the secondary of the 15,000-volt transformer. All of these spectra, except for certain changes in the relative intensities of the lines, resemble the spectrum of the iron spark in air, and contain no reversals.

The blue region of the spectrum of the transformer spark in air has also been observed at pressures up to twenty atmospheres, with results which harmonize well with those of Humphreys and Mohler, obtained with the arc at lower pressures. The iron lines remain fairly sharp, and are shifted toward the red by an amount which seems from preliminary measurement to be directly proportional to the pressure. The phenomena are thus very different from those observed in water.

In order to study the shifts of the lines of the spark in water the spectrum under various conditions has been photographed with a concave grating spectroscope of 21 ft. focal length. Portions of some of the spectra, in each case accompanied by a comparison spectrum of the iron spark in air, are reproduced in Plate XII. It will be seen that during the transition from the bright line to the dark line conditions represented in Plate XI, the spectra show both bright and dark lines simultaneously. The scale of the photographs is sufficient to render visible the shifts of the lines, which are now under investigation.

In repeating these experiments it will be interesting to follow the changes of the iron triplet $\lambda\lambda$ 3763.95, 3765.69, 3767.34. In the spark in air and in the 110-volt arc or high frequency spark in water, the two outer lines are much more intense than the central line. Kayser and Runge give all these lines the same intensity in their table of the arc spectrum of iron, but the photographic map which accompanies the table shows the central line to be decidedly fainter than the others. Hemsalech's table of spark lines gives the intensities as 8, 7, 8, and shows that self-induction increases the intensities of the two outer lines to 10, but does not affect the central line. With the transformer spark in water it is possible to obtain the lines of equal intensities. The general effect of the water is to strengthen the

PLATE XII.



PHOTOGRAPHS OF THE SPARK SPECTRUM OF IRON IN WATER.

Made with the 21-foot Concave Grating Spectroscope of the Yerkes Observatory, by George E. Hale.

central line and to weaken the other two. Intermediate effects are shown in Figs. 7 and 8, Plate XII, while Fig. 9 represents fairly strong absorption, which would be further intensified in a salt solution. The scale of the reproductions is sufficient to show the peculiar shifts of the various lines under these conditions. Professor Crew, who has very kindly confirmed some of these results in his own laboratory, informs me that the relative intensities of the lines of the triplet in the arc are not affected by an atmosphere of hydrogen or of oxygen.

Although it appears not improbable that the energy of the discharge is the principal variable concerned, a discussion of the results in the absence of quantitative data would be premature. It does not seem from the experiments so far made that the electrical conductivity of the solution is a dominant factor (*e.g.*, kerosene oil gives an effect closely resembling that obtained with a 4 per cent. $NaCl$ solution). Experiments to test the effect of self-induction (with and without iron cores in the coils) have given negative results, so far as the absorption is concerned, though the lines seem to stand out more clearly against the background of continuous spectrum when self-induction alone is used. A direct current spark in water and in a 5 per cent. solution of $BaCl_2$ gave similar effects at the positive and negative poles. These experiments, and those on self-induction, are regarded as inconclusive, and will be repeated with better apparatus.

The investigation will be continued with the assistance of Dr. N. A. Kent, to whom I am indebted for efficient aid in the work described above.

Yerkes Observatory,
March 1, 1902.

Minor Contributions and Notes

FURTHER OBSERVATIONS OF THE MOVEMENTS AND CHANGES IN THE NEBULOSITY ABOUT *NOVA PERSEI*.[1]

An examination was recently made, in the course of another investigation, of the series of negatives of *Nova Persei* secured with the Crossley reflector in February and March, 1901, by Messrs. H. K. Palmer and C. G. Dall. I found that a plate taken on March 29 with an exposure of ten minutes showed two faint rings of nebulosity about the *Nova*, as well as several masses in its vicinity. This negative carries back our knowledge of this interesting nebula nearly five months. The early observations of the nebula thus far reported are as follows:

1901 March 29, Lick Observatory.
1901 August 23, Heidelberg Observatory.[2]
1901 September 20, Yerkes Observatory.[3]

The following list of negatives of *Nova Persei*, showing the nebulosity, have been obtained with the Crossley reflector:

No.	Date 1901	P. s. t. of exposure	Duration of exposure
1	March 29	$7^h\ 57^m$ to $8^h\ 7^m$	$0^h\ 10^m$
2[4]	November 7	10 47 to 15 21	7 19
	" 8	10 0 to 13 0	
3	" 12	9 45 to 14 45	10 0
	" 13	9 29 to 14 29	
4	December 4	7 48 to 13 16	5 28
5	" 8	7 37 to 13 7	10 0
	" 11	7 29 to 11 59	
	1902		
6	January 2	6 18 to 11 18	10 0
	" 3	6 31 to 11 31	
7	" 10	6 27 to 11 57	10 30
	" 11	6 37 to 11 37	

[1] *Lick Observatory, University of California, Bulletin* No. 14.

[2] *Astronomische Nachrichten*, No. 3736. [3] Astrophysical Journal, **14**, 167.

[4] The exposure of November 7 was interrupted 15 minutes near the middle.

The following approximate coördinates of some of the principal condensations are deduced from the negative of December 8–11. These coördinates refer to the *Nova* and are intended for purposes of identification only.

NEGATIVE NO. 5.

Condensation	p.	s.
A	127°	8'.9
B	153	7.5
C	186	6.8
D	210	1.6
E	211	13.5
F	350	15.4
F_1	357	15.7
F_2	0	14.8
G	259	13.6
H	279	8.5
I	291	6.8
J	335	6.8
K	0	6.8

DESCRIPTION OF NEGATIVES.

No. 1, March 29.—The principal ring of nebulosity about the *Nova* is not circular, but is an irregular oval considerably flattened on the southwest side. The longer axis of the oval lies in position-angle 115°. Following are the distances of this ring from the *Nova*:

N. E.	2'.2
S. E.	2.1
S. W.	1.5
N. W.	2.2

The ring is best defined in the S. W. and N. W. quadrants, where it is 20″ in width. At the ends of the oval the nebulosity is more diffuse and not so well defined, resembling somewhat the Annular Nebula in *Lyra*.

Inside this principal nebulous ring is another much closer to the star, and much fainter. The smaller one is apparently a perfect ellipse, the *Nova* being in one focus. The major axis of this elliptical ring is in position-angle 60°, its length being 2'.5. The minor axis is 2'.0 in length. The star is situated ¾′ from the S. W. end of the ellipse. Condensation *D* appears to form the S. W. portion of this inner ring and to be a miniature of its present shape.

To the south of the *Nova*, at a distance of 3′, is a mass of very faint nebulosity exhibiting an arrow-headed structure similar to that of condensation *A*, and pointing in the same direction. By means of motions deduced from the later photographs this arrow-headed condensation is traced backward to the vicinity of that noted on the negative of March 29. The distances agree more closely than the directions, on the assumption of uniform rectilinear motion.

In the northeast quadrant there is a narrow wisp of nebulosity having the form of an arc of a circle of 5′ radius, with *Nova* at the center. This wisp is of uniform curvature and width, and extends from 0° to 90° of position angle.

No. 2, November 7–8.— It was an examination of this negative which led me to the discovery of motion of several of the principal condensations about the *Nova*. Having been taken on a less rapid plate than the later ones, it does not show all the detail that is visible on most of them. The principal condensations are very distinct, however, and much faint detail is seen in the region of 7′ radius surrounding the *Nova* (See *Bulletin* No. 10).

No. 3, November 12–13.—This negative, taken on a quicker plate and with a longer exposure, shows greater strength in the principal condensations, as well as more detail in the region about the *Nova*. This area is approximately circular, with a diameter of 15′, and is the region in which Mr. Ritchey of the Yerkes Observatory photographed nebulosity on September 20.

To the south of *Nova* are shown the principal condensations previously referred to, as well as a number of wisps between the outer edge of this area and the star. The displacement of condensation *A* is perceptible in the interval of five days. To the west of the star are several arrow-shaped masses pointing northwest. There are several narrow wisps of nebula to the north concentrically curved about *Nova*. One of these, at a distance of 7′, covers an arc of nearly 90°. Outside of this is a streamer extending from the region of *D* outward, perhaps spirally, counter-clockwise, to position-angle 0°, or over an arc of 140°.

To the east of *Nova* are a number of irregular masses, many of them connected together, but not indicating clearly any regular structure.

Outside of this comparatively well-defined area 15′ in diameter is another area of about 30′ diameter showing traces of very faint nebu-

losity. This is most distinct in the region from 270° to 90°, which is practically filled with it. Scarcely any trace of this exterior nebulosity is to be seen to the south and southwest on this negative. On the outer edge of this larger area, to the north, there are traces of several short wisps or lines (*F*), forming a portion of a rim as it were.

No. 4, December 4.—While this negative had an exposure of only $5^h\ 28^m$, and in a sky with some haze, as evidenced by the halation ring about the star, the principal features of the nebulosity are well shown. In condensation *A* there are signs of a separation of the fainter envelope from the brighter forked mass which it surrounds.

In position-angle 211′, and at a distance of 13′.5 from the star, there is a short wisp of nebulosity (*E*) that is not visible in the previous negatives. This wisp forms the arc of a circle about the star and is between 2′ and 3′ in length. Just inside of this wisp there are traces of one or two others, more or less concentric.

No. 5, December 8–11.—The motion in the interval of three days between the two parts of the exposure, caused by a period of stormy weather, was sufficient to blur the finer details. This is especially the case in condensation *A*. The wisps *E* and *F* are stronger and more pronounced than in the negative of December 4.

The faint outer nebulosity is most pronounced on this negative, being easily seen all around the *Nova*, but is strongest in the southeast quadrant, where it can be traced 18′ from *Nova*. Outside this area, and extending to the limits of the field to the east and southeast, are irregular masses of nebulous matter, some of them connected with the main area.

Perhaps the most interesting region of this outer nebulosity is that to the west of *Nova*. In this area, especially near the edge, is a perfect network of the finest detail on the plate. Composed of the faintest and finest thread-like filaments in all directions, it is useless to attempt a full description. Attention may be called, however, to an arrow-shaped mass (*G*) of these filaments to the west at a distance of 13′.6 from *Nova*. It is composed of several layers of filaments with others irregularly crossing, its axis being directed counter-clockwise. Some evidence of a similar network of these filaments is found in other parts of the outer nebula.

No. 6, January 2–3.—The conditions under which this negative was obtained were very unfavorable, the seeing being decidedly bad, with a high north wind which shook the telescope at times, notwith-

standing the wind screen. The star images are large and fuzzy and the fine details considerably blurred. However, the general features are well shown, and the wisps at *E* and *F* are easily distinguished. These show considerable displacement during the interval,

No. 7, January 10–11.—The conditions under which this negative was secured were good. The star images show that the focus of the telescope was not good at all times during the exposures, but the various features of the nebulosity are well recorded, although not perfectly sharp. The motions of condensations *A*, *B*, and *C* continue. The envelope about condensation *A* has separated farther from the brighter nucleus. The nucleus itself has separated into two portions, the whole being considerably fainter. Several new wisps and small masses have appeared to the south and west of *A*. Condensation *B* is little changed in form, but is weaker, as is also condensation *C*. Condensation *D* remains practically unchanged both in form and intensity.

Condensations *E* and *F* are quite well marked, as is also the western edge of the outer nebulous area.

SUMMARY.

A comparison of the negatives obtained between November 7 and January 10, inclusive, indicates a general expansion of the nebula in all directions. The motions of several of the best-defined masses of the inner circle of nebulosity, to the south of the star, are in a clockwise direction. To the west, at least one mass appears to have a counter-clockwise motion.

A comparison of the positions of the principal condensations, *A*, *B*, and *C*, as deduced from the plates of November 7–8 and January 10–11, indicates the following motions during the interval. Condensation *C* has undergone a change, principally in brightness, such that the latter position of this mass is somewhat uncertain.

Condensation	Direction of motion Position angle	Distance moved
A	104°	1′.2
B	116	1.0
C	178	1.6

In the outer ring of nebulosity the wisps marked *E* show a counter-clockwise motion, while the group marked *F* indicates a clockwise motion. In both cases a general expansion outward is shown, but the

larger component of motion is tangential and amounts to about 3′ in the interval December 8–11 to January 10–11.

From the rapid changes in form and intensity all determinations of position are subject to a large probable error. A comparison of the nebular rings recorded on March 29 with those obtained recently points to an intimate connection. The two groups of nebulous wisps, *E* and *F*, observed in December and January, show a radial component of motion for each group of ¾′ to 1′ in the interval of one month. It seems probable, therefore, that the two rings observed on March 29 have expanded into the later appearances, and that these two groups, *E* and *F*, are fragments of the outer ring of the earlier date.

A simple computation shows that the distances of the outer ring of March 29 and the two groups, *E* and *F*, from the Nova are approximately proportional to the intervals of time, from about February 20, assumed as the date of the outburst.

Upon the assumption of identity of these appearances the daily radial rate of recession is found, from the interval March 29, 1901, to January 2–3, 1902, to be 2ʹ.62 for the mass *E* in the southwest quadrant, and 3ʹ.00 for the mass *F* to the north. These values of the daily motions give February 16 as the date when mass *E* occupied a position apparently coincident with the star. For mass *F* we find in the same way February 17. Using the brighter nebular ring of March 29, we also obtain February 16 and 17 as the dates, respectively, when these portions of the ring occupied the same position as the star. The two sets of results agree within a tenth of a day, but this must be considered accidental, as the measures upon which they rest were made only to the nearest tenth of a minute.

If this nebula is expanding in all directions, and should continue to expand at its present rate, some of it should reach the solar system in 250 years.

It is planned to publish photographs and measures of the nebula at an early date.

C. D. PERRINE.

JANUARY 13, 1902.

A DETERMINATION OF THE CAUSE OF THE DISCREPANCY BETWEEN MEASURES OF SPECTROGRAMS MADE WITH VIOLET TO LEFT AND WITH VIOLET TO RIGHT.[1]

It has been observed here and elsewhere that in measuring spectrograms for determination of velocity in the line of sight, a systematic difference is found in the determinations, amounting to perhaps one kilometer per second on the average, depending on whether the plate is measured with the violet end to the left or to the right. More precisely, there is a tendency to set the cross-hair a little too far to the right (as seen in the microscope) on the comparison-lines, or to the left on the star-lines. For this reason each plate is measured once with the violet to the left and once with the red to the left, and the finally adopted value for the line-of-sight velocity with reference to the observer is the mean of the two determinations. The method in detail is this: First, the plate is set on the table of the measuring-engine with the violet end to the right (appearing to the left in the microscope-field), and suitable lines are measured throughout the length of the plate. In this position, readings of screw-divisions increase with increasing wave-lengths. The plate is then turned end for end and the same lines are remeasured. Suppose the readings for several lines in the first measurement are, respectively,

$$a_1,\ a_2,\ a_3,\ \text{etc.}\ ;$$

then, if settings were made with perfect accuracy, the readings for these same lines on the second measurement would be

$$b_1 = X - a_1,\ b_2 = X - a_2,\ b_3 = X - a_3,\ b_4 = X - a_4,\ \text{etc.}\ ,$$

where X is a constant depending on the two positions in which the plate was set relatively to the screw. In actual cases, however, $a + b$ is not a constant. Let $a' = x + b$, where x is any arbitrary number, preferably some round number not far from the average value $a + b$. Then $\frac{a + a'}{2}$ is the proper reading for the line (bearing in mind that only *differences* of screw-readings are of importance.) We have

$$\frac{a + a'}{2} = \frac{a + x - b}{2} = a + \tfrac{1}{2}\left[x - (a + b)\right]\ .$$

Therefore, if the sum of the two readings exceeds x by an amount ϵ,

[1] *Lick Observatory, University of California, Bulletin* No. 15.

we subtract from the first reading $\frac{1}{2}\epsilon$, or if it is less than x by an amount ϵ we add $\frac{1}{2}\epsilon$. The systematic difference already spoken of between measures with violet to left and with violet to right amounts to this: the average sum $a + b$ is greater for the comparison-lines than for the star-lines. For, if a_1 is the first reading on a comparison-line, a_2 on a star line, b_1 and b_2 the corresponding readings after the plate is reversed, then

$$a_1 - a_2 > b_2 - b_1, \text{ or } a_1 + b_1 > a_2 + b_2 .$$

Moreover,

$$(a_1 - a_2) - (b_2 - b_1) = (a_1 + b_1) - (a_2 + b_2) ;$$

so that if the mean value of the sum of the two readings is P for the comparison-lines, and Q for the star-lines, then $\Delta = P - Q$ may be taken as expressing in screw-divisions the difference between measurements of the Doppler-Fizeau shift as measured in the two ways. In my own measurements, Δ almost always lies between three and ten thousandths of a revolution of the screw, and usually is close to six thousandths; the pitch of the screw being 0.25 mm.

It was thought desirable to determine the cause of this effect, as it might possibly introduce systematic errors which are not eliminated by taking the mean; and a series of measurements was made with that end in view.

Three possible causes suggest themselves: first, the curvature of the lines, which is inappreciable in the narrow star-spectrum, but noticeable in the comparison-lines, especially if the slit is long; second, the fact that the star-spectrum is situated in the middle of the field, with the comparison-spectrum on each side of it; third, the fact that in one case the setting is made on a black line in a bright field, and in the other on a bright line in a dark field.

In order to determine the effect of curvature, the curvature was eliminated by fitting the spectroscope with a new slit which was itself curved in such a manner as to make the spectral lines straight. A trial plate taken with this slit showed no appreciable curvature of the lines, even when the slit-length was several times as great as is commonly employed. Using this slit, several plates were made for use in this investigation — in particular, one of the Moon and one of the star *β Herculis*.

To see whether the second possible cause suggested above really exerts any effect, some plates were taken with the absorption-spectrum and the comparison-spectrum interchanged in position; that is, the

comparison was placed in the middle of the field, with the absorption-spectrum (that of the Moon, or of the sky in the day time) on each side of it. It is obvious that if the effect in question is entirely due to the usual arrangement in position of absorption-spectrum and line-spectrum, the average sum $a + b$ should in these plates be greater for the former than for the latter.

To determine whether the effect is due to a psychological tendency to set too far to the right on a black line in a white field (or too far to the left in the reverse case), a good plate was secured of the iron-spectrum, both in its usual position and also in the middle of the plate where the star-spectrum usually goes. To prevent the outside lines from forming a continuation of the lines of corresponding wave-length in the center of the plate, the plate was shifted lengthwise before the exposure on the center was made, so that the plate resembled a spectrogram of a nebula or star with very great radial velocity, giving the iron lines alone as a bright-line spectrum.

Several other plates were taken of the spectrum of light reflected from the sky with certain unexposed gaps (left by the interposition of a diaphragm properly filed), in which gaps the strongest of the lines of iron were afterwards inserted; thus giving on the same plate bright lines on a dark field and dark lines on a bright field, without any lateral displacement, so that all would be measured in the same part of the field of the microscope. If the effect in question is due entirely to a tendency to set differently on bright and on dark lines, we should expect the value of Δ to be the same for these plates as for those in in which the comparison is placed on the sides; while the plate containing only emission-spectra would have the average sum $a + b$ the same for inside lines (in the usual position of the star-spectrum) as for those on the outside.

Plates of these various kinds were measured at various times, from the latter part of August to the early part of October. and the quantity Δ determined for each one. The results made it tolerably certain that the effect was due entirely to the third cause. Still, there were certain discrepancies which I attributed to a secular change in my methods of measurement, but which were of such a character as to make it seem possible that the second cause also operated in a slight degree. For this reason, it was thought best to measure them again, and also between every two to measure one or more good plates of the usual kind, thus eliminating as far as possible the effect of any secular

change in habits of measurement. The following table shows the results of these measures, which may be accepted as final. The first column gives the date when the plate was measured. The second gives the description of the plate. When the plate is one taken on our regular program (as in the case of all the ordinary plates, and of two taken with the curved slit but otherwise in the usual way), it is designated by the name of the star and the number and letter given to the plate in the regular observing book for the Mills spectrograph. The other special negatives which are used only for this investigation, and were therefore not entered on the regular records, are designated by the date when they were exposed, and by a letter to distinguish between plates taken on the same day.

The third column is a description of the character of the different plates. Those marked with an asterisk are the ordinary plates, used as a check on changes in habits of measuring. The notes in this

Date of measurement	Designation	Kind of plate	Δ
1901 Oct. 28	*α Tauri*, 2280 *D*	*	+6.2
28	October 3, *F*	*Fe* inside and outside.	+1.0
29	*β Capricorni*, 2197 *E*	*	+6.3
30	October 3, *E*	Sky and *Fe*, both in middle.	+8.2
31	*Venus*, 2290 *C*	*	+6.3
Oct. 31–Nov. 1	October 3, *D*	Sky and *Fe*, both in middle.	+9.3
Nov. 2	*Polaris*, 1853 *B*	*	+9.1
8	*ζ Ceti*, 1858 *B*	*	+9.1
9–11	August 30, D_2	Moon outside, *Fe* inside, curved slit.	+7.7
12	*Procyon*, 2292 *E*	*	+1.8
15	*η Geminorum*, 2304 *B*	*	+6.2
18	October 3, *F*	*Fe* inside and outside.	+2.0
19–20	*Venus*, 1894 *B*	*	+5.2
20	Moon, 1844 *A*	Moon outside, *Fe* inside.	+5.9
21–22	*Venus*, 1656 *A*	*	+5.9
22	Moon, 2249 *C*	Curved slit, otherwise as usual.	+3.8
23	*Venus*, 1997 *A*	*	+4.0
26–27	*Mars* 1976 *C*	*	+5.8
27–29	*β Herculis*, 2245 *E*	Curved slit, otherwise as usual.	+6.5
30	*Mars*, 2141 *C*	*	+5.7

column, referring to the other plates, scarcely need explanation. As an example, the note, "Moon outside, *Fe* inside, curved slit," means that the plate measured October 9–11 had the comparison spectrum in the middle, with the Moon-spectrum inclosing it on each side, and that the curved slit was used in taking this plate. When it is not specified otherwise, it will be understood that the straight slit was used. The

fourth column gives, in thousandths of a revolution of the screw, the the value of Δ — the difference between the average sum of the readings (with plate direct and with plate reversed) for the comparison-lines and for the absorption lines.

These results seem to point definitely to the conclusion that the effect is produced entirely by a tendency to set farther to the right on a dark line in a bright field than on a bright line in a dark field. It is true that the measurements of October 28 and November 18, in which the sum $a + b$ is a trifle less for emission-lines in the center of the plate than for the same kind of lines farther out, seem to throw doubt on this conclusion, but as there were comparatively few lines to be measured on this plate, and the results of the other plates agree fully with this conclusion, we may regard the discrepancy as accidental. Further evidence to this effect is found in the measurements by Mr. W. H. Wright, of this Observatory, of several photographs of the bright-line spectra of nebulæ. Here there are only emission-lines to measure, and we should expect, if our conclusion is correct, that there would be no difference between the average sum $a + b$ for the nebula lines and for the comparison-lines. His measurements on seven nebula plates give for the mean difference $+ 0.9$. In measuring ordinary star-plates Mr. Wright usually has a value of Δ at least twice as great as my own. We may therefore regard it as fairly certain that the difference between measures with violet to left and with violet to right is due solely to a tendency of the person measuring to set farther to the right on a dark line in a bright field than on a bright line in a dark field, without regard to the relative positions of the two in the field of view, or to the curvature of the lines. It may be said that this result was anticipated by us, but every care was taken to prevent personal bias from influencing the measures, and I believe them to be free from any such influence.

It certainly seems that no error is introduced by adopting the mean of measures made with violet to left and with violet to right, so that it is not worth while to seek any means of eliminating the discrepancy, especially as no ready method of accomplishing this presents itself.

The measurements of spectrograms of *Venus*, *Mars*, and the Moon made during the investigation were reduced by our usual methods and compared with the theoretical velocities, to see whether any connection could be established between the residuals and the corresponding values of Δ. No relation between the two was apparent.

Several other methods for investigating the origin of this right and left discrepancy presented themselves to us. A very simple one consists in measuring, in the usual manner, a contact positive copy of an ordinary negative secured with the Mills spectrograph. Since the bright and dark lines of the negative would be transformed into dark and bright lines, respectively, on the positive, the right and left discrepancies of the two plates should have opposite signs, if our suspicions as to their origin were correct. It was originally intended that this method should also be included in the program, but the results described above were so definite that further investigations along this line were considered to be superfluous.

In conclusion, I wish to acknowledge my indebtedness to Director Campbell (at whose suggestion the investigation was undertaken) and to Mr. Wright, for valuable advice and suggestions.

H. M. REESE.

December 16, 1901.

ORIGIN OF A DISTURBED REGION OBSERVED IN THE CORONA OF 1901 MAY 17-18.[1]

IN my preliminary report[2] of the observations of the Sumatra eclipse by the Crocker expedition from the Lick Observatory, I called attention to an unusual area of disturbance in the corona in the northeast quadrant. At the time of writing that report no observations of the Sun's surface were available from which to investigate the source of this disturbance. Through the courtesy of the astronomer royal, Royal Observatory, Greenwich, we have received a set of positives on glass of negatives of the Sun taken at Dehra Dûn, India, on May 17, 18, 19, 20, 21, 22, 26, and 28, 1901. These photographs are on a large scale, 7½ inches to the Sun's diameter, and furnish the desired observations. They show an intimate connection between activity on the Sun's surface as observed in the Sun-spots and faculæ, and the corona.

The photographs of May 17 and 18 show no spots or other evidences of activity on any part of the Sun's disk. This absence of spots was noticed before the eclipse at the station in Sumatra. The photograph of May 19, however, shows a medium sized spot which has just come into view around the east limb. On this date the spot is little more

[1] *Lick Observatory, University of California, Bulletin* No. 18.

[2] *Lick Observatory Bulletin* No. 9.

than a line, owing to foreshortening, ½′ in length, surrounded by faculæ. On the 20 it is ¾′ in length (*i. e.*, north and south), followed at a distance of ½′ by several small spots forming a close group. On all sides of the group, except the preceding or west side, is a large area of faculæ. The principal spot is compact, with well defined umbra and penumbra, and shows no more changes from day to day than are usually observed in the same period. The group of small spots following, however, shows traces of greater activity, principally growth.

Following are the coördinates of the principal spot deduced from the plates of May 19 and 28, the longitude being measured from the center of the disk:

			Greenwich Civil Time	Longitude	Latitude
1901	May	19	$3^h\ 30^m\ 37^s$	80°.7 East	+9°.0
		28	7 29 37	46°.7 West	+9.0

From these positions are deduced the following coördinates of the spot at the time of the eclipse in Padang:

			Greenwich Mean Time	Longitude	Latitude
1901	May	17	$17^h\ 40^m\ 37^s$	93°.8 East	+9°.0

From this it will be seen that the spot was on the opposite side of the Sun at the time of the eclipse and within 4° of the limb. Following are the position angles of the spot as projected on the limb, and of the apex of the disturbed area in the corona observed on the eclipse negatives:

	Position Angle
Sun-spot - - - - - -	60°.2
Apex of coronal disturbance - - -	60°.0

During the period of eleven days covered by the photographs, only this one group of spots was visible. In this time almost the entire solar surface was under observation.

We see from the above position angles that this region of Sun-spots occupied the same line of sight as the apex of the disturbed coronal region. While it is true that we have no means of determining the exact position of the coronal disturbance in the line of sight, attention was called in *Bulletin* No. 9, to the probability that its origin was near the Sun's limb. As both Sun-spot and disturbance are shown to have the same latitude, it can hardly be doubted that this unusual appearance in the corona was in reality immediately above the group of Sun-spots

and faculæ, and that it had its origin in the same disturbance of the Sun's surface. The long, thread-like prominence to the south, seen projected almost tangentially from the Sun's surface, appears likewise to have emanated from the same group of spots and faculæ.

These observations furnish very strong evidence of the intimate connection of all solar phenomena. Sun-spots, faculæ, prominences and corona all seem, in the present case at least, to have had a common origin.

The appearance of this disturbed region in the corona, and its undoubted connection with the group of spots on the surface so strongly suggested great activity that an investigation was made as to whether there had been a measurable displacement of any of the coronal masses in this region. The interval of time between photographs of the corona available for this purpose was but little over five minutes, yet if the velocities were large, 50 or 100 miles per second, such motion should be easily detected. The results give no certain indication of motion in the interval. The uncertainties of measurement of these coronal masses is so large, however, that a velocity of 5 or 10 miles per second would not be detected in so short an interval of time. We may conclude that the velocity across the line of sight was less than 20 miles per second.

The interval of one and one-half hours between the times of the eclipse in Mauritius and Padang should render a comparison of negatives secured at these two stations valuable in this connection.

C. D. PERRINE.

February 9, 1902.

Reviews

Handbuch der Spectroscopie. Von H. Kayser. Erster Band. Hirzel, Leipzig, 1900.

The year 1859 is not likely to be soon forgotten in the annals of science. For though many years separate Lamarck and Buffon from Darwin, and though nearly half a century intervenes between Fraunhofer and the work of Kirchhoff and Bunsen, we must still reckon from 1859, the establishment of the modern evolution theory and the discovery of modern spectroscopy. The forty years of history which have been made since then illustrate well the difference between ideas which deal only with "dead matter" and those which touch the life-history of the individual and of the race.

Since the *Origin of Species* was first published, the press has poured forth a stream of literature — of which a volume a day would probably not be an overestimate — dealing principally with the developmental hypothesis. On the contrary, one might almost count upon the fingers of his two hands all the treatises, popular or technical, dealing with the subject of spectrum analysis. Among the names which occur to everyone are Schellen, Roscoe, Lommel, H. W. Vogel, Schuster, Kayser (*Lehrbuch*), Scheiner, v. Konkoly, Young, Huggins, and Lockyer. Besides the volumes associated with these names, almost the entire literature of the subject is contained in highly specialized memoirs. Some faint idea of the extraordinary amount of spectroscopic work hidden away in the journals of chemistry, physics, and astronomy may be obtained from the fact that the author of the work under review finds no less than five octavo volumes necessary to contain a brief survey (*Uebersicht*) of what has already been achieved, and achieved principally in the last forty years.

Rarely has a more fortunate combination of talent and time ever been brought to bear upon such a task. For the author himself, one of the foremost spectroscopists of the world, frankly informs his readers that he has given to this work a large part of his time during the past ten years, believing that in so doing he was serving science to better advantage than in pushing his own investigations. Whether workers

in this science concur in this judgment, they must admit themselves under deep personal obligations to Professor Kayser for what must have been to him a deep personal sacrifice. To a man whose brain is full of unsolved problems, and who is well equipped for their solution, ten years is a long stretch; and the task is perhaps less than half completed.

The present volume deals only with the history of the subject and with the description and theory of the apparatus employed. But the completeness of the treatment is such that if the other volumes are carried out in the same manner, it will be practically possible to say that a certain experiment has, or has not, been tried according as it is, or is not, "found in Kayser."

The first chapter clears the ground for all those which follow by placing before the reader an outline history of the entire subject. This sketch, occupying some 128 pages, takes up all the more important advances included between the work of Newton and the discovery of Zeeman. The arrangement is in nearly chronological order of the men who made the advances.

The second chapter deals with the three principal methods of producing luminous vapors, namely, the flame, the arc, and the spark. The eminent propriety of beginning the actual discussion of the subject with such a chapter will appeal to everyone who agrees with the logic of one who makes the capture of the hare the first item in the recipe for rabbit pie.

The phase of the subject here treated is not the mechanics of luminosity, but rather the various radiant sources at our disposal in the laboratory. So far from being a mere description of apparatus, every page is marked by keen judicial opinions and exhaustive references. But amidst all this array of devices for producing light, the description of which occupies more than a hundred pages, nothing perhaps is so impressive as the large number of contradictory views and *apparently* contradictory facts, compelling the author to insist, time and again, upon our almost entire ignorance of what is going on in either arc or spark. Speaking of the discharge in a vacuum tube (p. 203) he says: "Wir bewegen uns fast durchweg auf dem Boden von Hypothesen, denen andere ziemlich ebenso gut berechtigte gegenüberstehen." Again (p. 243), speaking of the fickle character of this discharge: "Es darf freilich nicht vergessen werden, wenn man diese Veränderlichkeit der Geisslerröhren bespricht, dass sehr Vieles von unserer

mangelhaften Kenntniss der Gasspectra herrührt." And still again (p. 248), speaking of the effects of pressure, temperature, foreign gases, etc.: "Aber wir wissen über alle diese Verhältnisse noch so gut wie gar nichts."

The complexity of the process going on in a spark between pure carbon poles in ordinary (moist) air is strikingly illustrated by the list of spectra (p. 214) obtained by Eder and Valenta under these conditions, viz.:

1. The line spectrum of carbon.
2. The banded spectrum of carbon.
3. The cyanogen bands.
4. The line spectrum of air.
5. The banded spectrum of water vapor, together with hydrogen and oxygen lines.
6. The banded spectrum of nitrogen at the positive pole.
7. The emission spectrum of ammonia when the electrodes are moistened.
8. The spectrum of carbon monoxide in the aureole of the spark.
9. And, in closed vessels, the absorption spectrum of nitrous acid.

The student who first attempts the separation of some of these spectra is brought very keenly to realize the wide gulf which at some point exists between spectroscopic theory and spectroscopic practice.

The third chapter, written by Dr. H. Konen, of Bonn, is entirely devoted to one of the two dispersion pieces, viz., the prism. Besides a complete account of the history and literature of the subject, one finds here an exceedingly clear discussion of dispersion and deviation along the lines laid down by Czapski in his *Theorie der optischen Instrumente;* and, what is perhaps more important to the spectroscopist, a full account of the results of Helmholtz and Rayleigh concerning resolving power, purity, and intensity. A compendium so complete as this on the various forms of prisms ought certainly to prevent, in the future, the frequent duplication of work which this and the other chapters of this volume have made evident. One form of prism originally devised by Brewster is mentioned as having been reinvented by no less than three different men.

A fourth chapter of nearly a hundred pages devoted to the diffraction grating completes the discussion of dispersion pieces. Here we find four sections dealing in turn with the manufacture of gratings, plane gratings, curved gratings, and the echelon of Michelson. So far as your reviewer is aware, this is the only single volume, in any

language, which presents a fairly complete account of what is known about gratings. Fraunhofer's work naturally forms the introduction; and those who have not read Fraunhofer's original papers (or Ames' translation in Harper's *Scientific Memoirs*) will be astonished at the thoroughness with which he covered the entire field of the grating and with the variety of forms which he manufactured and studied.

Here, too, will be found Rowland's powerful paper on the plane grating (*Astronomy and Astro-Physics*, 1893). The reader who approaches this paper only through the German will remain blissfully unconscious of how much time Professor Kayser has saved him by filling in a number of gaps and by correcting numerous typographical errors in the original. Then follows Cornu's elegant method of discussing the focal properties of both plane and curved gratings in one general equation, a paper which was translated in *Astronomy and Astro-Physics*, 1894.

Instead of Rowland's original handling of the curved grating (*Phil. Mag.*, 1883) the author wisely adopts the totally different and more elegant treatment of Runge, already familiar to the spectroscopic world through Kayser's article in Winkelmann's *Handbuch*. Near the end of this chapter the author offers, from his own experience, some excellent suggestions regarding the adjustment of the concave spectrograph.

The fifth chapter, dealing especially with the design and theory of the spectroscope, contains, in its first section, a description of practically every type of spectroscope ever made, together with numerous ones that never have been made. The second section, dealing with the ideas of purity and resolving power, as introduced by Helmholtz and Rayleigh and extended by Schuster, is one of the most valuable in the entire volume. Here, too, will be found a systematic account of the masterly manner in which Wadsworth has cleared up and perfected this whole subject.

Space permits us barely to mention some of the more important topics treated in the remaining sections of this chapter, namely, the interferometer — and Michelson's analysis of individual lines — the work of Higgs, Abney, and Schumann, the measurement of spectrograms, the automatic comparator of Kayser, and the bolometer.

The sixth and last chapter deals with the latest and finest achievements of spectroscopy considered as a quantitative science: and if we admit that the question *how?* can seldom be answered until after

the question *how much?* has been answered, we may reckon this the most fundamental chapter in the volume. Your reviewer is not familiar with any opinion in the literature of physics which combines in greater degree a judicial attitude of mind, critical acumen and complete impersonality than does the author's survey of the work which has been done on the measurement of the absolute wave-length of light. Naturally Bell's determination is given first place among all values obtained by means of a grating. However, excellent reasons are advanced for thinking that the highest possible accuracy attainable with this instrument is one-tenth of an Angström unit. Indeed, the discrepancy between the values of Bell and Michelson, for the absolute wave-length, may be considered a closed chapter. For here one may find the facts concerning the grating and may find also that Michelson, in the superb and powerful method which he employed at Breteuil, left no room for an error as great as the one-hundredth part of an Ångström unit, in his final figure.

The standard relative values of Rowland, together with the corrected wave-lengths for the iron lines recently given by Kayser, are next taken up. The volume closes with a brief review of measurements in the ultra-violet and in the infra-red. An ample index of authors and subjects saves one that final disappointment which too often accompanies the closing of an English or a French book.

Concerning the volume as a whole, a reader gets the impression that the author has gone through the entire periodical literature of physical science with a drag net from which nothing has escaped.

The result is that these pages contain some matters which will probably prove new to the most accomplished spectroscopists of the world. In short, the work is of such a character that its possession is not only desirable but indispensable to every serious student of this science.

H. C.

NOTICE.

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed. If a request is sent *with the manuscript* one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

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THE

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ON THE DISCREPANCY BETWEEN GRATING AND INTERFERENCE MEASUREMENTS.

By Louis Bell.

For several years past it has been well understood that there is a wide discrepancy between the value for absolute wave-length on which Rowland's maps are based and that obtained by Michelson through the use of his beautiful interference methods. Moreover, the difference is of a size and character that, as I shall presently show, precludes its being properly chargeable to experimental errors of either method or to differences in the standards of length to which these measurements have been referred.

The first step toward an explanation is to determine the actual amount of the difference as it stands and the extent to which it may be influenced by known sources of error. This is not so easy as it would seem to be. Rowland's standard wave-lengths[1] are based upon an assumed weighted mean value for the wave-length of D_1, derived in the main from the writer's grating measurements.[2] They were obtained by the coincidence method, applied with enormous skill and care. The precision of

[1] *A. and A.*, **12**, 321, 1893.

[2] *Phil. Mag.*, (5) **25**, 255, 350, 1888.

measurement is slightly different for different lines, but in general the weight of the evidence indicates that the error in case of the main solar standards will rarely exceed 0.005 tenth-meter and ordinarily fall within 0.002 tenth-meter. Michelson's wave-lengths,[1] on the other hand, were obtained with vacuum tubes charged with *Cd* vapor as a source of light. These lines are very weak and dubious as reversed in the solar spectrum, and are not among Rowland's solar standards. They are, however, given as standards measured from the arc, and thus a connection is established between the absolute values of wave-length as obtained by the two methods in question.

Another path to the same result is that established by Perot and Fabry[2] in their estimation of certain lines of *Hg*, *Zn*, *Cu*; *Ag*, *Na*, and *Li* compared directly by the interference method with Michelson's red cadmium line. Unhappily the lines of the first named element are not definitely present in the solar spectrum, and those due to the other elements measured are, except in the case of *Na*, few and weak. The D lines, three lines of *Zn* at λ 4680, 4722, and 4810, and a faint *Cu* line at λ 5105 are available for comparison, and Rowland has the red *Li* line as a fairly good arc standard.

All the Perot and Fabry figures, like those of Michelson, are for 15° C. and 760 mm, so that before they can be compared with the Rowland tables the latter, standard at 20° C. and 760 mm, must be reduced for temperature difference.

For this purpose I have followed the values of Lorenz[3] for the refraction and dispersion of air. On this basis the wave-length of D_1 as used by Rowland, becomes, at 15° C. and 760 mm

$$D_1 = 5896.132.$$

Reducing in similar fashion other Rowland standards to 15° C., and deriving from them the value of D_1 by comparison with the wave-lengths established through the interference methods, one

[1] *Mém. du B. int. des Poids et Measures*, II, 1895.

[2] *C. R.*, 130, 492, 1900.

[3] *Wied. Ann.*, 11, 70, 1880.

can derive the following table of values for D_1 based on interference measurements:

Derivation of wave-length	Wave-length of D_1	Difference from mean	Remarks
M., absolute Cd_R, via R. Table of Standards	5895.967	+0.027	Comparison is with
M., absolute Cd_G, via R. Table of Standards......	5895.950	+0.010	arc lines in R.
M., absolute Cd_B, via R. Table of Standards......	5895.926	−0.014	Tables.
P. & F., *Zn* 4680, M. Cd_R, via R. Table of Standards	5895.923	−0.017	Comparison is with
P. & F., *Zn* 4722, M. Cd_R, via R. Table of Standards	5895.925	−0.015	R. solar standard, P.
P. & F., *Zn* 4810, M. Cd_R, via R. Table of Standards	5895.926	−0.014	& F., spark in vacuo.
P. & F., D_1 direct comparison with Cd_R.........	5895.932	−0.008	R. solar, P. & F.
P. & F., D_2 via R. Table of Standards..........	5895.939	−0.001	flame.
P. & F., *Cu* 5105 R. Table of Standards.........	5895.953	+0.013	R. solar, P. & F. spark in V.
P. & F., *L* 6708 R. Table of Standards..........	5895.959	+0.019	R. arc, P. & F. flame.

Average $D_1 = 5895.940$ 0.014 Mean difference.

Comparing this mean value with that assumed by Rowland from the grating measurements, we have

Rowland (Gratings) $D_1 = 5896.13$
Michelson (Interference) $D_1 = 5895.94$

Difference - - - - 0.19 tenth-meters.

That is, Rowland's wave-lengths must be reduced by very nearly one thirty-thousandth to bring them into accord with Michelson's results. This difference-ratio gives absolute corrections proportional to the wave-length.

The next question involves the relative experimental errors of the two methods under comparison. The advantage lies assuredly on the side of the interference method, but to an amount which cannot be evaluated until some one repeats Michelson's work with all possible care. A single set of measurements, even by so skillful an operator as Michelson, cannot carry concurrent proof of its own precision, and the work, including even the readings by Benoit, was practically unchecked save by repetition with no conditions changed. The use of the three *Cd* lines furnished material assistance in avoiding errors, but until the work is independently repeated the precision of the method is slightly uncertain. Michelson himself (*loc. cit.*) appreciated this, and in spite of marvelously close agreement

in the figures actually obtained considers that the uncertainty probably may extend to several thousandths of a tenth-meter, *i. e.*, his Cd_R, given as $\lambda_R = 6438.4722$, might be subject to a variation of several units in the next to the last decimal place. Benoit[1] estimates the possible error at about one one-millionth. The work of Perot and Fabry, just discussed, would indicate at least this amount of possible variation, probably extending to more than a unit in the second decimal place. The work of Hamy[2] along the same line shows the possibility of very much larger errors. For Cd_G, given by Michelson as 5085.824, Hamy obtained 5085.903, and, although Hamy found this line triple, its components are so close that the above difference is not adequately explained by this difference of appearance. Bad results are obtainable by any process, so that I am not inclined to attach great weight to this aberrancy. There is common, however, to the results of both Michelson and Perot and Fabry, a source of incertitude which is of serious import.

It has been shown by many researches within the past few years that the vibration periods to which the spectral lines are due are affected by the density of the medium in which the vibration takes place, being quicker *in vacuo* than in free or compressed air or other fluid. More particularly it has been shown by Mohler[3] that under the conditions obtaining in Michelson's vacuum tubes, the apparent wave-length of the *Cd* lines was decreased by 0.014 ± 3 tenth-meters, the corresponding shifts for *Zn* being 0.007 ± 3 tenth-meters, and for *Hg* 0.005 ± 2 tenth-meters. The shift seems to bear for the lines of each element concerned a uniform ratio to the wave-length, but, however this may turn out to be, the net result of the observation is that Michelson's *Cd* wave-lengths, as referred to air at $15°$ and 760 mm, are too short by about 0.01 tenth-meter. The results of Perot and Fabry being referred back to Michelson's Cd_R, obtained *in vacuo*, presumably, and based themselves, except for the *Li* and *Na* lines, on sources *in vacuo*, but under con-

[1] *Rap. Cong. Int. de Phys.*, 1900, **1**, 70.

[2] *C. R.*, **130**, 489, 1900.

[3] ASTROPHYSICAL JOURNAL, **4**, 176, 1896.

ditions bearing an unknown relation to those found in Michelson's *Cd* tubes, are subject to corrections of the same order of magnitude but of unknown amount. This change of vibration period, with the pressure about the radiant, is a very serious matter, especially in certain phases of stellar spectroscopy, and needs a thorough overhauling. Applying the correction indicated to Michelson's *Cd* wave-lengths, the wave-length of D_1 in air at 150 and 760 mm appears to be very nearly 5895.95, as the result of the interference measures thus far made. How far further measures may change this result is a little uncertain, but so masterly was Michelson's original work that it would be rather surprising if the outstanding uncertainty involved more than one or two units in the last decimal place above. In other words, the value of D_1 should certainly fall among the values just tabulated. Its exact location must be slightly dubious until the matter of pressure shifts in the metallic lines involved has been worked out. The work of Huff[1] on Cd_B and of Haschek[2] on various spectra shows that the wave-frequency corresponding to a given line is subject to variation of a very complicated kind, so that comparisons involving solar, spark, and arc spectra, under various conditions, may easily be affected by indeterminate errors amounting to 0.01 tenth-meter or more.

As regards purely experimental errors, grating measurements of absolute wave-length are fairly satisfactory. So far as the probable errors of the experiments with a single grating go, a wave-length determination may be carried on with a high degree of precision. As an example, I may cite my own work with G III (*loc. cit.*). I have fortunately at hand my original records, and deducing the probable error of the comparisons of G III with the standard of reference S_2^a, I find that it amounts to rather less than than one two-millionth without weighting or applying Pierce's criterion. The probable error of the angular determinations similarly treated is 0.00000055, so that the probable error of the result, exclusive of the reference of S_2^a to A_0

[1] ASTROPHYSICAL JOURNAL, 14, 41, 1901.

[2] ASTROPHYSICAL JOURNAL, 12, 181, 1900.

(or other ultimate standard of length) amounts to very nearly 0.005 tenth-meters. The results with the other gratings are of about the same precision as regards the angular determinations, but the glass gratings gave somewhat larger probable errors in the length determination. But so far as experimental errors are concerned the result should, from any grating, be in error by less than one five hundred-thousandth. Kurlbaum[1] and Thalén[2] confirm this degree of angular precision so far as their results for a single line and order go, about as theory would indicate from the number of their observations. In other words, with a good modern grating and spectrometer, angular readings should be coherent to 0.01 tenth-meter or less as regards probable error. The measurement of the distance between the terminal lines of n grating spaces, however, is a very different matter. The things to be compared differ greatly in four important particulars: mass, thermal conductivity, coefficient of expansion, and character of the defining lines, each affecting the precision of the measurements in differing degree according to circumstances. The two materials used for gratings are both bad in this respect. Glass is used only for very small gratings, and, while its coefficient of expansion is rather close to that of platinum-alloy and steel standards, it has considerably less than 1 per cent. the conductivity of either, and is transparent besides. Speculum metal, used for the larger gratings, has an expansion coefficient varying slightly in different specimens, but very close to that of brass, than which its conductivity is somewhat less. It therefore is easy to compare with bronze standards, but needs very good temperature control when used with other standards.

The difference in defining lines on various standards is liable to cause material errors. Gratings have beautifully thin, sharp lines compared with which those on most standards look like plough-furrows. This encourages errors of setting, since the coarse lines become ill-defined under any power which permits very precise setting.

[1] *Wied. Ann.* 33, 159, 381, 1887.

[2] *Nova Acta, Ups.* 1899, p. 1. (Date on title of paper itself is 1898.)

As to the total magnitude of the errors in experimental work on grating measurements, there is considerable information available. A good example is found in the work on my standard S_2^1, which resulted in locating a change of length. In this case the comparison by Professor W. A. Rogers with the several double decimeters on his standard R_2 (bronze) and my own determination from a steel standard A_4 which had been compared by Rogers with R_2, differed by only 0.3μ in 200 mm. Benoit[1] thinks that an exactitude of 0.2μ can be guaranteed in a comparison between lengths under favorable conditions, or a little better than that with short standards. The probable error of the Berlin comparison of S_2^1 with the standard R_{78} was 0.15μ, and hence by this comparison the probable error in the length of G III or G IV should not exceed 0.01 tenth-meter when reduced to wave-length.

Now this Berlin standard R_{78} was directly compared with the gratings used by Müller and Kempf[2] and by Kurlbaum. It, therefore, affords a means of directly comparing the results derived from ten gratings used for absolute measurements. Müller and Kempf's gratings were about 20 mm broad, and the probable errors of the length determinations were not far from 0.2μ or about 0.05 tenth-meter in wave-length. Kurlbaum's gratings were 42 mm and 43 mm broad, and the probable error of the length determination is of the order 0.02 tenth-meter.

Collecting the results from the ten gratings in question as referred to R_{78} and applying no corrections thereto except to reduce all to 15° C., the wave-length of D_1 is as follows:

M. and K.	"2151"	5896.46
M. and K.	5001	5896.14
M. and K.	8001	5895.97
M. and K.	8001 L	5896.33
Kurlbaum	I	5895.84
Kurlbaum	II	5895.96
Bell	I	5896.24
Bell	II	5896.18
Bell	III	5896.32
Bell	IV	5896.16

Mean = 5896.160, or omitting "2151," = 5896.126

[1] *Rap. Cong. Int. de Phys.*, 1900, 1, 66.

[2] *Publ. Potsdam Observ.*, V, 1886.

Between the extremes there is a difference of no less than 0.62 tenth-meter, almost one ten-thousandth! Casting out "2151" entirely as exceptionally bad and recognized so to be, and making a liberal allowance for the probable errors just examined, there still remains an outstanding variation of more than 0.3 tenth-meter which cannot by any reasonable process be charged to experimental errors of any kind. It represents, in fact, the magnitude of the errors in ruling, the difference between the mean grating space as measured and the grating space optically effective in producing the spectra upon which the angular measurements are made. The analysis of the error is worth following up.

In general, the ruling of a grating is not uniform, and a grating of total breadth B, instead of having n spaces, each of $1\,\mu$ in width will have

$$B = o\,(b\mu) + p\,(c\mu) + q\,(d\mu) + \cdot\cdot\cdot\cdot\,.$$

One compares his standard of length with the first member of this equation, but obtains his spectra from the gratings defined by the second member. In practice there is one greatly predominant grating space, say $b\mu$, o being a very large fraction of n.

The theory of errors in ruling gratings has been investigated rather fully by Rayleigh,[1] Rowland,[2] Cornu,[3] and others, but there is much relating to the effect of these errors on absolute and relative wave-length measurements which does not at once appear from the equations. For ordinary ocular observations the chief practical requirement is that the errors, whatever they may be, shall not perceptibly injure the general definition or produce visible false lines, assuming the full aperture of the grating to be in use. This merely demands that the term in $(b\mu)$ above shall predominate so far as to drown out the spectra due to the other grating spaces.

This condition may coexist with very serious errors, which would appear on thorough investigation of the grating. It is these optically ineffective errors which cause the most serious trouble in spectrometry. A grating in which the errors were so large and so distributed as to give blurred definition, false lines,

[1] *Enc. Brit.*, **24**, 437 *et seq.* [2] *A. and A.*, **12**, 129, 1893. [3] *C. R.*, **116**, 1421, 1893.

and large variations in focus would not be used in these days for any delicate or important work.

The commonest error in ruling is a simple periodic variation in the grating space, due to eccentricity of the screw-head, periodic errors in the screw itself, or bad bearings. Its period is generally the revolution of the screw and the amplitude of the fluctuation in grating space is usually rather small in carefully ruled gratings, so that the definition is seldom perceptibly injured. The well-known effect of this particular error is to produce "ghosts" and to diffuse a little light (Rowland, *loc. cit.*).

In good modern gratings ghosts are inconspicuous save now and then in the high orders, and they are from their symmetrical position so easily distinguished as to give little trouble to the experienced spectroscopist. In photographing metallic, or other bright line spectra, ghosts may commonly be detecte deven in gratings giving the most superb definition. The ultra-violet *Mg* lines are most effective ghost-raisers[1], and the grating that will not respond to them on prolonged exposure is a treasure. So far as measurements are concerned the ordinary periodic errors are not serious if B is, as usual, an integral number of periods.

An error almost equally common is the simple linear error, usually chargeable to variations of temperature. If considerable in magnitude and extent it ruins the definition and throws the spectra on the two sides widely out of focus. The latter effect may exist even when the definition is good, and has been investigated by Cornu (*loc. cit.*). The danger of this particular error lies in the fact that it seldom affects the whole ruled surface and commonly appears at one end only. For instance in a grating 10 cm broad the error may only affect 1 cm or less to a perceptible degree. It may be found by going over the grating with a slit a few millimeters wide and looking for displacement of a spectral line. It is rarely perceptible when the full aperture is used, either in the definition or focus, and has to be looked for carefully; when small in extent it does not affect the principal spectrum at all but is, of course, included in measuring B and

[1] Especially the one at λ 2852.

may lead to a serious error in absolute measurements. Once located the extent of the error introduced in B may be found by the process of micrometrical calibration which I used, or by computation from the change in focus, following Cornu. Either process will enable the greater part of it to be eliminated, but a residual error due to minor variations of the same kind of too small extent to be readily eradicated is likely to remain unless in exceptionally good gratings. The definite errors in B which I found produced by such errors in the grating space amounted in one grating to 0.45μ in 10 cm.

These common abnormalities must be watched for in photographic work, for they may easily give rise to false lines, particularly in the higher orders where the increased dispersion leads one to look for additional lines. False lines due to this cause can of course be eliminated by stopping successive portions of the grating, beginning at the ends, which should first be under suspicion. Relative measurements with the concave grating can hardly suffer much from errors of this kind unless in using the method of coincidences a line of the higher order is shaded off by such a false line so as apparently to displace it.

In using closely ruled gratings, wherein space and groove do not differ greatly in width, spectra of faint orders may suffer especially from false lines, since the extinguishment factor will not be the same for the normal and the aberrant grating spaces so that one may confuse the real and false lines, or they may so run together as to give no sign of trouble in the character of the resultant line. In gratings such as one would use for careful measurements these phenomena would very rarely rise to material amounts, and the apparant shifting would almost universally be far within the small micrometric errors.

Far graver are the single or recurrent, non-linear errors such as the one which I found in grating III[1], used for my wavelength measurements. This instance was the appearance, in a grating otherwise of wonderful uniformity so far as could be detected, of a group of lines of spacing widely abnormal, involving

[1] *Am. Jour. Sci.* (3) **35**, 361, 1888.

an error of 2.5μ in B within the space of less than one hundred lines. Only about twenty lines were concerned in the worst of the variation so that it is well within bounds to say that the grating space at this point was aberrant by about 10 per cent. The lines too were here scored deep and rough. The calibration centimeter by centimeter showed only trifling errors elsewhere.

Errors of this kind and magnitude are rare, but cannot be detected by anything save the microscope and not always even so. In this instance the slit test showed nothing, and while it is possible that a photograph of a bright line spectrum with long exposure might have given a false image it would have been so wide from the real line that the connection would not have been apparent.

I am inclined to think that in errors of this kind the discrepancy between grating and interference measurements has its source. A very small recurrent group of slightly aberrant grating spaces is amply sufficient to account for the whole observed difference. Suppose for example that in a grating 10 cm broad and ruled at $s=0.0025$ mm, like my grating IV (*loc. cit.*), there were in each millimeter a group of ten lines spaced even so little as 0.0025μ too widely. Then each group would represent an error of 0.025μ in B and the accumulated error would be one forty-thousandth in the resulting wave-length.

But neither the slit test, calibration, or difference in focus would give the slightest clue to such an error. Photography would show the existence of a special case of ghosts but could not lead one back to an error beyond the reach of the micrometer.

The requirement for such a fault is a recurrent aberrance of a few lines in the same sense, periodic as regards the engine but non-sinusoidal, so that the effect is cumulative as regards B. The spectra from the main grating space alone are visible but B is increased by a relatively considerable quantity. Such an error could be produced by a hard streak along the axis of the screw, bearing the merest trifle unequally upon the nut perhaps even in passing a split, or by a very minute inequality in the thrust bearing.

Even the accidental recurrence of errors enormously less than that found in grating III would produce a large error in B, but they would hardly persist through many gratings from the same engine. It is sufficient to call attention to this type of error, not reckoned with in the ordinary consideration of gratings, but easily sufficient to render gratings useless for absolute measurements.

Some new light is thrown on grating measurements by the research of Thalén,[1] who worked with a Rowland grating which was measured before and after the angular measurements, at the International Bureau of Weights and Measures. Thalén found for D_1 the value 5895.946, which agrees with the interference measurements in a most striking manner. But since the two measures of Thalén's grating made three years apart differ by very nearly one five hundred-thousandth, this extreme precision must be considered more apparent than real. One might naturally infer that Thalén was fortunate enough to obtain a grating practically without errors. His measurements however are in doubt by an amount considerably greater than the above value for D_1 would indicate, since, as Kayser[2] points out, he found in measuring various lines in absolute measure a nearly constant difference from Rowland's values quite irrespective of wave-length, while obviously he should have found a constant *ratio*. The unusual feature of Thalén's research was his extension of the angular measures over a very wide temperature range, thus concurrently determining both the deviation and the temperature coefficient of the grating. This innovation is of dubious usefulness, since his value for E from all the measures was 5270.315, while reducing the observations near $15°$ by the coefficient of expansion deduced from other lines observed at widely different temperatures gave $E = 5270.378$.

Considering now the wide discrepancies found in wavelength measurements referred to the same standard of length, the relatively high precision of the angular measurements involved, the known and great effect of errors in ruling and the

[1] *Nov. Act. Ups.*, 1898. [2] *Handbuch der Spectroscopie*, 1, 707.

probability of the particular kind of error to which I have here drawn attention, I am forced to the conclusion that the diffraction grating is, and is likely to remain, entirely insufficient for the determination of absolute wave-length.

So far as relative measurements are concerned the method of coincidences is so precise as to leave little to be desired. Such possible discrepancies as may exist in Rowland's results, are with rare and dubious exceptions, well within the experimental errors of any method used for checking them. Rowland's values were so thoroughly cross-referenced that there was small chance left for large accidental errors.

In a most interesting recent paper Perot and Fabry[1] have instituted a direct comparison between certain solar lines and the green *Cd* ray at λ 5086. From their interference measurements they have derived an apparent systematic error in Rowland's tables, expressed in an irregular curve already published elsewhere.[2] The variations from a uniform ratio are rather small, in the neighborhood of 0.01 tenth-meter for the most part, and the curve evidently contains the summed errors of Rowland's and their own investigations.

As the solar lines examined do not correspond with any fundamental lines determined by interference methods it is difficult to obtain any check on the results. One can be derived, however, through Cd_B measured by Michelson, and *Zn* λ 4810 measured by Perot and Fabry themselves.

It happens that these two lines when reduced through Rowland's table of standards lead to the same value for D_1 also, of course, showing exact agreement as to their relative wave-lengths with Rowland's table.

Now, if Perot and Fabry's curve of ratios represents the facts, its value at these wave-lengths, applied to the wave-lengths found for the two lines mentioned, ought to lead directly back again to Rowland's values for the same bright lines.

In other words, if Rowland's values for this pair of lines are correct as regards relative wave-lengths, then Perot and Fabry's

[1] *Ann. de Ch. et de Phys.*, January 1902, p. 98. [2] *C. R.*, 133, 154.

ratio determined with respect to the solar tables on which these two wave-lengths are based, must apply, provided Perot and Fabry introduced no error in passing from their green cadmium line to their solar wave-lengths. If Rowland's relative values for this pair are wrong, then the relative values of Perot and Fabry, and Michelson for the same lines, must also be wrong, since they agree with Rowland's.

In point of fact the ratio in question leads to a value 0.02 tenth-meter too small for Rowland's lines. Two other lines of *Zn*, λ 4680 and 4722, lead to almost exactly the same discrepancy.

Of course, the question of pressure shifts as between the arc in air and the spark at reduced pressure enters any such comparison, but if it should prove an adequate explanation it would furnish proof conclusive that wave-lengths cannot safely be carried to a higher degree of precision than just indicated without exact knowledge of and correction for the pressure shift in the source used.

Another check on the validity of these corrections lies in the comparison between Perot and Fabry's values for certain bright iron lines and the same lines reversed in the same solar spectrum. From their tables,[1] one can cull nine pairs of corresponding lines. In these the shift of the solar lines varies from $+0.019$ tenth-meter to -0.019 tenth-meter, again denoting errors or physical differences demanding explanation, of about the same order of magnitude as those just recorded and those indicated in the table of values for D_1 previously given.

Finally reference must again be made to the wide discrepancy between the results of Hamy and others for the green *Cd* line λ 5086. This amounts to 0.079 tenth-meter. If differences in the source, or in the appearance of the line in the interferometer are competent to produce a difference like this, then Cd_G is a very shaky sort of standard. It is worth noting that the reversal of this line in the Sun, if Rowland's tentative line is really a reversal, corresponds very closely to Hamy's value for the head line of

[1] *Ann. de Ch. et de Phys.*, January 1902.

his triplet. A similar condition has been noted by Rowland[1] in the case of some of the *Mg* lines.

In view of these facts Perot and Fabry's corrections to Rowland's tables seem to be based on inadequate evidence. There are doubtless plenty of minute errors in this table but it has yet to be shown that the interference methods are sufficiently developed to evaluate them with certainty.

To clear up the wave-length matter we need most of all an entirely independent determination by Michelson's method to show its working limits of error; second, a very thorough study of pressure shifts to define standard working conditions; and finally, a careful study of the constancy of wave-length in the solar lines, following up Jewell's admirable preliminary paper.[2]

For absolute measurements the grating must now be considered as out of the game. For relative measurements it has less equivalent dispersion but greater brilliancy and sharpness of definition than the interference methods. Personally I do not think that the available evidence points to a greater real precision than about 0.01 tenth-meter for either method in the present state of the art. At about this point one reaches a dubious ground among pressure shifts and similar small corrections where the values become very uncertain. To the interpretation of these phenomena much future work must be addressed, for they may furnish the key to the great problems of molecular mechanics.

BROOKLINE, MASS.,
March 14, 1902.

[1] *A. and A.*, **12**, 321 *et seq.*

[2] ASTROPHYSICAL JOURNAL, 3, 89, 1896.

THE APPARATUS FOR THE ELECTRIC HEATING OF THE POTSDAM SPECTROGRAPH, NO. III.[1]

By J. HARTMANN.

THE effects of changes of temperature become very disturbing in the case of all accurate measurements made with prism spectroscopes. This is particularly the case in the photography of stellar spectra, for then the spectrograph attached to the refractor is exposed to all the variations in the air temperature in the well-ventilated dome during exposures commonly of an hour or more. The first plates of this kind, taken by Vogel and Scheiner in 1888 for determining the velocity of stars in the line of sight, indicated the necessity of paying particular attention to the temperature changes, and Professor Vogel gives in the *Publicationen des Astrophysikalischen Observatorium* (**7**, 24, 1892), a precise statement of all the phenomena coming into question.

The effect of a change in the temperature during an exposure is in the first place due to the change in the index of the prisms, and in the second to the thermal expansion of the metallic portions of the apparatus. If the temperature of the prisms rises, the deviation increases, and the spectrum is displaced on the plate in the direction of shorter wave-lengths. The amount of this displacement is only slightly altered by the expansion of the metallic parts. If such a change of temperature occurs during the exposure of a star spectrum, the lines will not only become diffuse, but they may also be displaced with respect to a comparison spectrum taken on the same plate, causing a systematic error in the determination of the star's velocity. To avoid this error Professor Vogel then arranged to give to the comparison spectrum a number of short exposures

[1] Translated from advance proofs, sent by the author, from the *Zeitschrift für Instrumentenkunde*, **21**, 313–325, 1901, to which acknowledgments are also due for the cuts.

distributed uniformly through the whole exposure to the star spectrum. All the displacements undergone by the star spectrum during the exposure must thus be transferred to the comparison spectrum, and although the lines of both spectra would now appear somewhat lacking in sharpness, there would be no danger of a systematic error in the relative positions. That this arrangement was effective in greatly diminishing the influence of temperature variations is best proven by the accuracy of the results then obtained. It did turn out to be necessary to limit the exposure to about an hour, since the increasing broadening of the lines with longer exposures made accurate measurements more and more impossible.

The displacement of the spectrum due to the variations of temperature in one evening is so considerable that it may be several times as large as that due to the motion of the star. The displacement of the spectrum per degree of change of temperature can be computed from the values of the temperature coefficient of the indices of the different kinds of glass, as determined by Müller,[1] Pulfrich,[2] and others.

The following table gives the displacement $d\lambda$ for 1° C. for the lines $H\gamma$ and D with the three flint prisms employed by Müller, and with compound prism No. 6. v is the velocity of a star which would produce an equal displacement of the lines:

TABLE I.

Displacement of lines for 1° C. change of temperature.

Prism.	$H\gamma$, $\lambda = 4341$		D, $\lambda = 5893$	
	$d\lambda$	v	$d\lambda$	v
Flint No. 1	0.28 t. m.	19.4 km.	0.52 t. m.	26.5 km.
Flint No. 2	0.37	25.4	0.68	34.6
Flint No. 3	0.29	20.2	0.42	21.4
Compound No. 6	0.35	24.2	0.73	37.2

[1] G. MÜLLER, "Ueber den Einfluss der Temperatur auf die Brechung des Lichtes in einigen Glassorten, im Kalkspath und Bergkrystall," *Publ. des Astroph. Obs.*, **4**, 149, 1885.

[2] C. PULFRICH, "Ueber den Einfluss der Temperatur auf die Lichtbrechung des Glases," *Wied. Ann.*, **45**, 609, 1892.

The table shows that very large displacements occur, and that they vary greatly for the different kinds of glass. The conditions are much more favorable for crown glass, for the displacements are considerably less, and, indeed, become zero at a point near D, changing their direction beyond that point toward the red. The displacement for quartz is nearly the same as for flint glass but with opposite sign. If it was desired to construct a spectroscope as insensitive as possible to changes of temperature, prisms of crown glass or a combination of quartz and flint prisms could be employed.

The figures in the table show only that part of the displacement which is produced by the change of index of the glass. As the position of the spectrum in a compound spectroscope is also affected by the expansion of the metallic parts, it would be of interest to investigate the effect of temperature changes on the completed spectrograph. To this end I have made the following experiment with four spectroscopes. After a spectrograph had stood for a long time in a room of uniform temperature, an iron spectrum was photographed. The room was then heated, and after the instrument had again attained, some hours later, a steady temperature about 15° higher than before, a second iron spectrum was taken on the same plate, which had meanwhile remained entirely undisturbed. The experiment was then repeated with a decreasing temperature. The results are collected in Table II. There is one flint prism of about 60° angle in spectrographs *D* and No I, three in No. III, and two compound prisms in the older instrument *A*, with which Vogel and Scheiner made their plates in the years 1888 to 1890.

These measured displacements deviate considerably from the figures given in Table I, as was to be expected in view of the difference of the kinds of glass and the concurrent effect of the metallic parts. They are of the same order of magnitude for instruments *D*, *A*, and III, and only spectrograph I exhibits a different behavior. For this the displacement is wholly neutralized at $\lambda 4252$ by the expansion of the metallic parts; and the rest of the spectrum is only slightly affected by changes of tem-

perature. This compensation does not occur so favorably in case of rapid changes, as the metallic parts will not always have just the temperature of the prisms.

TABLE II.

Displacement of lines for 1° C. change of temperature.

λ	Spectrograph *D*		Spectrograph I		Spectrograph III		Spectrograph *A*	
	t. m.	km.	t. m.	km.	t. m.	km.	t. m.	km.
3800	.....	....	+0.057	+4.5	.....	....	.....	....
3900	.....	....	+0.046	+3.5	.....	....	.....	....
4000	.....	....	+0.035	+2.6	.....	....	.....	....
4100	+0.251	+18.3	+0.023	+1.7	+0.212	+15.5	+0.233	+17.0
4200	+0.265	+18.9	+0.009	+0.6	+0.215	+15.4	+0.241	+17.2
4300	+0.279	+19.4	−0.008	−0.6	+0.217	+15.2	+0.250	+17.4
4400	+0.293	+20.0	−0.024	−1.6	+0.220	+15.0	+0.259	+17.6
4500	+0.307	+20.5	−0.041	−2.7	+0.222	+14.8	+0.267	+17.8
4600	.....	....	−0.059	−3.8	.....	....	.....	....
4700	.....	....	−0.076	−4.8	.....	....	.....	....
4800	.....	....	−0.095	−5.9	.....	....	.....	....
4900	.....	....	−0.114	−7.0	.....	....	.....	....
5000	.....	....	−0.134	−8.0	.....	....	.....	....

In the course of these experiments I tried to answer a second question, viz.: In what way and how soon are the prisms affected by the variations of the external temperature. On another occasion[1] I derived certain theorems bearing on the behavior of bodies not in thermal equilibrium. If Newton's law of cooling holds for such a body, this differential equation will be true:

$$\frac{dA}{dt} = \gamma(A - W), \tag{1}$$

where A is the temperature of the body and W that of the surrounding air, and γ is a constant characteristic of the temperature sensitiveness of the body. By integration we get

$$\frac{\log(A - W) - \log(A_0 - W)}{t - t_0} = M\gamma, \tag{2}$$

where M is the number 0.43429. . . . . We may use equation (2) to test whether or not a body follows the above simple law

[1] *Zeitschrift für Instrumentenkunde*, 17, 14, 131, 1897.

in its changes of temperature. For this purpose we bring the body, at another temperature, into a space at the constant air temperature W, and then we determine the temperature A of the body at suitable intervals. The law of cooling is fulfilled if we always get the same value of the product $M\gamma$ in (2) from all these observations.

In order to render possible the measurement of the momentary temperature of the prisms, I have utilized the change of the dispersion itself as a criterion, since it is so large that the temperature of the prisms can be determined to within a few tenths of a degree by it. The experiments were performed as follows: Spectrograph III was first left for a long time in a space at the constant temperature of $+15^\circ.8$ C., and when it was certain that all parts of the apparatus had assumed this temperature, four plates of the iron spectrum were made (Nos. 1 to 4 in Table III). Then, at the time t_0, the instrument was carried into an adjoining room, which was maintained by heating at a constant temperature of $+26^\circ.0$. Another plate was taken after 20 minutes, and again after 40 minutes, and 1, 2, 3½, and 6 hours. Nineteen hours later, when the instrument had fully assumed the new temperature, plates 11 and 12 were made, and after 24 hours plates 13 to 16. The distance D between the two

TABLE III.

Exposure	Plate No.	$t-t_0$ minutes	θ	D	A	$M\gamma$	A'	O.—C.
1......	III 396	....	$+15^\circ.8$	65.196	+15.2	....	$+15^\circ.8$	$-0^\circ.6$
2......	397	....	15.8	217	16.3	....	15.8	+0.5
3......	402	....	15.8	209	15.9	......	15.8	+0.1
4......	403	....	15.8	210	15.9	....	15.8	+0.1
5......	396	20	20.7	237	17.3	−0.00338	18.0	−0.7
6......	397	40	22.6	293	20.1	−0.00594	19.7	+0.4
7......	398	60	23.8	313	21.1	−0.00531	21.0	+0.1
8......	399	120	25.4	368	23.9	−0.00569	23.6	+0.3
9......	400	210	25.9	392	25.1	−0.00500	25.2	−0.1
10......	401	360	26.0	411	26.0	....	25.9	+0.1
11......	399	1140	25.8	403	25.6	....	25.8	−0.2
12......	402	1140	25.8	408	25.9	....	25.8	+0.1
13	398	1440	26.8	423	26.7	....	26.8	−0.1
14......	400	1440	26.8	418	26.4	....	26.8	−0.4
15......	401	1440	26.8	438	27.4	....	26.8	+0.6
16......	403	1440	26.8	428	26.9	....	26.8	+0.1

iron lines at λ 4202 and 4495 was measured on all the plates. The reduction of this series of observations is contained in Table III, in which θ denotes the reading of thermometer belonging to the spectrograph, its bulb in the interior of the prism-box. (I shall again advert to the behavior of this thermometer.) D is the measured distance between the two iron lines, expressed in revolutions of the micrometer screw of 0.5 mm pitch, and A is the prism temperature computed therefrom.

The computed values were obtained as follows: We get from exposures

1–4	$D = 65.2080$ revs. at $A =$	15°.8
11 and 12	65.4055	25.8
13–16	65.4268	26.8

Hence for 1° rise of temperature we get a change of 0.01982 revs. in D, whence follow the values of A. The values of the product $M\gamma$ were computed by (2) from exposures 5 to 9, made during the rapid change of temperature. There is no marked progressive change in these figures, and only the first one differs seriously from the others, in consequence of some accidental disturbance. As the mean of the whole series, we get

$$M\gamma = -0.005238,$$

from which again by (2) the theoretical temperatures A' are calculated, which the prisms would have had if they had more closely followed the law of cooling. The values of $O.—C.$, or $A—A'$, in the last column, show that the prism temperature is exceedingly well represented by these computed values.

A second series of observations was made with falling temperatures, in order to fully assure this result. The results of the measurements, which were of the distance between the lines λ4228 and λ4529, are given in Table IV.

The separate values of $M\gamma$ show again an excellent agreement, the mean being

$$M\gamma = -0.00420,$$

with which value the calculated temperature A' represented the observed prism temperatures within the errors of observation.

It is not surprising that the numerical value of $M\gamma$ differs somewhat from that found from the first series, since the value of these constants is in a high degree dependent upon the motion

TABLE IV.

Exposure	Plate	$t-t_0$ minutes	θ	D	A	$M\gamma$	A'	O.–C.
1	III 404	..	+26°8	50.405 revs	+27°0		26°8	+0°2
2	405	..	26.8	399	26.6		26.8	−0.2
3	406	15	23.4	382	25.5	−0.00379	25.3	+0.2
4	407	30	21.7	371	24.7	308	24.1	+0.6
5	408	45	20.7	351	23.4	368	23.0	+0.4
6	409	60	19.8	337	22.4	375	22.0	+0.4
7	410	90	18.3	307	20.4	433	20.5	−0.1
8	411	120	17.5	280	18.6	519	19.4	−0.8
9	412	180	16.7	263	17.4	489	17.9	−0.5
10	413	240	16.3	265	17.6	350	17.1	+0.5
11	414	900	15.7	236	15.6		15.7	−0.1
12	415	1440	15.7	240	15.9		15.7	+0.2

of the surrounding air, as I have shown elsewhere. I take as mean of the two series

$$M\gamma = -0.0047 .$$

Inasmuch as the temperature of the prisms follows very closely the law of cooling, as shown by these series of observations, we may now employ the theorems I have proposed in order to obtain an accurate conception as to the course of the variation of the prism temperature. Special interest would be attached to the answer to these two questions: How does the prism temperature change (1) with a sudden change of the external temperature, (2) with a uniform change of the air temperature? The first case occurs when the instrument is brought from a cold into a warm room and is to be used in the latter, or if the dome is opened and aired before beginning exposures of stellar spectra in the evening. A nearly uniform fall of the air temperature will then commonly occur during the exposures. The reply to the first question is contained in Table V, which is calculated with $M\gamma = -0.0047$ in equation (2). $A - W$ is as before the difference in the temperature of prisms and air; t is the corresponding time.

As an illustration of the use of the table, suppose the air temperature quickly fell 4° after opening the dome; then the value of t corresponding to this $A - W$ would be approximately $2^h\ 30^m$. The table then teaches that after another hour, at $t = 3^h\ 30^m$, the prism temperature will be 2° higher than that of the external air; and after two hours will still be 1° higher. In round numbers we may say that the difference between the temperature of the prisms and of the outer air decreases by one half per hour (more precisely in 64 minutes).

TABLE V.

$A-W$	t	$A-W$	t	$A-W$	t
	h m		h m		h m
20°00	0 00	6°77	1 40	0°56	5 30
18.07	0 10	5.46	2 00	0.41	6 00
16.11	0 20	3.94	2 30	0.21	7 00
14.45	0 30	2.85	3 00	0.11	8 00
12.97	0 40	2.06	3 30	0.06	9 00
11.64	0 50	1.49	4 00	0.03	10 00
10.45	1 00	1.08	4 30	0.01	12 00
8.41	1 20	0.78	5 00	0.00	18 00

The second question as to the change of the prism temperature with variable air temperature is answered by formula 4, on p. 15 of my paper already referred to, viz.,

$$\log(W_1 - x) = M\gamma t + \log(W_1 - x_0).$$

Here $x = \gamma(A - W_0 - W_1 t)$; the subscript 0 is given to the values for $t = 0$, and W_1 is the change in the air temperature in one minute. To illustrate the use of this formula I compute the following example:

Let the air temperature fall rapidly from +15° to +10° after opening the dome, and thereafter fall 1°2 hourly, or 0°02 per minute. Then we have

$$A_0 = +15^\circ \qquad W_0 = +10^\circ \qquad W_1 = -0^\circ 02$$
$$M\gamma = -0.00470 \qquad \gamma = -0.0108$$
$$x_0 = 5\gamma = -0.0540 \qquad W_1 - x_0 = +0.0340.$$

Table VI shows the resulting course of change of the temperatures.

TABLE VI.

t	Temperature of air, W	Temperature of prisms, A	$A-W$
h m			
0 00	+10°00	+15°00	+5.00
0 30	+ 9.40	+13.53	+4.13
1 00	+ 8.80	+12.30	+3.50
1 30	+ 8.20	+11.22	+3.02
2 00	+ 7.60	+10.31	+2.71
3 00	+ 6.40	+ 8.70	+2.30
4 00	+ 5.20	+ 7.29	+2.09
5 00	+ 4.00	+ 5.97	+1.97
6 00	+ 2.80	+ 4.72	+1.92
8 00	+ 0.40	+ 2.27	+1.87
10 00	− 2.00	− 0.14	+1.86
12 00	− 4.40	− 2.55	+1.85

We see that under the assumed conditions, which might easily occur in observing practice, the prism temperature changes 2°7 in the first hour and 2° in the second hour. This change, moreover, occurs in such a way, as I showed previously, that the difference $A - W$ approaches a fixed limit, in this case 1°85.

The readings θ of the thermometer attached to the spectrograph by no means indicate the true temperature of the prisms during a rapid fall, as is clearly shown in Tables III and IV; in Table III the difference $\theta - A'$ is as large as 2°9, in Table IV as 2°4. As was to be expected, the readings of this thermometer do *not* follow the law of cooling.

These rapid changes of the prism temperature cannot, of course, take place uniformly throughout the entire mass of each prism, but for rising temperature the heat will first warm the thinner parts of the glass — near the edges — and the interior of the prism will not attain this temperature until later. Meanwhile, the prisms are not optically homogeneous, as is clearly shown by the form of the lines of the spectrum. On plates 5, 6, and 7 of Table III, made at rising temperature, there is a faint shading on the violet side of every line; but on plates 3, 4, and 5 of Table IV, taken with falling temperature, these shadings lie, *per contra*, on the side toward red. Inasmuch as the shading always

lies on the side toward which the whole spectrum is moving on account of the temperature change, it follows that the portion of the prism which produced this diffuse shading has taken on the new temperature sooner than the rest of the mass of glass; and this portion is evidently the thinnest part, near the refracting edge. It must, however, be pointed out that this deterioration in the sharpness of the lines from the thermal strain of the prisms is not very large, even in case of the very strong changes of temperature given above — the lines can still be set upon quite accurately. The loss of sharpness due to the gradual shifting of the whole spectrum is incomparably larger.

In view of what has been said, no further doubt can remain that the prism temperature must be maintained constant for all plates of stellar spectra which are to be accurately measured. Several astrophysicists have accordingly provided their spectrographs with heating apparatus. Wrapping the instrument in poor conductors, such as felt or woolen covers, can only delay the escape of heat; with a long continued uniform fall in the temperature of the surrounding air, such a wrapping will indeed be entirely useless, for, as appears from Table VI, the change of temperature will proceed at the same rate both within and without.

Deslandres[1] was the first to introduce the artificial heating of a spectrograph, at the Meudon Observatory. He surrounded the whole apparatus with a strong metallic case with double walls, between which constantly flowed a stream of water from the city mains. Since the temperature of this water was pretty constant, he thus succeeded in maintaining the temperature of the prisms at nearly the same point for days at a time. In spite of the simplicity of this arrangement, it is hardly to be recommended, as it greatly increases the weight of the apparatus, and does not insure the perfect constancy of the temperature.

Lord[2] provided the stellar spectrograph, of the McMillin Observatory, at Columbus, with electric heating. Spirals of

[1] *Bulletin Astronomique*, 15, 49, 1898.

[2] ASTROPHYSICAL JOURNAL, 8, 65, 1898.

wire, which could be heated by a current of about six amperes, were attached at suitable places on the metallic frame of the apparatus; the whole is wrapped in a double layer of felt, through which projected a thermometer, the bulb of which was within the prism-box. The observer reads the thermometer from time to time, and turns on the current for a short interval as soon as a fall in the temperature is noticed.

The heating apparatus applied by Campbell to the Mills spectrograph of the Lick Observatory is quite similar, except that he made the important improvement of placing the bulb of the thermometer outside instead of inside of the prism-box, and in the large wooden box which covered the whole spectrograph. In this way the observer can correct for any small change of temperature occurring *outside* of the prism-box by promptly turning on the current. Careful observation of the control thermometer made it possible to maintain the temperature inside the prism-box within 0°.1 C. Inasmuch as Campbell employs only a single thermometer attached to one wall of the felt-protected outer box, for determining its temperature, there is a danger of a stratification in the temperature within the box during the exposure of each star. The thermometer in the prism-box would then give a constant reading during the exposure to the particular star; but on pointing to another part of the sky, with the spectrograph in another position, the prism temperature might change from the effect of another air stratum, even if the readings of the outer thermometer were kept constant by heating.

In order to avoid taxing the observer's attention too much with the temperature control as it is sufficiently occupied with accurately holding the star on the slit of the spectrograph, I have arranged the heating for the new spectrograph (No. III) of the Potsdam Observatory so that the temperature of the air surrounding the whole instrument is automatically kept constant. The kindness of Professor Hagen gave me an opportunity of becoming acquainted at the *Reichsanstalt* with the different kinds of thermostats which have proven to be practical. The device

made according to my specifications by Mr. Toepfer, of Potsdam, is shown in Figs. 1 and 2.

Professor Vogel has given a precise description of the spectrograph in the ASTROPHYSICAL JOURNAL (**11**, 393, 1900). As this paper may be inaccessible to many readers of the *Zeitschrift für Instrumentenkunde*, I will first briefly describe the spectrograph.

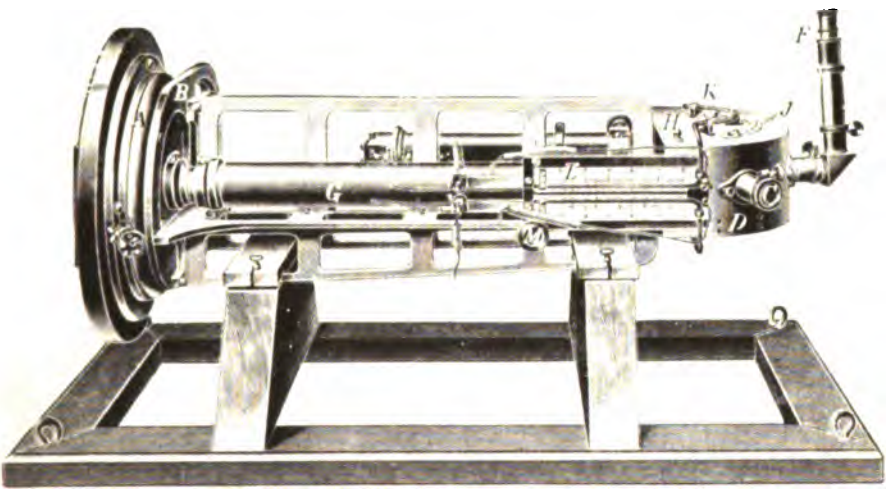


FIG. 1.

Fig. 1 shows it mounted on the wooden support as it is used for plates made in the laboratory. It is attached to the adapter of the 80 cm refractor by the strong cast-iron plate *A*, as shown in Fig. 2. An iron plate *B*, 10 mm thick, is firmly screwed to the plate *A*, and itself supports a very rigid truss-work of iron plates. To this the optical parts are attached. *G* is the collimator, of 32 mm aperture and 48 cm focus. The prism-box *D* contains three excellent prisms of the heavy silicate flint O 102 of Schott & Co., furnished by Steinheil. Their refracting angle of 63° 27′ is so chosen as to make the total deviation for $H\gamma$ accurately 180°, and the prisms are rigidly set for minimum deviation for $H\gamma$. The camera *E* has an objective of 40 mm

aperture and 41 cm focus; it can be rotated about an axis close in front of the last prism surface, and can be rigidly screwed to the base plate in five different positions, in which the rays D, F, $H\gamma$, $H\delta$, and K successively fall at the center of the plate. This camera can also be wholly removed and a longer one of 42 mm aperture and 56 cm focus can be substituted. The guiding telescope *F* receives the light reflected from the first surface of the second prism, after it has been dispersed by the first prism, and furnishes a permanent control on the accurate position of the star in the slit. For faint stars this telescope can be slipped into the tube *C* and then the image of the slit as directly reflected from the first prism surface is observed. At *H* there is a small slit through which may be slid the various diaphragms used in testing and focusing the apparatus. The whole prism-box consists of stiff sheet nickel and offers an excellent protection against radiant heat. The bulb of the thermometer *J*, the readings of which were designated above by θ, is situated directly under the surface of the prism-box.

For controlling the air temperature outside of the prism-box, two mercury contact thermometers are attached, with their long semicircular bulbs at a distance of 2 cm from the two base surfaces of the prism-box. The upper bulb is visible at *K*. The capillaries of the thermometers are led after several bends to the side of the collimator, where they are attached to their scales *L* parallel to each other. Platinum wires are melted into the glass just below the ends of the scales, making contact with the mercury column. Two platinum wires of 0.3 mm diameter are introduced at the open ends of the capillaries, which are of 0.5 mm diameter, and can be set by the rack and pinion *M* at any desired points on the scales.

If the spectrograph is to be used for stellar spectra it is inclosed in the light wooden box seen in Fig. 2, which is attached by six long screws to the edge of the plate *B*, but otherwise does not touch the spectrograph. Two electric heaters are attached on the inside of this box at points opposite to the semicircular bases of the prism-box. One of these heaters is

visible on the opened door in Fig. 2. It consists of a wooden frame *OO* carrying two glass rods, between which a length of 20 meters of German-silver wire of 0.24 mm diameter makes a large number of turns. Spiral springs keep the wires taut at all temperatures. The frame is attached at a distance of 1 cm from

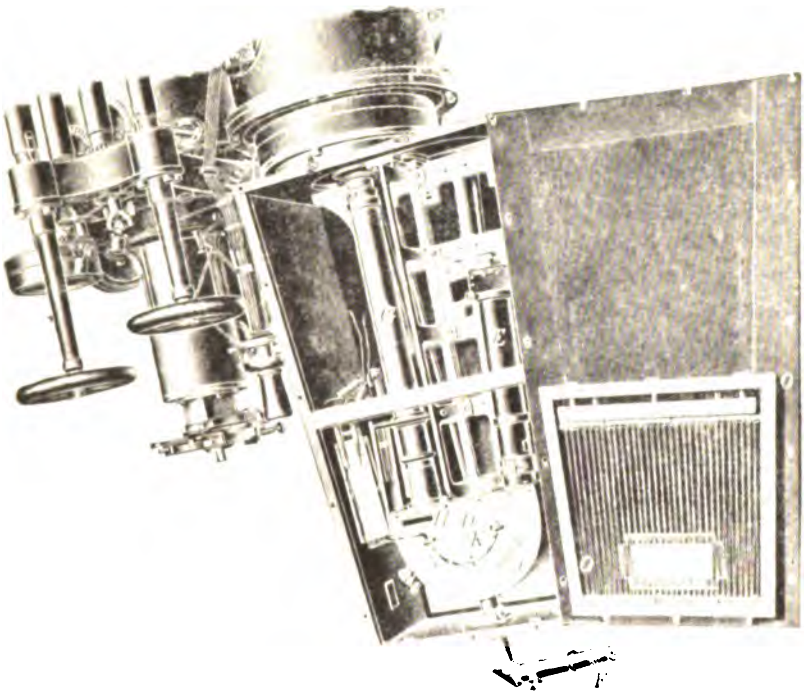


FIG. 2.

the wall of the box; and on the side toward the spectroscope, also at a distance of 1 cm from the frame, a cover of bright sheet metal is fastened, inclosing the whole heater. (In Fig. 2 the cover is removed to make the heating coil visible.) There is, therefore, a free circulation of the air about the heaters, which is an important matter, as otherwise there may easily be marked heat strata, and consequently unequal heating of the different parts of the apparatus. In order to produce a circulation of air in the box by the heater itself, I limited the extent of the heater

to that part of the box which lies at the lowest point during the observations, as is seen in the figure, instead of covering the whole surface of the box by the heater. On turning on the current the warm air will first rise and displace the cooled air at that point. The thermometers do not rise and break off the current until the warmed air has risen to them (at K and similarly below the prism-box). In order to still further exclude injurious air stratification in the box, I separated the heating into two wholly independent parts. The upper thermometer K regulates only the current in the heater on the cover, while the thermometer beneath the prism-box independently regulates the lower heater. If both thermometers were therefore set for the same temperature before the beginning of the observations, it is certain that just this temperature will be continuously maintained on both sides of the prism-box.

The ends of all the conducting wires—two from each heater and two from each thermometer, or eight in all—are connected together in binding posts outside of the box. The eight wires leading to these posts are united in two cables of four strands each, one for the upper and one for the lower heater. The heating wires are distinguished by their color from those leading to the control thermometers. The binding posts for the thermometers contain apertures too small for holding the wires of the heating currents, so that confusion in attaching the wires to the binding posts cannot occur.

The cables are five meters long, and can follow all the motions of the refractor; they lead to a switch board attached to the observing chair, the arrangement of which is shown in Fig. 3.

This switch board also has two precisely similar, separate halves, the right corresponding to the upper and the left to the lower heating circuit. The ends of the cable wires are inserted at the plug contacts $O_1O_2O_3O_4$ and $V_1V_2V_3V_4$, the larger size of the plugs for the heating current preventing mistakes. The current from a single Leclanché cell is led in by the posts A. It passes through the electromagnets of the two relays E_1 and E_2, set at zero current, and to the posts O_1O_2 and V_1V_2, which

are connected by the cables with the control thermometers. If the mercury columns of the thermometers are in contact with the platinum wires, the relay current is closed, the armatures of the magnets are attracted and the heating current is broken off. If one of the thermometers falls below the temperature for which it is set, the proper relay current is broken and the released armature closes the corresponding heating current. The relay current is made as small as possible, only 0.011 amperes, in order

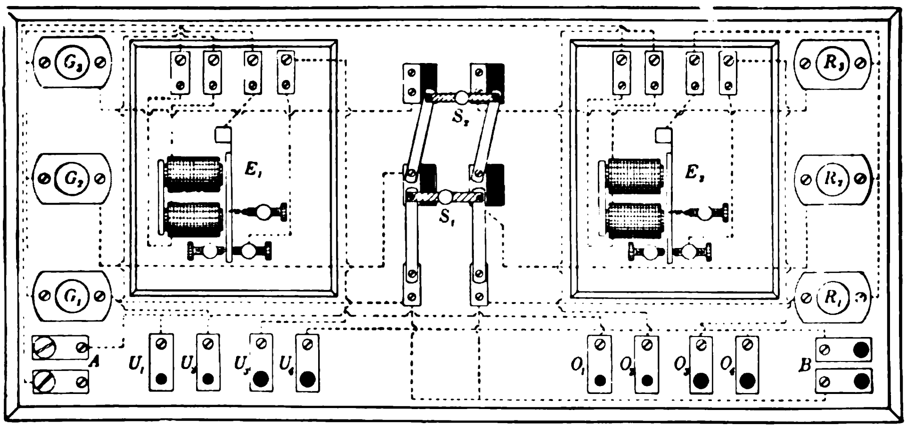


FIG. 3.

to diminish as far as possible the sparking and consequent oxidization at the mercury surface. The heating current is taken from the 110 volt observatory mains and led to the switch board by the two binding posts *B*. It is first conveyed to the two double switches S_1 and S_2 which throw in three different resistances as needed, changing the current strength. I use for resistances ordinary incandescent lamps, which are screwed into the sockets $R_1R_2R_3$ and $G_1G_2G_3$. In socket R_1G_1 and R_2G_2 I employ lamps of 110 volts and 16 c. p.; in R_3 and G_3 lamps of 65 volts and 10 c. p. When both switches are turned to the right (as S_2 in Fig. 3), the current only goes through the lamp R_1 or G_1, from thence to the relays, and if these are closed, through the cable to the heater, being of strength 0.47 ampere. If the cir-

cuit is closed at S_1 by turning the switch to the left, as shown in the figure, the current divides between the lamps in parallel R_1 and R_2 or G_1 and G_2, rising to 0.62 amperes. On throwing in the third pair of lamps R_3 and G_3, we finally get a current of 0.72 amperes.

The use of the lamps for resistances furnishes a convenient check on the correct working of the heater. In order to further reduce the brightness of the lamps, which are already rather weak with the small current, and at the same time to readily distinguish between the two heaters, I colored the lamps of the upper heater red, and of the lower heater green. The observer can always tell whether everything is working properly, without looking at the switch board, by noting the successive lighting up of the red and green lights at uniform intervals.

I arranged the dimensions of the contact thermometers so that 1° C. has a length of 3 mm. When the heaters are in action the variations of the thermometers do not quite reach 0°.1. An almost absolutely constant temperature must prevail in the interior of the prism-box, as these slight variations of short period are considerably diminished by the metal wall and the inner mass of air. The readings of the thermometer J in fact almost never show a change of over 0°.1. The following series of observations serves to illustrate the satisfactory operation of the apparatus, which has been in use for a year past. The evening of November 8, 1900, was clear, and I began observations at 4 : 30 P. M., obtaining the following readings θ of thermometer J, and of a thermometer W indicating the temperature of the air in the dome.

Time	W	θ
4h 30m	+10°.2	10°.9
5 30	9.2	10.9
5 45	9.1	10.9
6 30	8.1	10.8
7 40	7.7	10.8
9 30	7.2	10.8
10 30	6.7	10.8

The heating remained in action over night, and when I again began observing next morning the temperature had not changed at all, being at

$17^h\ 0^m$	$+4°.5$	$10°.8$
18 0	3.2	10.8

Only an automatic heating device could have kept the prism temperature absolutely constant for so long a time while the air temperature fell 7°.

In conclusion let me say a few words as to the mode of setting the platinum contacts of the metal thermometers. The intention is to maintain the temperature of the prisms constant from the time of making this setting. For this the contacts should be set to correspond with the instantaneous temperature of the prisms, which is, however, unknown, since the readings of the thermometer *J* do not accurately record the prism temperature, as has been shown above. I have previously demonstrated[1] that a body obeying the law of cooling always has very nearly the temperature that prevailed in the surrounding air $1/\gamma$ minutes earlier. For spectrograph III $1/\gamma = 92$ minutes, and hence, if no other disturbances have meanwhile occurred, the prisms will be nearly at the temperature the air had one and one-half hours before. A Richards thermograph, attached to the refractor near the spectrograph is used for determining this temperature. After some experience it is possible from the reading of the thermograph, in combination with the thermometer *J*, to set within 0°.1 of the instantaneous temperature of the prisms.

ASTROPHYSICAL OBSERVATORY, POTSDAM,
October, 1901.

[1] *Zeitschrift für Instrumentenkunde*, 17, 16, 1897.

ON THE SPARK DISCHARGE FROM METALLIC POLES IN WATER.[1]

By Sir Norman Lockyer.

During the appearance of the new star in the constellation *Auriga*, which was discovered in January 1892, the Kensington photographs was the first[2] to show that several of the brighter lines were accompanied by absorption lines on their more refrangible sides.

This appearance I explained on the hypothesis that we were dealing with at least two bodies, one giving a radiation, and the other an absorption spectrum, the differential movements of which could be determined by the changes of wave-lengths observed.

In a paper[3] published in the year 1899, Dr. J. Wilsing made the suggestion that, in view of the great velocities shown by the large displacements of the lines in the spectra of new stars, and the occurrence of these displacements in the same direction, some other cause of them was probably at work, and he suggested that the cause might be high pressure, which drives the line towards the red.

THE FIRST OBSERVATIONS OF NON-SYMMETRICAL EMISSION.

The non-symmetrical development of emission lines is of frequent occurrence in ordinary arc spectra. Typical photographs of such phenomena were referred to by me in illustration of papers communicated to the Royal Society more than a quarter of a century ago on peculiarities of emission and absorption spectra.[4]

The following extracts from parts of these communications will serve to indicate the facts observed at that time:

[1] From advance proofs, communicated by the author, of a paper read before the Royal Society, on March 6, 1902 (received January 31, 1902).

[2] *Roy. Soc. Proc.*, **50**, 434. [3] Astrophysical Journal, **10**, 113, 1899.

[4] *Phil. Trans.*, **164**, Part II, 805–813, 1874; *Roy. Soc. Proc.*, **28**, 428–432, 1879.

PHOTOGRAPHS SHOWING NON-SYMMETRICAL LINES.

I. Spectrum showing two *Ag* lines at about wave-lengths 4054.3 and 4210.0. Both lines are fluffy and reversed; the less refrangible line is much more strongly expanded on its more refrangible side, and is carried up to a much greater height as a radiation line than its other side. The more refrangible line is more symmetrical, but presents the same phenomenon to some extent, only in the opposite direction, its less refrangible side being the most developed.

II. Spectrum of *Rb*, showing line at wave-length 4202. Here the two ends of the line are produced by radiation alone, the central portion showing absorption on its more refrangible side, with fluffy shading on its less refrangible side.

Afterwards, when higher dispersions became available, the investigations of Messrs. Humphreys and Mohler on the effect of pressure on spectrum lines[1] showed that the actual wave-length of a line was increased by pressure; thus Humphreys[2] states "the wave-lengths of all fine and sharp lines, *and also of the reversals of heavy ones*, increase with increase of pressure around the arc, no matter how the lines may spread out, symmetrically or chiefly towards either side."

In the case of pressures of twelve atmospheres, a shift of scarcely 0.05 tenth-meter was observed by Messrs. Humphreys and Mohler. Eder and Valenta,[3] in their work on the spark spectra of argon and sulphur under pressure, obtained a displacement amounting to as much as one tenth-meter. With flame spectra of the easily volatile metallic salts, small displacements, averaging 0.4 tenth-meter, were observed by Ebert,[4] and were explained by him as being due to an unsymmetrical broadening of the lines towards the red.

Dr. Wilsing thought that such investigations suggested[5] "the direction which must be taken in the experiments for pro-

[1] ASTROPHYSICAL JOURNAL, 3, 114–135, 1896; 4, 175–181, 249–262, 1896; 6, 169–232, 1897.

[2] ASTROPHYSICAL JOURNAL, 6, 183, 1897.

[3] *Denkschriften der K. Akad. der Wiss. zu Wien.*, 64, 1–39; 67, 97–151.

[4] *Wied. Ann.*, 34, 34–90, 1888.

[5] ASTROPHYSICAL JOURNAL, 10, 115, 1899.

ducing shifts of lines without motion in the sight-line, and ultimately for producing double spectra."

Wishing to avoid the experimental difficulties necessarily connected with the employment of high pressures, he made use of the fact that very high tensions are produced when electric *sparks* are discharged in liquids.

He employed a large induction coil, with a spark gap inserted in the secondary circuit, in connection with a battery. With the passage of each spark "a blinding discharge took place between the electrodes in the water, giving a very intense continuous spectrum crossed by faint lines." The discharge spectra in water and air were photographed on the same plate with a spectrograph, the scale of the spectrum being about 50 mm between λ4800 and λ4600, and the accuracy of the determination of the wave-lengths of the sharp lines could be obtained within a few hundredths of a tenth-meter. Further, several plates were employed which were secured with a grating spectrograph of high dispersion, and with a large prism spectrograph.

Dr. Wilsing investigated in this way the spectra of the metals iron, nickel, platinum, copper, tin, zinc, cadmium, lead and silver, and arrived at the conclusion that "now there occur displacements of lines and double lines which are in every respect similar to those in the spectra of *Nova Aurigæ*." Pressure, then, according to Dr. Wilsing, is the cause of the duplication and broadening of the lines in the spectra of new stars.

The great importance of this result for stellar spectroscopy rendered it imperative to repeat the experiments, and I at once commenced them, using the large Spottiswoode coil, capable of giving a 42-inch spark in air, controlled by placing a large glass plate-condenser in the secondary circuit, so that a spark of length 3 mm was obtained in air, and about 0.5 mm in water. The photographs of the more intense lines in the water-spark spectrum showed very distinct reversals.

The work was postponed a little later owing to this coil being no longer available, but it was again resumed with a smaller (10-inch) coil while waiting for a new large one which is under construction.

With this coil the investigation has been extended by photographing the spark spectra of several other metals in water, and these have furnished material for a more general classification of the attendant phenomena.

The coil used for producing the discharge, being capable of giving only a 10-inch spark, had a 1-gallon Leyden jar placed in parallel with the secondary circuit. The spectrograph employed was a large concave Rowland grating of 6 inches (152.4 mm) diameter, ruled with 14,438 lines to the inch (568.44 per mm), and having a radius of curvature of 21 feet 6 inches (655 cm). The first order spectrum was employed, arranged to photograph the region of the spectrum from λ 3800 to λ 4800, occupying a length of 18 inches (457 mm) on the plate. Distilled water was used in all cases.

Of the metals so far examined (iron, silver, lead, copper, zinc, and magnesium), only three—iron, zinc, and magnesium—show reversals of the principal lines, and those of zinc are very weak.

In all cases the lines of the spectrum of the spark in water are much broader than the corresponding lines in the spectrum of the air-spark. From an examination of the different photographs, however, showing many lines of varying degrees of intensity, it appears that the broadening is, for the most part, of a similar nature to that observed in the arc spectrum in air when an excess of material is introduced between the poles.

THE PHENOMENA PRESENTED BY THE SPARK IN WATER.

A. GENERAL.

In the cases of iron and magnesium, many lines undergo complete reversal, for example, the following:

Iron		Magnesium
λ 4045.98	λ 4325.94	λ 3829.50
4063.76	4383.72	3832.45
4071.91	4404.93	3838.44
4271.93	4415.29	
4308.06		

As shown by the enlargements, this reversal is not always symmetrical with the original bright line, and the part of the emission line on the red side of the reversal is the brighter. It will be evident that in such cases if the exposure is insufficient for the less intense component to be photographed, the appearance of a bright line in a position greatly displaced towards the red will be presented, as is shown in the line of iron at λ 4260.64.

In the case of copper, we have stopped apparently at such an intermediate stage, and the phenomena observed thus appear to agree more closely with those described by Dr. Wilsing. In this case no reversals have actually taken place, and the only lines seen in the water-spark spectrum present the appearance of broad bands, considerably displaced towards the red, and having their more refrangible edges rather sharply defined by absorption, which is not otherwise manifested, while the less refrangible edges are very diffuse.

With zinc two of the lines in the strong group of three in the blue-green region show reversal, the absorption line being nearly normal, separating parts of the emission line of very different intensities. These lines, λ4722.34 and λ4810.72, are much more intense on the red side of the central absorption line. In the remaining line of the triplet at λ 4680.32 there is no reversal, but the maximum of intensity of the emission line is also shifted towards the red.

B. CLASSIFICATION OF THE DIFFERENT PHENOMENA PRESENTED.

Considering the photographs obtained with various exposures and conditions, the phenomena observed may be grouped as follows: (1) Broadened bright line. (2) Broadened bright line with central absorption line. (3) Broadened bright line with non-symmetrical absorption (maximum of emission towards red).

1. *Broadened bright line.*—This appearance is well shown in the spectrum of copper and the under-exposed spectrum of iron.

The broadened line is not of uniform intensity throughout its

PLATE XIII.

Mg 3829, 3832, 3838

b

a

a

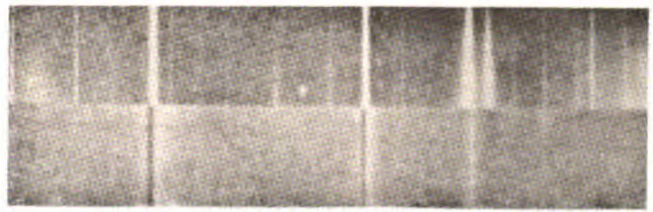
b

1

b

a

Mg 4481



a

2

b

4383 4404 4415

SPARK DISCHARGE IN WATER (LOCKYER.)

breadth, being stronger on the blue side, which is terminated almost abruptly, while the border towards the red is more diffuse.

2. *Broadened bright lines with central absorption.*—This is well shown in the central line of the violet triplet of iron at λ 4063.76.

3. *Broadened bright line with non-symmetrical absorption (maximum of emission towards red).*—The best examples obtained of this type of reversal are in the spectra of iron. The

Type	Example	Remarks
1. Complete reversal (strong)....	4415.29 (*Fe*)	Both components of bright line shown strongly, red side most prominent.
2. Complete reversal (weak).....	4415.29 (*Fe*) (another photograph)	Both components of bright line shown, red side much the stronger.
3. Partial reversal (weak)........	4282.57 (*Fe*)	Appearance of a bright line with a dark border on more refrangible side.
4. Partial reversal (weaker)......	4282.57 (*Fe*) (another photograph)	Bright component pre dominant, dark line only just visible.
5. No reversal..................	4315.26 (*Fe*) and many other weak lines.	....

strong line at λ 4260.64 in the water-spark shows the most decided asymmetry, the less refrangible component of the underlying bright line being seven or eight times stronger than the part on the violet side of the absorption line. There appears to be no suggestion, either of the line being duplex, so that the asymmetry cannot be explained as due to the interaction of two neighboring reversals of varying intensities.

In the case of the absence of any line at λ 4481.30 in the spark in water, it may not be owing to the balance of absorption and radiation, but to a special peculiarity of this line. From

many considerations, I regard λ4481.30 as a high temperature line only, and therefore it may be that the cooling action of the water envelope surrounding the water spark entirely prevents the production of this radiation.

From these considerations it appears evident that, if proper exposures be given, lines may be photographed in the spectrum of iron, say, which show all the phenomena described by Dr. Wilsing, but so related to each other and the complete stage—that of reversal, symmetrical or unsymmetrical—that it is impossible to regard them as anything abnormal. A typical set of lines illustrating these points, beginning with complete reversal with maximum of emission towards red is as shown in above table.

C. VARIATION OF INTENSITIES.

The most prominent lines in the water-spark are not always the chief lines of the air-spark. This is well shown in the spectra of iron and copper.

Many of the lines in the spark of iron, if their intensities are compared under the two conditions of sparking, show distinct inversions. A typical instance of this occurs with the lines at λ 4422.74 and λ 4427.48. With the spark in air λ 4427.48 is quite twice as strong as λ 4422.74, whereas in the water-spark there is scarcely any trace of a line at λ 4427.48, the 4422 74 line being, however, easily seen.

Another example, slightly less prominent, is found in the lines at λλ 4315.26 and 4337.22. With the spark in air these lines are almost equal in intensity, but in the water-spark λ 4315.26 has about three times the intensity of λ 4337.22.

In the case of copper, in the ordinary spark the most prominent lines are those at λ 4275.32 and λ4651.31. In the water-spark spectrum the line at λ 4587.19 is almost as strong as either of the lines just mentioned, although in the ordinary spark it is much weaker.

APPLICATION TO STELLAR SPECTRA.

I will next consider the bearing of these results on the explanation of certain features of the spectrum which is charac-

teristic of new stars. It has been seen that in the water-spark the position of the absorption undergoes little if any change of position, while in the case of non-symmetrical reversals, a bright line may be observed greatly displaced towards the red. In the new stars, on the other hand, the absorption lines are greatly displaced, the accompanying bright lines occupying in comparison normal positions. The facts are as follows:

In the case of *Nova Aurigæ* the emission lines had practically normal wave-lengths, but the displacement of the dark line at $H\epsilon$ was about 10.7 tenth-meters towards the violet, indicating a velocity of approach of about 500 miles per second.

The recent new star in *Perseus* exhibited the same normal positions of the bright lines, and indications of even greater displacements of the dark lines, at one time amounting to 15 tenth-meters at $H\epsilon$, representing a velocity of approach of the body producing the dark-line spectrum of over 700 miles per second.

These values differ enormously from those produced by pressure. The amount of shift produced by subjecting the light-source to pressure is given by Humphreys and Mohler, in the paper above referred to, as follows:

λ	Shift in tenth-meters	Atmospheres
4045.98	0.009	6
4045.98	0.02c	11¼
4383.72	0.016	9¾
4383.72	0.026	11½

We find then that the known direct effect of pressure on the radiation or absorption lines is the same, in quality, in water as in air; that is, we get displacements in the *opposite* direction to that we observe the dark lines to occupy in the spectra of *Novæ*, and we find further that the amount of shift observed in the spectra of new stars differs not only in this respect but also in degree, thus:

Spark in water	New stars
1. Absorption lines least shifted.	Absorption lines most shifted.
2. Radiation lines most shifted.	Radiation lines least shifted.
3. Absorption shift small.	Absorption shift enormous.

It would thus appear that the pairs of bright and dark lines shown in the spectra of new stars do not arise from the cause which produces the appearances presented in the spectrum of the spark in water.

My thanks are due to Mr. C. P. Butler, who obtained and discussed the photographs of the spark spectra, and who, together with Dr. Lockyer, assisted me in the preparation of the paper, and to Mr. F. E. Baxandall, who checked the wavelengths of the lines discussed and studied the behavior of the lines representative of the different phenomena.

PHOTOGRAPHIC WORK OF THE EXPEDITION FROM THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

TOTAL SOLAR ECLIPSE, MAY 17–18, 1901, SAWAH LOENTO, SUMATRA.

By HARRISON W. SMITH.

AMONG the various expeditions which assembled in Sumatra for the purpose of observing the recent eclipse, the party sent out by the Massachusetts Institute of Technology was unique in that it was prepared to make scientific observations apart from the eclipse and independent of the weather. The valuable set of pendulums of the United States Coast Survey was loaned for the use of the party, and pendulum observations were made at a new station in Sumatra as well as at a station in Singapore, where similar observations had previously been made by other observers. The outfit of the party included also a magnetometer and a dip-circle for detecting variations in the Earth's magnetism during the eclipse. The results of these observations are recorded in the *Technology Quarterly* for March, 1902.

Professor Alfred E. Burton was in charge of the expedition, and was accompanied by Mr. George L. Hosmer, of the Institute, Mr. Gerard H. Matthes, of the United States Geological Survey, and the writer. It is the purpose of this article to describe the photographic work of the expedition. By sailing from New York direct to Genoa, where we met the steamer "Koningin Regentes" of the Netherlands Steamship Co., we were enabled, notwithstanding the distance, to make the journey with very little difficulty. This route was not only the most direct, taking us by way of the Suez Canal and the Red Sea to Padang, the capital of the West Coast, near the central line of the path of the Moon's shadow, but was also the most convenient, for it involved only one transfer of our instruments after leaving New York.

We had, in addition, the pleasure of becoming acquainted, on board the "Koningin Regentes," with the Netherlands party consisting of Professor W. H. Julius, Professor A. A. Nijland, Mr. Wilterdink, and Mr. Hubrecht, and with the English party, Professor H. F. Newall and Mrs. Newall, of Cambridge, Mr. F. W. Dyson, of the Greenwich Observatory, and Mr. Atkinson. After a most enjoyable voyage, we arrived on April 6 at Padang, where we were received with unbounded hospitality by the officers of the Netherlands government and by the United States consular agent. We are under great obligations to these gentlemen for the invaluable assistance which they freely rendered to us during our whole visit, but more particularly for their suggestions in aiding us to select a site for our station. The decision in this important matter had purposely been postponed until our arrival, for, although we had carefully studied the meteorological reports which the Netherlands government had specially prepared for the assistance of astronomers, we wished to have the opportunity of considering the local conditions on the spot. After consultation with the governor and with Mr. Delprat, the director of the government railroads, we soon decided on a location at the end of the railway at Sawah Loento, near the Oembilien coal mines. This point is well beyond the ridge of the coast range of mountains, and thus, to a certain extent, protected from the rains of the coast. At Sawah Loento we had the assistance of Mr. van Lessen, the manager of the mines, and of Mr. Sieburgh, the resident *comptrolleur*, from whom we obtained building materials in abundance and the labor of the convicts at the mines. Our station was on a ridge about a mile and a half from the town; near us was Dr. Mitchell, of the United States Naval Observatory expedition, while about two miles distant Professor Newall had established his camp.

Our photographic outfit included three lenses of 76 mm (3 inches) aperture and 343 cm (135 inches) focus, made by Lundin, of Cambridge, two of which were loaned by Professor E. C. Pickering. We carried also a 3-inch lens of 102 cm (40 inches) focus, which, though not adjusted for photographic work, had

been successfully used in Georgia in photographing the faint elongations of the equatorial streamers. This lens was designed for use both with ordinary plates and with isochromatic plates and a color screen. Of the other three lenses, one was to be used for the corona while two were for photographing regions of the sky east and west of the Sun in the search for intra-mercurial planets. The cameras were long, square boxes of pine sheathing, and were taken apart before leaving Boston and shipped in crates; this method of construction being found very light and especially satisfactory on account of the rigidity of the cameras after being mounted. All four cameras were fastened to one frame-work six feet (183 cm) wide by eight feet (244 cm) high, which hung from a polar axis supported on brick piers eleven feet (335 cm) in height. The fact that our station was only a few miles from the equator, and that the eclipse occurred a short time after noon, made this method of mounting very convenient. The polar axis was a steel shaft 32 mm (1 ¼ inches) in diameter on which the frame was supported by two brass straps passing over it. Thus the whole cluster of cameras — two on one side of the axis and two on the other — formed one rigid structure so balanced as to swing freely through the necessary angle. Before the cameras were placed in position they were focused on a dark window in a blue house at a distance of about two miles. A base line was measured and the distance accurately determined by triangulation ; the focus was then corrected for parallel rays, and finally the cameras were placed in position and the focus tested by obtaining trails from *Arcturus*, whose declination is very nearly the same as was that of the Sun at the time of the eclipse. The trails, however, showed that the focus was practically correct.

The driving of the cameras was accomplished by means of an electric motor acting through a train of gear wheels on a tangent-screw. One end of the screw passed through a nut fixed at the end of the frame at a distance of 259 cm (102 inches) from the center of the polar axis, while the other end was suitably supported on a brick pier. Fig. 1 shows the arrangement of the

motor, gearing, and tangent-screw, with a corner of the frame, before the cameras were placed in position. The motor was maintained at the proper speed by means of an exceedingly ingenious method of clock control devised by Mr. Willard P. Gerrish, of the Harvard College Observatory, after a suggestion

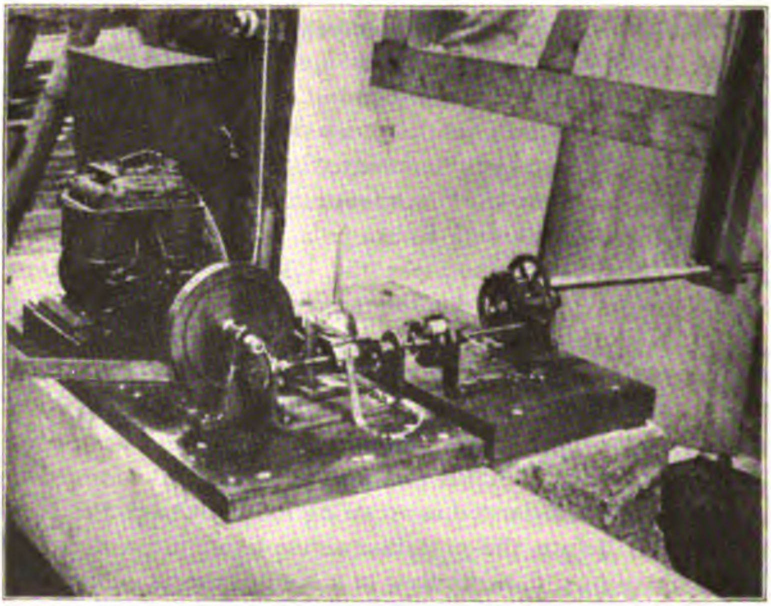


FIG. 1.

by Professor E. C. Pickering, which may be described as follows: A flexible coupling connects the motor to a comparatively heavy fly wheel, which in turn is joined by a worm and gear to a shaft so designed that it will make exactly one revolution per second when the tangent-screw is being driven at the desired speed. This shaft, which may be seen at right angles to the motor shaft, is provided with a make and break contact (about half way between the bearings) so arranged that a brush resting on a disk, composed half of brass and half of vulcanite, closes a circuit during one half of a revolution and opens it during the other half. Fig. 2 is a diagram of the apparatus and the electrical connections, show-

ing at K this contact disk, at M the motor, at R a relay, and at P the pendulum of the controlling clock, which beats half seconds. Attached to the end of the pendulum rod is a strip of platinum which, during each swing of the pendulum to the left of its middle position, cuts through a meniscus of mercury, thus alternately opening and closing the relay circuit at intervals of one half second. In the figure the meniscus of mercury is shown, for clearness, slightly below the platinum tip. This relay contact and the contact K are electrically connected in series with each other and with the motor, and are then joined to a storage battery of a size sufficient to furnish considerably more power than necessary to drive the motor at its proper speed. Thus the motor is driven by a series of impulses succeeding each other at intervals of one second, for the motor can only receive current when both the relay and the contact K are closed. The figure represents the condition of things when the pendulum is just on the point of closing the circuit while the brush of the contact disk is on the point C; so that current will flow until the disk, revolving in the direction represented by the arrow, has brought the point D under the brush, thus breaking the circuit. If the acceleration represented by the arc BD is sufficient to maintain the proper average speed, the point B will again be under the the brush when the clock closes the circuit one second later; if, however, this acceleration is insufficient the motor will drop behind a little and when the circuit is closed the point A, for

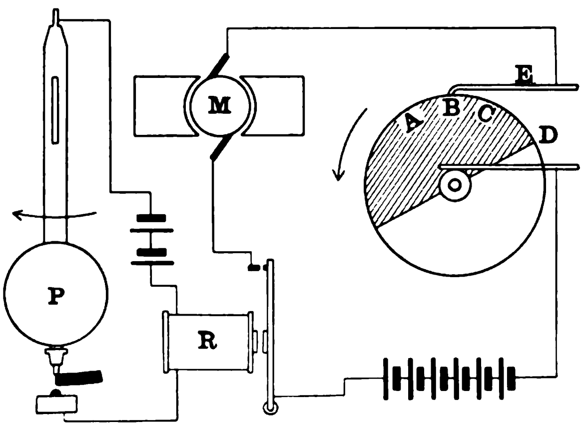


FIG. 2.

example, will be under the brush so that the acceleration will continue for a greater interval represented by the arc *AD*. If, on the other hand, the acceleration were too great, the point *C* would have arrived under the brush when the circuit was closed and the acceleration would continue during the interval *CD* only. The device constitutes a form of electrical cut-off in which the clock closes the circuit while the motor opens it after taking the amount of energy required, thus maintaining the average speed absolutely constant. The speed, of course, rises and falls each second, but these fluctuations may be reduced to any desired amount by increasing the weight of the fly wheel and by a slight modification of the electrical connections. Let the motor be connected permanently to the battery, but with a resistance, inserted in series with the armature, of such magnitude as to reduce the speed of the motor somewhat below its normal rate, the control device may then be used to short-circuit this resistance, thus producing a small acceleration each second and gaining the desired regulation with very small fluctuations in speed. A portable storage battery of six five-ampere cells was found sufficient to drive the motor. Through the courtesy of the ship's officers the battery was charged on the steamer just before reaching Padang, and was maintained in perfect condition by means of a bichromate battery. This method of driving the cameras was found most satisfactory by the writer and gave no trouble whatever. The clicks of the relay and the hum of the motor during the short intervals in each second when it is receiving current, produce a rhythmical sound by which one very soon learns to judge accurately whether or not the motor is "in step" with the clock. For this reason, the method possesses a distinct advantage over many of the other forms of driving devices in that, having once completed the adjustment, there is no possibility (aside from an exceedingly improbable change in the rate of the clock) that the apparatus will fail to work correctly without the operator becoming instantly aware of the fact.

Of the three cameras of 343 cm focus, two were used for photographing the sky near the Sun, and were designed to take two

8×10 plates (20×25 cm), each slightly tilted to improve the focus. One very long exposure was to be made with these cameras, so that it was necessary to cut down the number of exposures for the other camera, which was intended to photograph the corona, as it was not considered advisable to attempt any manipulation on one camera while the others were exposing. Unfortunately the long exposure was of no advantage, since the cloud was so dense on each side of the Sun that scarcely any stars are to be found on the plates. Of the exposures with the third camera, the first, of one-half second for the prominences at second contact, and the last, of twenty seconds, were successful. The other negatives were badly fogged by diffused light from the clouds and are of no value. The two referred to are, however, good, clear negatives. The prominences, although not large, are interesting, and there is evidence of a very violent disturbance in the equatorial region on the easterly limb of the Sun. This disturbance is also plainly visible on the negative of twenty seconds' exposure, and is accompanied by a marked crossing of the filaments radiating out, from a point near the equator, to a distance of eight or nine minutes of arc from the Moon's limb. A particularly striking dark arch appears over a prominence on the easterly limb. The polar streamers extend nearly one-half diameter from the Moon's limb and the equatorial streamers on the west, fully one diameter. This negative was undoubtedly exposed at an opportune moment, when an opening in the clouds was passing over the Sun, for it shows very little indication of fog. Toward the end of totality the sky presented a most striking appearance; the diffused light from the clouds about the Sun seemed almost as brilliant as the corona itself, while near the southern horizon the clouds suggested the last colors of sunset. It was particularly noticeable that, notwithstanding the much greater duration of totality, the darkness was apparently no more intense than it had been in Georgia the year before. At no time was the lamp in the camera-shed a necessity.

The plates were all developed in a very weak Ortol developer

for one and one-half to two hours. Fortunately ice was easily obtainable from Padang, for otherwise it would have been impossible to prolong the development to such an extent; in fact, it would have been difficult to develop at all, since the water in the dark-room was not observed below 80°.5 F. (27° C.). The Seed double-coated plate was used for the corona, as it was found to have yielded excellent results at the eclipse of 1900. With suitable precaution, no more difficulty was experienced in manipulating these plates than the ordinary plates, and the writer is firmly convinced that the negative contains detail in the inner corona which can be reproduced by careful shading of the negative during printing that would have been lost by over-exposure in an ordinary single-coated plate.

In addition to the work already described, an attempt was made to photograph the shadow bands by a method which the writer believes has not previously been tried. Although the results were not satisfactory, they were such as to make it appear that the method was worth another trial, especially since the bands were not sufficiently distinct to give the method a thorough test. In brief, it consists merely in exposing a sensitive plate directly to the bands themselves rather than attempting, with a camera, to photograph them as they appear on a white screen. The exposing is done by means of a sort of large focal plane shutter. Three 8×10 plates were placed in the bottom of a shallow box, giving an area 10×24 inches. Over this was a light-tight screen of two pieces of rubber gossamer cemented together on the rubber side. The screen is attached at each end to a curtain roller, a narrow slot is cut across it at the proper point, and the operation of exposing consists in rapidly drawing this slot over the sensitive plate by means of rolling the screen up on one roller while it is being drawn off from the other. The boxes, for several were used, were laid flat on the ground, and the exposures were made when the bands appeared, but the diffused light was so great that it was hardly to be expected that the bands would appear, and, in fact, the plates indicate over-exposure. As the exposures averaged

about 1/100 second, it is probable that 1/200 to 1/500 second, or, perhaps 1/1000 second would be sufficient for plates of moderate speed. Plates of different speed were used; but as they all show over-exposure it would probably be advisable in another attempt to err on the side of too short rather than too long exposure. It would be better also to substitute for the the cloth a metallic screen that could be moved at a more uniform rate, and it was found also that a cloth screen twelve inches or more wide, with a slot across nearly its entire width, did not run as smoothly as could be wished, while a metallic screen, on the other hand, could be made to slide freely and would not be affected by having a slot of any width cut across it.

The apparatus was tested successfully on artificial shadow-bands before leaving Boston, and it is largely on this account that mention of it is made here in the hope that it may be again tested at a later eclipse.

NOTE ON A PERSONAL EQUATION IN MEASURING PHOTOGRAPHIC SPECTRA.

By B. HASSELBERG.

IN *Bulletin* No. 15 of the Lick Observatory Mr. Reese describes a systematic discrepancy between measures of a stellar photograph made in different directions, going to show that there is a definite tendency to set the cross-hair of the microscope a little farther to the right (as seen in the microscope) on the dark lines of the comparison spectrum, than on the bright lines in the spectrum of the star. Having by special experiment ascertained that the discrepancy in question, according as the measures are made in the direction from violet to red or vice versa, is not to be attributed to any curvature of the comparison lines, nor to the relative position of the spectra, Mr. Reese concludes that the cause of the phenomenon is an entirely, or almost entirely, physiological one, the perception of a dark line in a bright field being somewhat different from that of a bright line in a dark field. As I have had similar experiences in the course of my researches upon the arc spectra of the metals I think it may be of some interest to give a succinct account of the results arrived at, more particularly as my perception of the spectral lines seems to be contrary to that described by Mr. Reese.

In measuring the photographs of the arc-spectra of metals I have hitherto always placed the plates on the measuring engine in such a way that the measures with increasing readings proceed from red to violet, the cross-hair in the microscope apparently moving from left to right. In this position of the plate, which may be marked I, the solar spectrum is seen to be placed above that of the metal. Every metallic line within a group limited by two solar lines, in a mutual distance generally not exceeding 20 tenth-meters, is thus referred to these solar lines and its

wave-length deduced simply by linear interpolation. With the view, however, of further controlling the results obtained, a solar line in every group is also occasionally measured; the agreement of the wave-length of this with the value given by Rowland shows the correctness of the wave-lengths deduced for the metallic lines. Now, this agreement is always very close indeed, but on examining the small residual differences, they are invariably found to be to a certain degree systematic, the wave-lengths obtained by me being in almost every case a little too small, as compared with the values from Rowland's table. As a specimen of this peculiar circumstance the following list of solar lines, measured in connection with my researches on the arc spectrum of molybdenum may be adduced:

TABLE I.

H.	R.	H.—R.	H.	R.	H.—R.	H.	R.	H.—R.
5816.558	.601	−0.043	4358.657	.670	−0.013	4066.508	.524	−0.016
5763.200	.218	− .018	51.200	.216	− .016	50.820	.830	− .010
01.766	.772	− .006	37.193	.216	− .023	45.977	.975	+ .002
5662.737	.744	− .007	08.075	.081	− .006	30.323	.339	− .016
38.477	.488	− .011	07.904	.907	+ .007	10.298	.327	− .029
5560.430	.434	− .004	4288.319	.310	+ .009	07.409	.429	− .020
5497.723	.735	− .012	67.108	.122	− .014	3996.118	.140	− .022
66.582	.609	− .027	48.381	.384	− .003	68.611	.625	− .014
45.255	.259	− .004	25.605	.619	− .014	52.099	.103	− .004
5353.575	.571	+ .004	22.363	.382	− .019	48.227	.246	− .019
22.234	.227	+ .007	04.138	.132	+ .006	41.897	.878	+ .019
5250.808	.817	− .009	02.197	.198	− .001	33.822	.825	− .003
34.787	.791	− .004	4185.040	.058	− .018	16.868	.879	− .011
4810.710	.724	− .014	63.802	.818	− .016	3892.073	.069	+ .004
4772.998	73.007	− .009	56.962	.970	− .008	73.900	.903	− .003
4691.573	.602	− .029	36.675	.678	− .003	52.710	.714	− .004
36.025	.027	− .002	18.696	.708	− .012	3792.472	.482	− .010
4598.286	.303	− .017	14.605	.606	− .001	86.300	.314	− .014
71.261	.275	− .014	00.878	.901	− .023	66.799	.801	− .002
4449.325	.313	+ .012	4098.327	.335	− .008	44.253	.251	+ .002
25.598	.608	− .010	76.102	.101	+ .001	29.954	.952	+ .002
4395.198	.201	− .003	71.893	.908	− .015	17.523	.539	− .016
74.621	.628	− .007	70.912	.930	− .018			

It is evident from the inspection of this table that the numbers of the third column include not only the accidental errors of observation, but also a systematic difference, the value of which may be found by taking the mean of all the differences

H. – R. This mean is H. – R. = – 0.009 tenth-meter, by which amount my solar wave-lengths are systematically too small. On applying this as a systematic correction to the differences above, they may be regarded as representing only the accidental errors of observation, from which the probable error of a solar wave-length as determined on my plates relatively to the system of Rowland comes out $\epsilon = \pm 0.007$ tenth-meter.

Now the existence of such a systematic error or personal equation is by itself hardly surprising; the question, however, as to its origin might well be of such a nature as to elicit differences of opinion. Among the suppositions that may be put forward in this respect, the most natural seems to be to assume that the setting of the cross-hair of the miscroscope on the spectral lines takes place somewhat differently, according to the direction in which the measures are executed. With the view to test this hypothesis more closely a series of solar lines, the wave-lengths of which had been determined in the position I of the plate on the occasion of the researches upon the spectrum of tungsten now in progress, was again measured in the opposite position of the plate, or in the direction from violet to red with increasing readings. This position may be designated as II. The results obtained are embodied in the first two columns of the following table, containing in addition the differences II—I and the mean results, together with their comparison with the corresponding values of Rowland.

It appears from the numbers of the third column that there exists between the determinations in the two positions a difference, certainly very small indeed, but nevertheless clearly pronounced, which indicates a systematic correction of +0.003 tenth-meter to be added to the measures in the position I. On comparing the means in the fourth column with Rowland it is seen, however, that this correction does not suffice to explain the systematic difference between my determinations and those of Rowland, but that there remains a mean discrepancy of –0.006 tenth-meter thus far not accounted for. Could it be assumed that the measures of Rowland were systematically too great by

this amount, the agreement of our values would be perfect. Besides that it may be remembered that the systematic difference

TABLE II.

I	II	II−I	(I+II)/2	R.	H.−R.
4852.730	.736	+0.006	.733	.743	−0.010
36.050	.059	+ .009	.055	.059	− .004
10.704	.715	+ .011	.710	.724	− .014
4789.838	.839	+ .001	.838	.849	− .011
72.995	73.002	+ .007	.998	.007	− .009
56.697	.702	+ .005	.700	.705	− .005
36.022	.021	− .001	.022	.031	− .009
18.596	.605	+ .009	.600	.601	− .001
4690.309	.310	+ .001	.310	.317	− .007
78.341	.347	+ .006	.344	.347	− .003
61.707	.712	+ .005	.710	.712	− .002
38.180	.189	+ .009	.184	.193	− .009
19.456	.464	+ .008	.460	.468	− .008
4594.291	.295	+ .004	.293	.297	− .004
71.263	.269	+ .006	.266	.275	− .009
58.818	.819	+ .001	.818	.827	− .009
41.693	.681	− .012	.687	.690	− .003
12.904	.908	+ .004	.906	.906	.000
4491.528	.565	+ .037	.546	.570	− .024
56.777	.780	+ .003	.778	.794	− .016
43.358	.356	− .002	.357	.365	− .008
17.858	.868	+ .010	.863	.884	− .021
00.548	.546	− .002	.547	.555	− .008
4379.377	.394	+ .017	.385	.396	− .011
68.054	.067	+ .013	.060	.071	− .011
44.439	.451	+ .012	.445	.451	− .006
27.269	.263	− .006	.266	.274	− .008
13.029	.038	+ .009	.033	.034	− .001
08.078	.080	+ .002	.079	.081	− .002
07.908	.917	+ .009	.912	.907	+ .005
4299.140	.147	+ .007	.143	.149	− .006
74.332	.341	+ .009	.336	.348	− .012
58.475	.490	+ .015	.482	.477	+ .005
39.501	.511	+ .010	.506	.525	− .019
20.493	.506	+ .013	.500	.509	− .009
01.078	.075	− .003	.077	.089	− .012
4182.916	.914	− .002	.913	.922	− .009
63.796	.815	+ .019	.806	.818	− .012
42.017	.025	+ .008	.021	.025	− .004
25.771	.781	+ .010	.776	.776	.000
06.583	.576	− .007	.580	.583	− .003
4087.243	.245	+ .002	.244	.252	− .008
70.409	.419	+ .010	.414	.431	− .017
52.078	.083	+ .005	.080	.070	+ .010
25.962	.972	+ .010	.967	.972	− .005
15.753	.763	+ .010	.758	.760	− .002

between my measures of solar lines as obtained in connection with the spectrum of molybdenum, and the values of Rowland, is the same as in the case of my researches upon the spectrum of tungsten, or H. — R. = —0.009 tenth-meter.

If it be considered that in measuring spectral lines of the *same* appearance, as in the cases above, a systematic error or personal equation nevertheless can exist, it becomes very probable that such a personal equation in a still higher degree may also be met with in case that a metallic spectrum is to be compared with the solar spectrum, the objects to be observed then being of an appearance quite contrary one to the other. The correctness of this view may be inferred from the inspection of the following table, giving the wave-lengths of a group of lines in the arc spectrum of tungsten, as measured in the two positions of the plate.

TABLE III.

I	II	II-I	I	II	II-I
4449.182	.164	— 0.018	4422.010	21.958	— 0.052
45.324	.297	— .027	21.168	.135	— .033
44.650	.592	— .058	20.627	.608	— .019
42.004	41.976	— .028	19.429	.387	— .042
39.904	.845	— .059	18.967	.934	— .033
38.469	.441	— .028	18.609	.584	— .025
37.070	.070	.000	15.891	.845	— .046
35.903	.874	— .029	15.233	.246	+ .013
32.379	.314	— .065	14.042	13.987	— .055
28.663	.607	— .056	13.173	.142	— .031
27.533	.508	— .025	12.347	.343	— .004
26.087	.052	— .035	11.871	.834	— .037
25.064	.030	— .034			

It is immediately seen that the values in the second column constantly fall short of those in the first, the mean discrepancy being — 0.033 tenth-meter. Thus the systematic error or personal equation in every case numerically amounts to 0.016 tenth-meter to be subtracted in the first position of the plate and added in the second. Now, as in the position I, the measures proceed from red to violet or from left to right in the microscope and the resulting wave-lengths at the same time are too great, it follows that the settings on the metallic lines must have been

made a little to the left of the true position. On the second position of the plate the left and right correspond to violet and red, and as in this case the resulting wave-lengths come out too small, it appears that even now the settings are systematically erroneous in the same direction.

As far as I am concerned, then, the personal equation in the present case is with regard to the direction quite the contrary of that observed by Mr. Reese, but this circumstance need not be regarded as surprising, the whole phenomenon without doubt depending only on physiological peculiarities. As to the amount of the error it may on the other hand be remarked as a curious incident that if the above measurements had been intended for determination of radial velocity, the corresponding correction would in conformity with the result obtained by Mr. Reese have been almost exactly 1 kilometer.

ACADEMY OF SCIENCE, STOCKHOLM,
February 1902.

SOME RESULTS WITH THE BRUCE SPECTROGRAPH.

By WALTER S. ADAMS.

THE VARIABLE VELOCITY OF *o PERSEI* IN THE LINE OF SIGHT.

FIVE spectrograms of the fourth magnitude star *o Persei* ($\alpha = 3^h\ 38^m$; $\delta = +31°\ 58'$) recently obtained by the writer show a large variation in its radial velocity. The results of the measurements are as follows:

1902	February 19	- - - - - -	+ 134 km
	February 21	- - - - - -	− 77
	March 4	- - - - - -	+ 128
	April 2	- - - - - -	− 117
	April 3	- - - - - -	− 4

The plate of April 2 is under-exposed, and the value given by it is consequently somewhat less accurate than the others in the list. The spectrum of this star is of the Orion type, but the intensities of the helium lines are much less than in the representative stars of the group, while the *Mg* line λ 4481 is hardly visible.

The ranges of variation in this star and in the star *η Orionis*, whose variability was announced in December, are the largest which have hitherto been found among spectroscopic binaries having one component dark.

THE VARIABLE VELOCITY OF *δ LIBRAE* IN THE LINE OF SIGHT.

The well-known *Algol* variable *δ Librae* ($\alpha = 14^h\ 56^m$; $\delta = -8°\ 8'$) exhibits a considerable variation in its radial velocity. The following spectrograms have been secured:

1902	March 4	- - - - - -	+ 36 km
	March 12	- - - - - -	− 123
	April 2	- - - - - -	− 97
	April 3	- - - - - -	+ 38

The spectrum of this star has been assigned by Miss Maury to Group VIIa of the Harvard classification, but the metallic lines are exceedingly broad and faint, with the exception of the

Mg line λ 4481, which though broad is of moderate strength. The measures are consequently uncertain by several kilometers.

THE RADIAL VELOCITY OF *SIRIUS*.

A considerable number of plates of *Sirius* have been obtained by the writer during the winter in the course of investigations on the photographic efficiency of the spectrograph, and the density of the negative best suited to bringing out the fine lines of the class of spectra to which the star belongs. I have measured ten of these with a view to securing as accurate a value as possible of its radial velocity, and the results are given in the table below. Both of the cameras in use with the instrument have been employed, the series letter *A* indicating the Zeiss anastigmat of 449 mm focus, and *B* the Hastings triplet of 607 mm focus. Numerous checks in the way of plates of *Venus* and the Moon have been applied.

Series No.	Date	No. of Lines	Velocity
B 258	1901 December 18	12	−7.2 km
B 263	December 31	13	7.1
A 301	1902 January 4	16	7.0
A 302	January 4	16	6.6
B 271	January 9	14	6.8
B 278	January 16	17	6.8
B 285	January 24	18	6.9
A 311	February 10	11	6.9
A 329	March 3	13	6.9
B 291	March 12	15	6.5
	Epoch 1902.06		−6.87 km

This result, in combination with the value of −15.6 km for the epoch of 1890.09 obtained by Vogel and Scheiner at Potsdam, renders possible a computation of the parallax of the system of *Sirius* through the change in the radial velocity of the principal star during the interval. The method has been treated by several writers, among others Rambaut and Monck.[1] Using the most recent orbital elements by Zwiers,[2] and the results found by

[1] *Sidereal Messenger*, 9, 289, 1890.

[2] *Kon. Akademie Wetenschappen Te Amsterdam*, Proceedings of meeting, May 27, 1899.

Auwers from meridian observations for the motion of the center of gravity of the system, we obtain for the parallax

$$\pi'' = \frac{1.84}{-6.9+15.6} = 0\overset{''}{.}21\ .$$

The latest value of the parallax of *Sirius*, as found by Gill from heliometer measures, is $0\overset{''}{.}37$. The difference is no doubt partially due to the error of the earlier spectroscopic measures, but unless we assume this error to amount to 4 km, the above result would lead us to infer that the true parallax of *Sirius* lies somewhat below the heliometer determination.

THE WAVE-LENGTH OF THE *Mg* LINE λ 4481.

On account of the prominence of the line λ 4481 in the spectra of stars of the Orion and Sirian types an accurate determination of its wave-length is of considerable importance for radial velocity work in this region of the spectrum. So far as the writer has been able to learn, however, no such determination exists, owing no doubt to the difficulty of securing accurate measurements upon the broad and diffuse line given by the magnesium spark in the laboratory. Accordingly it seemed best to determine its value directly from suitable stars, and for this purpose *Sirius*, *γ Geminorum*, and *θ Leonis* were selected, in all of which the line is sharp, narrow, and of great brilliancy. Ten plates of *Sirius*, three of *γ Geminorum*, and two of *θ Leonis* gave the following results, after elimination of radial velocity:

Sirius

B 258	4481.407
B 263	.386
A 301	.382
A 302	.413
B 271	.407
B 278	.405
B 285	.384
A 311	.389
A 329	.410
B 291	.397

γ Geminorum

B 236	4481.388
B 243	.408
B 249	.414

θ Leonis

B 272	.413
A 308	.396
	4481.400

The value 4481.400 has been employed in the determination of the radial velocities of *o Persei* and *δ Librae* given in the present paper.

YERKES OBSERVATORY,
April 9, 1902.

SOME OBSERVATIONS ON THE RESOLVING POWER OF THE MICHELSON ECHELON SPECTROSCOPE.[1]

By P. ZEEMAN.

1. ON A recent occasion[2] I gave a few observations on this subject. The acquisition of some new data induces me to return to it in this place.

In his "Investigations in Optics," Lord Rayleigh[3] expressed the wish that spectroscopists in possession of powerful instruments would compare the actual resolving power with that of which they are theoretically capable, and remarked that a carefully arranged succession of tests of gradually increasing difficulty would be of especial value.

I remembered these remarks as I tested the very original echelon invented by Michelson.

The echelon at my disposal, made by Hilger, London, consists of thirty plates each about 7.8 mm thick, made of light flint-glass, set with 1 mm steps. A clear aperture of 1 mm is left beyond the width of the largest glass plate. The number of apertures n, operative in the formation of the spectrum, is hereby one more than the number of plates. The mounting was somewhat improvised. Telescope and collimator belonging to a Kirchhoff spectroscope were employed. The telescope had object-glasses of 50 cm focus and 38 mm aperture. It is evident that when the mounting is made especially it is advisable to have glasses of shorter focus, so as to get greater intensity.

Denoting by $d\lambda_1$ the difference of wave-length of spectral lines when they are just distinguishable as separate in the spec-

[1] Communicated by the author as advance proof of a paper to appear in the Proceedings of the *Amsterdam Academy of Sciences.*

[2] BOSSCHA, *Collection of Memoirs, Archiv. Néerl.*, Sér. II, **6**, 319, 1901.

[3] *Phil. Mag.*, 1879, 1880.

troscope, by t the thickness of the plates of glass, and by n the above mentioned number, then we have

$$q_t = \frac{d\lambda_1}{\lambda} = \frac{\lambda}{knt}, \tag{1}$$

if

$$k = (\mu - 1) - \lambda \frac{d\mu}{d\lambda}.$$

The resolving power is given by

$$r = \frac{\lambda}{d\lambda_1} = \frac{knt}{\lambda}. \tag{2}$$

For the green line $\lambda = 5460$ we obtain in the case of our echelon,

$$r = \frac{0.63 \times 31 \times 7.8}{5460 \times 10^{-7}} = 280000 \text{ and } q_t = \frac{d\lambda_1}{\lambda} = 3.6 \times 10^{-6}.$$

In the calculation of k I use the following values of the refractive indices given to me by Hilger:

$$\mu_C = 1.5713$$
$$\mu_D = 1.5753$$
$$\mu_F = 1.5853$$
$$\mu_{G'} = 1.5936$$

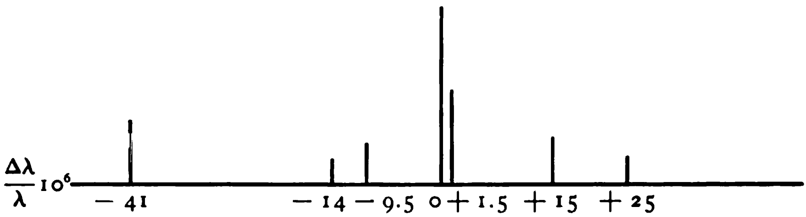
Henceforth I will denote by q_t the theoretical value of the limit of resolution calculated according to (1), by q_e the experimental value. By means of a Hoffmann direct vision spectroscope the light of the vacuum tubes (driven by a Ruhmkorff) undergoes the necessary preliminary analysis. In some cases absorbing media were therefore sufficient. In some experiments the mercury arc-lamp of Fabry and Perot was used.

2. The very intense *green* line (λ 5460) of *mercury* was investigated first. Using the echelon in a position in which two strong lines of equal intensity corresponding to successive orders of the radiation were visible, I could distinguish also five faint, very narrow lines between the principal ones. The distance between two pairs of these lines was very small.

As I could not find a table of the wave-lengths of these feeble radiations, I addressed myself to Messrs. Fabry and Perot. I am very much obliged to Messrs. Perot and Fabry for their kindness

in investigating for me anew the green radiation of the mercury arc *in vacuo*.

The following scheme represents the constitution of this very complex radiation according to their observations. The ordinates are *approximately* proportional to the intensities.



The numbers given are only approximate, especially (− 14) and (− 9.5).

The radiation (+ 1.5) was observed by Fabry and Perot only in the radiation of a Michelson tube; it is too close to the principal radiation to be seen separately in the arc light. In the photographic reproduction in the ASTROPHYSICAL JOURNAL[1] of the interference fringes of the green mercury line, the radiation (− 41) coincides with the radiation (+ 15) and is therefore invisible.

I could distinguish very clearly the radiations (− 9.5) and (− 14) as separate lines. For these radiations $q = \frac{d\lambda}{\lambda} = 4.5 \times 10^{-6}$ or $r = 222{,}000$ and hence q_e rather smaller; calculation gave $q_t = 3.6 \times 10^{-6}$.

Using the *green* line of *thallium*[2] I very easily distinguished the faint radiation at a distance $\frac{d\lambda}{\lambda} = 21 \times 10^{-6}$ from the principal radiation, but I could not see as a separate line the the one determined by $\frac{d\lambda}{\lambda} = 3 \times 10^{-6}$.

Hence q_e exceeds 3×10^{-6} but is smaller than 21×10^{-6}.

[1] FABRY and PEROT, ASTROPHYSICAL JOURNAL, 13, 272, 1901.

[2] FABRY et PEROT, *Ann. de Chim. et de Phys.* (7) 16, 134, 1899.

Indeed for the thallium radiation (λ 5440)

$$q_t = \frac{5440 \times 10^{-7}}{0.63 \times 31 \times 7.8} = 3.6 \times 10^{-6}.$$

For the *green* (λ 5086) line of *cadmium* it was just possible to see that this line is a double one. The distance of the components is according to Fabry and Perot[1] $\frac{d\lambda}{\lambda} = 5 \times 10^{-6}$. For $\lambda = 5086$ I calculate $q_t = 3.2 \times 10^{-6}$. Hence with the above mentioned echelon it is possible to approach rather nearly the limit of the theoretical resolving power.

3. Perhaps the best series of tests of gradually increasing difficulty can be obtained by observation of the change of spectral lines in magnetic fields of gradually increasing intensities, a Nicol between source and apparatus being used in order to reduce the complexity of the radiation. In this manner all values between, *e.g.*, 0.001 tenth-meter to about 1 tenth-meter can be obtained. Corresponding herewith are the values $q_t = 0.2 \times 10^{-6}$ and $r = 5{,}000{,}000$; resp. $q_t = 200 \times 10^{-6}$ and $r = 5000$. The performances of echelons and interferometers and of ordinary spectroscopes with a few glass prisms lie between the limits indicated. This test I have not yet applied systematically to the above mentioned echelon.

In order, however, to show its fitness I will use some observations of Lord Blythswood and Dr. Marchant.[2] In their § 6, "Results Obtained of the Zeeman Effect on the Chief Lines of the Mercury Spectrum," p. 397, these authors communicate observations with an echelon spectroscope concerning the difference in wave-length between the components of the outer components of the sextet of the blue line (λ 4358) of mercury. The following table is an extract:

H	$\delta\lambda_3$
5.000	..... tenth-meters
12.100	.....
12.900	0.052
20.000	0.098 ?
21.300	0.09
23.400	0.098

[1] *Loc. cit.* p. 137.

[2] *Phil. Mag.* **49**, 384, 1900.

For a value of the field between 12.100 and 12.900 the splitting up of the lines becomes sufficient to make them appear as separate lines *on a photograph* (upon which the measurements were taken). Two lines can, of course, be *seen* separated at a rather smaller distance.

Thus now $q = \frac{0.052}{4358} = 11.9 \times 10^{-6}$ and q_e is rather smaller. For the echelons of these observers we have $t = 7.5$, $n = 15$.

With these data I calculate $q_t = 5.3 \times 10^{-6}$.

Thus it appears from the data given in this paper that it is possible to manufacture echelons performing nearly as well as they are theoretically expected to.

THE MECHANICAL EQUIVALENT OF THE UNIT OF LIGHT.[1]

By KNUT ÅNGSTRÖM.

A FIRST attempt at determining the mechanical equivalent of a source of light had been made by Julius Thomsen[2] as early as 1865, and in 1889 a more precise determination of our present unit was carried out by O. Tumlirz.[3] The total radiation (Q) was measured with a variety of air thermometer, the ratio of the luminous portion to the whole radiation, or the luminous effect of the radiation $\left(\frac{L}{Q}\right)$ was determined in the manner earlier used by Melloni and Julius Thomsen, by causing the infra-red rays to be absorbed by a layer of water, and thus separated from the luminous rays.

In my former determination[4] of the total radiation of the Hefner amyl-acetate lamp, I became convinced that it was considerably greater than given by Tumlirz.

The method of measuring the luminous efficiency by water absorption is readily seen to be incorrect in theory, and must lead to too high values of the luminous radiation.[5]

Since an accurate knowledge of our unit of light is of great importance in connection with many problems of physics, I decided to resume the question. The treatment of the Hefner lamp is as heretofore in two parts: (1) the determination of the whole radiation; (2) the determination of the ratio of the luminous and total radiation.

The normal lamp with Hefner flame used was obtained from

[1] From advance proofs from the author of an article also to appear in the *Physikalische Zeitschrift.*

[2] *Pogg. Ann.*, **125**, 348, 1865.

[3] *Wied. Ann.*, **38**, 640, 1889.

[4] *Wied. Ann.*, **67**, 647, 1899.

[5] Tumlirz found the total radiation of a candle at one meter to be 0.0000148 gram-calories per second, and the luminosity to be 2.4 per cent. of this.

Siemens and Halske. For determining the total radiation I employed the compensation-pyrheliometer[1] of my design; this was an excellent instrument, with strips of manganin; for the details of its construction I would refer to the paper above cited. The equality of the temperature of the strips was obtained with the aid of a highly sensitive reflecting galvanometer, and the strength of the heating current with a milliampere meter from Siemens and Halske. The total radiation was measured at two different distances, 50 and 100 cm from the lamp, and the results were in complete agreement. As mean of several determinations this result was obtained:

Total radiation (Q) at a distance of one meter $= 0.00129$ gram-calories per minute, or 0.0000215 gram-calories per second. The error of this determination probably does not exceed 3 per cent.

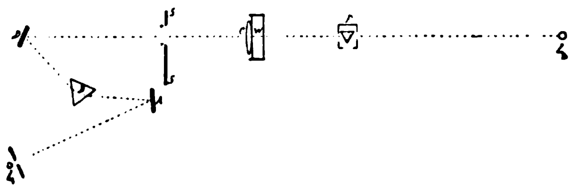
The luminous effect of the radiation was also determined by a new method. That employed by Langley, determining the energy spectrum by the bolometer and obtaining the luminous effect by integrating the resulting curves, is doubtless correct in principle, but is beset with considerable difficulties, especially with weak sources of light. Moreover the prisms and lenses or mirrors of the spectroscope exert a selective effect on the radiation, which is especially conspicuous in the extreme infra-red, and which affects the result in a manner difficult of computation. The following method is free from this source of error:

The radiation of the source under investigation is dispersed by a spectroscope. The invisible portions of the spectrum are cut off by screens, while the luminous rays, on the other hand, are united by a cylindric lens to form a white image on the head of a photometer. A second similar source of light is so arranged as to throw a photometrically equal amount of light directly upon the photometer screen. We are thus dealing with two radiations, physiologically precisely equal in intensity and composition, the first containing luminous rays only, while the second is the corresponding total radiation. If a bolometer or

[1] *Wied. Ann.*, **67**, 633, 1899.

a thermopile is substituted for the photometer screen the energy of the two radiations, and hence their ratio, can be determined.

The arrangement of the experiment will be rendered clear by the figure. L_1 and L_2 are the sources of light; ABD is a mirror spectroscope; S a screen; C a cylindric lens; W a water tank to prevent the radiation from warming the screen; P the photometer screen, which can be exchanged for a sensitive thermopile.



All these parts are attached to a large optical bench two meters long, and hence can be easily adjusted. The screen S can be moved by a micrometer screw perpendicularly to the length of the bench, and was set so as to cut off the spectrum beyond λ 0.76 μ.

In the investigation of sources whose luminous radiation is produced by the incandescence of carbon particles, the source L_1 may be replaced by an incandescent lamp and the current regulated until the color of the light is the same as that of the source to be investigated. This is especially advantageous in the cases where the luminous radiation is weak, and was accordingly employed by me in the investigation of the Hefner light.

I obtained as the mean of many determinations of the luminous effect of the Hefner lamp,

$$\frac{L}{Q} = 0.90\ \% \ (+\ 0.04\ \%)\,.$$

From these determinations of Q and $\frac{L}{Q}$ may now be computed the energy corresponding to our unit of light (energy of luminous radiation on 1 sq. cm at 1 cm distance), and unit of luminosity (energy of luminous radiation on 1 sq. cm at 1 meter's distance). We obtain: 1 unit of light $= 0.009 \times 0.0000215 \times 100^2 = 1.94 \times 10^{-3}$ gram-calories per second $= 8.1 \times 10^4$ ergs per second. One meter candle $= 0.009 \times 0.0000215 = 1.94$

$\times 10^{-7}$ gram-calories per second = 8.1 ergs per second. Hence the mechanical equivalent of the unit of luminous intensity is, in round numbers, 8 ergs per second.[1]

I have thus far only had an opportunity to investigate the luminosity of the radiation of the Hefner lamp and the acetylene flame. For the latter I obtained as the mean of five determinations,

$$\frac{L}{Q} = 5.5\ \% .$$

This has been previously investigated by Stewart and Hoxie.[2] They obtained by an improved absorption method $\frac{L}{Q} = 10.5$ per cent., a result clearly too large on account of the sources of error of the method. But a later determination by Stewart,[3] using the method of integrating the energy curve obtained spectrometrically, confirmed the former high value. This is, however, to be explained by an error he makes in the treatment of his results.[4]

On avoiding this error we obtain from Stewart's observations a value of the luminosity between 5 and 6 per cent., in complete agreement with that found above.

It has been long known that the luminosity of the radiation of our ordinary sources of light is very low; but it would seem that the earlier determinations — with the possible exception of those of Langley — were from two to three times too high.

[1] See the statement in DRUDE'S *Lehrbuch der Optik*, p. 445, 1900, where the computations are based on Tumlirz's values.

[2] E. L. NICHOLS, *Physical Review*, 11, 219, 1900.

[3] *Physical Review*, 14, 257, 1901.

[4] He introduces the measures made in a prismatic spectrum into a normal spectrum, and divides each observation by the square of the slit-width expressed in wave-lengths. In this way he really twice converts his prismatic spectrum into a normal one.

UPSALA,
January 1902.

SELECTIVE ABSORPTION AS A FUNCTION OF WAVE-LENGTH.

By GEORGE E. HALE.

READERS of my note on the spark spectrum of iron in liquids, which was published in the last number of this JOURNAL, can hardly have failed to notice that the first reversals of the iron lines appeared at the more refrangible end of the spectrum, and that as the conditions became increasingly favorable to elective absorption, bright lines of greater and greater wave-length passed over into dark lines. This relationship between selective absorption and wave-length does not apply to all of the iron lines, for some of them remain bright even under circumstances highly favorable to reversal. But if we except such lines, and also certain others, including those of great intensity (which reverse long before the fainter lines in the same region), we may say that selective absorption begins in the ultra-violet and gradually advances into the less refrangible part of the spectrum.

In order to ascertain whether similar results can be obtained with other metals, with the assistance of Dr. Kent, I have photographed the spark spectrum in water of the following metals: titanium, magnesium, cadmium, copper, nickel, cobalt, lead, and aluminium. Speaking generally, and omitting reference to certain doubly reversed lines and other interesting details requiring special investigation, it may be said that the absorption phenomena of most of these metals are very similar to those previously found for iron: the first reversals occur in the ultra-violet, and only as the conditions become more favorable to absorption do dark lines appear in the less refrangible region. Some of the spectra contain very few dark lines, but the absorption phenomena of cobalt and titanium are fully comparable with those of iron.

It thus appears that in the case of some of the metals the law of selective absorption resembles that of general absorption. It seems probable that other elements will give similar results, particularly in view of Campbell's discovery that in the spectra of certain stars the ultra-violet hydrogen lines are dark, while those of greater wave-length are bright.[1] The effects of partial reversal which Campbell describes are very similar to those frequently obtained with the spark in liquids.

It should be added that in 1893 Professor Frost suggested an explanation of such stellar spectra, based on the assumption that the law of selective absorption resembles that of general absorption. In a recent number of this JOURNAL (December 1901), Professor Kayser has given a somewhat more detailed explanation, derived from a simple application of Kirchhoff's law.

YERKES OBSERVATORY,
April 19, 1902.

[1] ASTROPHYSICAL JOURNAL, 2, 177, 1895.

Minor Contributions and Notes.

EARLY OBSERVATIONS OF ALGOL STARS.[1]

One of the most important uses of the collection of photographs at Cambridge is to determine early dates of minima of stars of the *Algol* type. Nearly all of the early photographs were taken with the 8-inch Draper Telescope. The number has been greatly increased during the last three years by supplementing its work with anastigmat lenses. In the *Astron. Nach.*, **156**, 313, Mr. Williams announces that the star 78.1901, R. A. $= 20^h\ 18^m\ 4^s.0$, Dec. $= +42^\circ\ 46'.4$ (1855), is a star of the *Algol* type. One hundred and seventy-seven photographs of this region were contained in the Harvard collection, the first being on September 19, 1885. On ten of these the star was distinctly fainter than its normal brightness. Measures with the twelve-inch Meridan Photometer gave the maximum magnitude of this star 10.47. Assuming this value, a light curve, whose coördinates are given in Table I, was determined by Professor Wendell from observations with the fifteen-inch telescope.

TABLE I.

LIGHT CURVE.

Ph.	Decrease.	Increase.
d		
0.20	10.47	10.47
0.18	10.55	10.50
0.16	10.67	10.57
0.14	10.81	10.75
0.12	11.07	11.03
0.10	11.15	11.11
0.08	11.45	11.38
0.06	11.80	11.75
0.04	12.22	12.16
0.02	12.76	12.64
0.00	13.05	13.05

The various determinations of the minima are enumerated in Table II. The year, month, and day are given in the first column, the

[1] *Harvard College Observatory, Circular* No. 64.

Greenwich Mean Time in the second, the Julian Day and fraction in the third, and the photographic magnitude in the fourth. The fifth column gives the time from minimum, indicated by the magnitude according to the light curve, assuming that the normal photographic magnitude is 10.2. On three of the plates the star was not seen, although stars of the magnitude 11.7 were visible. This indicates that the star was within $0^d.06$ of minimum. The last four lines relate to visual observations, three by Mr. Williams and the last one by Professor Wendell. The letter m is entered in the fourth column, and the value in the fifth is of course zero. The adopted formula for the times of minima is 2,410,000.20 + 3.45083 E, which agrees closely with that given by Mr. Williams. The value of E is given in the sixth column, and the deviation O–C in the seventh. These deviations must be corrected by the values in the fifth column, although they are sometimes thus increased. In the first line the correction may have any value from $+0^d.06$ to $-0^d.06$, but even the latter leaves the deviation $+0^d.009$. In the second line the magnitude 10.83 shows that the time of minimum must have preceded or followed the time of observation by $0^d.121$.

TABLE II.

OBSERVED MINIMA OF 78.1901.

Y.	M. D.	G. M. T.	J. D.	Magn.	Curve.	E.	Unc.	Corr.
		h. m.	d.		d.		d.	d.
1890	5 16	19 56	1504.831	<11.7	±.006	436	+.069	+.009
1891	1 13	10 50	1746.451	10.83	−.121	506	+.131	+.010
1892	9 18	14 07	2360.588	<11.7	±.06	684	+.020	.000
1893	7 29	18 04	2674.753	10.67	−.143	775	+.160	+.017
1894	9 23	14 03	3095.585	<11.3	±.08	897	−.010	.000
1895	8 10	14 52	3416.619	10.98	−.107	990	+.097	−.010
1896	8 20	16 32	3792.689	<11.7	±.06	1099	+.027	.000
1897	9 25	14 46	4193.615	10.86	−.120	1215	+.657	+.537
1898	1 13	10 43	4303.447	11.62	−.062	1247	+.062	.000
1899	10 3	13 43	4931.572	10.52	−.162	1429	+.136	−.026
1901	8 24	14 27	5621.602	m	.00	1629	.000	.000
1901	9 7	9 43	5635.405	m	.00	1633	.000	.000
1901	9 14	7 29	5642.312	m	.00	1635	+.005	+.005
1901	11 1	14 47	5690.617	m	.00	1649	−.002	−.002

It is by no means certain that the best value of the period, $3^d.45083 = 3^d\ 10^h\ 49^m\ 12^s$, has been found. A change of one or two seconds would, however, increase the residuals perceptibly. It is curious, and perhaps suspicious, that the correction has always been taken with the negative sign, that is, that the star when photographed was in each of

the ten cases increasing in light. One result, that on J. D. 4193, seems entirely wrong. The minimum apparently took place half a day too late. Some of the large residuals occur when the star is bright, or when it is changing slowly. We should expect that they would be the most uncertain. It is probable that more accurate results can be obtained when the photographic light curve is better known, and also by correcting for aberration.

In the *Astron. Nach.*, 157, 79, Dr. Schwab announced that the star $+19°3975$, 93.1901, R. A. $= 19^h\ 14^m\ 26^s$, Dec. $= +19°25'.4$ (1900), was a variable of the *Algol* type. From the *Harvard Annals*, Vols. XXIV and XLV, it appears that the photometric magnitude of this star when at full brightness is 6.50, the range in the measures on four nights being less than a tenth of a magnitude. An examination of the Harvard photographs showed that we had 155 images in which its brightness was nearly normal, and thirteen in which it was near minimum. For the observations near minimum, Table III gives in successive columns the year, month, and day, the Greenwich Mean Time, the Julian Day and fraction, the approximate magnitude, the assumed valued of E, and the uncorrected residual found by subtracting the time computed by the formula, 2,410,011.6 + 17 E.

TABLE III.

OBSERVED MINIMA OF 93.1901.

Y.	M.	D.	G. M. T. h.	G. M. T. m.	J. D.	Magn.	E.	Unc.
			h.	m.	d.			d.
1887	8	3	15	35	0487.649	7.06	28	+0.05
1890	11	5	10	48	1677.450	9.15	98	−0.15
1891	7	13	16	28	1927.686	8.68	113	−4.91
1893	8	4	15	2	2680.626	7.06	157	+0.03
1893	8	4	15	14	2680.635	7.06	157	+0.04
1894	10	15	12	8	3117.506	7.35	183	−5.09
1895	8	22	13	22	3428.557	8.74	201	−0.04
1895	8	22	14	25	3428.601	8.11	201	0.00
1895	9	8	14	22	3445.599	8.40	202	0.00
1897	9	24	13	31	4192.563	7.06	246	−1.04
1900	5	22	19	33	5162.815	7.80	303	+0.22
1901	5	12	19	4	5517.794	8.74	324	−1.81
1901	7	12	17	13	5578.713	9.15	327	+8.11
1901	7	12	17	28	5578.728	9.11	327	+8.13
1901	10	8	13	14	5666.551	8.68	333	−6.05
1901	11	1	6	30	5690.27	m	334	+0.67
1901	11	28	12		5717.50	m ?	336	−6.10
1902	1	4	12		5754.5	m ?	338	−3.10

The photograph on J. D. 3428 is especially valuable. The spectrum, which is of the first type like other *Algol* stars, trailed over the plate, and showed that the star was at first of about the ninth magnitude, after about half an hour suddenly becoming brighter until it attained the eighth magnitude at the end of the exposure. Two lines have, therefore, been given to this plate. When the law of variation becomes known this will probably give an accurate value of the time of minimum. The last line but two represents the observation of Dr. Schwab, the last line but one that of Professor Wendell. On this last date, the star had apparently nearly recovered its full brightness, the range of the observations not exceeding two-tenths of a magnitude. The last line represents a somewhat uncertain observation by Mr. White when the star was near the horizon. The range in light appears to be greater than that of any other *Algol* star. The period, 17 days, given by Dr. Schwab, is longer than that of any other *Algol* variable, and does not satisfy the observations. No subdivision, or other value of the period, appears to give better results. By a slight change in the period, and by applying a correction for the light curve and for aberration, about half of the residuals in the last column of Table III could be reduced nearly to zero. The others are so large, amounting to several days, that their careful study becomes a matter of very great importance. If these discordances are due to a third body this star will become one of the most interesting in the sky. Evidently it should be carefully watched. This has been done at Cambridge ever since its variability was announced. On two dates, November 18 and December 22, 1901, when minima were expected, according to the formula of Dr. Schwab, clouds prevented observations, and on a third day, December 5, 1901, no diminution in light was perceptible. During the coming year it is proposed to look at it early and late on each clear evening at Cambridge and Arequipa. The value of this work would be greatly increased if observers in other longitudes would co-operate. A continuous watch might thus be kept upon it, and no minima missed. The observations needed are very simple. The star when at full brightness is easily seen with an opera glass, and it is only necessary to select two adjacent stars, one a little brighter, the other a little fainter, as $+19^\circ$ 3956 and $+18^\circ$ 4043, and see each night if the brightness is normal. Any observations when the star is faint will be very valuable. If the observer is not accustomed to the method of Argelander, it will only be necessary to name one or two stars which

are a little brighter and others a little fainter at a given hour and minute. These observations should be repeated two or three times an hour to detect changes. The stars may be identified by a sketch of the region, if desired. The value will be greatly increased if any kind of photograph can be taken, attaching a hand camera to an equatorial or even allowing the stars to trail through the field. The detailed observations should be forwarded at once by mail, and if possible the time of minimum cabled. For instance, "Observatory, Boston, Schwab twenty-first sixteen, Smith," would be understood to mean that Mr. Smith found Schwab's variable at minimum on the twenty-first of the current, or preceding, month at sixteen hours after Greenwich Mean Noon. The message sent from here, "Schwab, May twenty-first," would indicate that a minimum was predicted for that date, and that observations on that evening were greatly desired. It is hoped that by the aid of astronomers in Asia and Australia it will be possible to follow the star continuously during the present year, so that no minima will be missed, or if so, only during a limited period, as the month of July, 1902, when the star is in opposition to the Sun.

EDWARD C. PICKERING.

JANUARY 18, 1902.

NOTE.—The following observations of Schwab's variable have been obtained at the Yerkes Observatory by Mr. J. A. Parkhurst:

Y.	M.	D.	Gr. T. h.	m.	J. D.	Mag.		
1902	2	3	23	40	5784.986	8.31	vis. comp's	5 stars
1902	2	3	23	50	5784.993	8.69	photometric	2 stars
1902	2	4	0	28	5785.019	8.34	photometric	2 stars

The photometric results depend upon the HCO Meridian Photometer (Vol. XXIV) magnitudes of

	M.
B. D. + 19°3971	9.23
19 3972	8.20

NOTICE.

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed.

Authors are particularly requested to uniformly employ the metric units of length and mass; the English equivalents may be added if desired.

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THE

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ON THE RADIATION OF MERCURY IN THE MAGNETIC FIELD.[1]

By C. RUNGE and F. PASCHEN.

WE have undertaken in this paper an investigation for the lines of the spectrum of mercury of the connection between the Zeeman effect of the action of a magnetic field upon the light-vibrations and the distribution of the lines in series.[2] This connection was some time ago pointed out by Thomas Preston,[3] but it has not been known to what extent and with what accuracy he demonstrated it. He speaks in his published papers only of the series in the spectra of magnesium, cadmium, and zinc, and in these spectra he has only given the character of the magnetic resolution of the second subordinate series. The investigation of the mercury spectrum by A. A. Michelson[4] refers only to the visible portion in which there is no recurrence of series lines,

[1] Translated from advance proofs from the authors from the Appendix to the *Abhandlungen der K. Akademie der Wissenschaften zu Berlin vom Jahre 1902*. Read at the session on February 6, 1902.

[2] As to series consult the report of RYDBERG, *Rapport au Congrès internat. de Physique*, II, 200, 1900.

[3] *Nature*, 59, 248, 1899.

[4] ASTROPHYSICAL JOURNAL, 7, 136, 1898.

and the work of Reese,[1] who studied some of the mercury lines, hardly touches upon the questions we are treating. Kent[2] alone takes up the question, but his results are not, however, in accord with our observations.

For producing the spectrum we employed a large Rowland concave grating of 6.5 meters radius, in a solid mounting. An iron frame of strong U-rails (Fig. 1) rests at *A*. *B*, and *C* on three concrete pillars. The semicircle *AB*, of 6.5 m diameter, constitutes a horizontal table of about 30 cm width on which the slit and camera could be set up at will. The grating is at *C*, and the slit at *A*. Two wooden cameras, each 2 m wide, served for photographing the spectrum. With these placed beside each other, a continuous series of photographic plates, in length four meters, could be exposed, so that one exposure furnished the whole spectrum in several orders. In adjusting the grating we found that the spectrum by no means fell on the "Rowland circle" passing through the slit, the grating and the center of curvature, the deviations amounting to as much as 5 cm. This is explained by Cornu, as is well known, by an increase in the distance between the rulings from one side of the grating to the other.[3] It is often desirable to photograph a line simultaneously in several orders, in order to convince oneself of the reality of faint components, for false side lines often occur from faults of the slit or from inaccurate focusing of the camera. Real components must appear in the different orders at different distances according to their wave-lengths. If they do so appear, their reality is very probable. The fixed mounting of the Row-

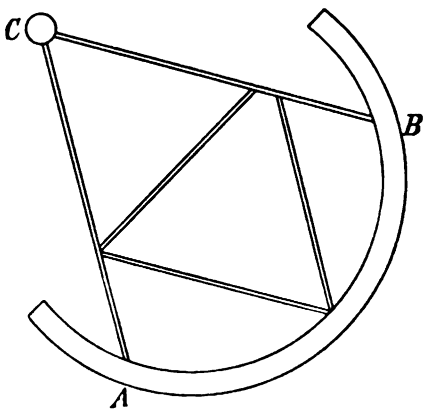

FIG. 1.

[1] ASTROPHYSICAL JOURNAL, 12, 120–135, 1900. [2] *Ibid.*, 13, 289–319, 1901.

[3] See KAYSER'S *Handbuch der Spectroscopie*, I, 441.

land grating has the further advantage that the adjustment does not change, and that greater independence from jars of the building is secured. It was further of importance to us that the magnetic field should be the same for the lines simultaneously photographed. The field-strengths at different exposures may be compared, however, if only one line is common to both exposures. This could be easily arranged in view of the great extent of the region simultaneously photographed. We have largely used this arrangement in the investigation of other elements, and we intend soon to publish the results of this investigation.

For producing the magnetic field we use a Dubois[1] semicircular magnet from Hartmann and Braun, which the Berlin Academy of Sciences was kind enough to place at our disposal.

The source of light is doubtless the most important point in the investigation. We have used Geissler tubes with mercury electrodes in the form suggested by F. Paschen.[2] In the course of the investigation we found it necessary to make several alterations in it. The light which falls upon the grating should come solely from those parts of the source of light which are in the most intense part of the magnetic field. To accomplish this the Geissler tubes were given the form shown in Fig. 2. The capillary crosses the strongest part of the field perpendicularly, and a diaphragm, *AB*, permits only the light from this part to reach the slit. In order to have the same advantage in the case of the ultra-violet rays, a tube closed by a window of fluor-spar was attached at the middle of the capillary (Fig. 3). A quartz window cannot be used on account of the rotary dispersion of quartz, if the polarization of the component is to be determined. Two pieces of equal thickness of right- and left-handed quartz would have to be employed in that case. We placed a calc-spar plate in front of the window of the tubes for investigating the polarization. If an image of the capillary is now projected upon the slit by quartz lenses, it will be separated by the calc-spar into two images, polarized perpendicularly to each other. By a slight change of

[1] *Annalen der Phys.*, 1, 199, 1900. [2] *Physikalische Zeitschrift*, 1, 478, 1900.

the adjusting screws of the lens support we could throw either one of these two images upon the slit. In the correct position of the calc-spar, the one image was composed of light having the electric vibrations in the source of light parallel to the lines of force, while the electric vibrations of the other image are perpendicular to the lines of force. The rotation of the plane of vibration by the passage through the calc-spar by the quartz lenses is without effect upon the result.

The connection between the Zeeman effect and the series is exhibited by the fact that all lines of one series—*i. e.*, all lines

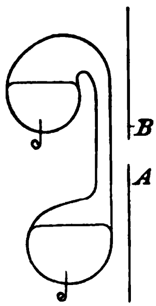

FIG. 2.

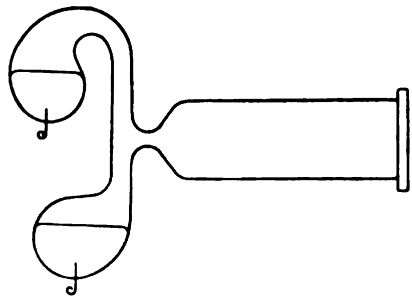

FIG. 3.

whose vibration numbers are expressed by the same formula, when the number of the order runs through the series of whole numbers—are separated by the magnetic field in the same manner, but lines of different series in a different manner. The measurements themselves show best how this is to be understood. Our measures all refer to the same field-strength, although all the exposures were not made at the same field-strength. They can be reduced to the same field-strength, inasmuch as it appears that at different field-strengths the distances of the components of a line remain in proportion, so that the components always give the same image, and only the scale of the image increases with the field-strength. An apparent exception to this rule occurs in case of a few fainter components seen near the strongest lines. It can hardly be doubted, however, that these components do not belong to the lines themselves,

but to satellites which lie very near to them without the magnetic field. If the components of a satellite could be observed alone, it would presumably come out that their distances also remained in proportion when the field-strength changed, although their distances from the component of the principal line did not remain in proportion. A greater dispersion than that of the Rowland grating would be necessary, however, for the satisfactory solution of this question, as the components of the principal lines far too easily cover up the components of the satellite, especially with low field-strengths.

Our observations further show that the increase of the scale of the separation with rising field-strength is always in the same ratio for all lines of the spectrum: if the scale of separation of a line increases in the ratio $a : b$ with a rise of field-strength, then the scale of separation of every other line increases in the same ratio. We have tested this fact for the mercury lines for field-strengths from about 12,000 to about 25,000 c.g.s. units. These observations contradict the statement of N. A. Kent,[1] who, like H. M. Reese, asserts a different behavior for the three zinc lines λ 4680, 4722, and 4811, which show the same separations as the mercury lines at λ 4047, 4359, 5461. According to them, the scale of the separation of λ 5461 does not increase as fast as that of λ 4047 and 4358, when the field-strength rises above 18,000 units. But the objection can be made to the statements of Kent and Reese that they did not obtain the type of the mercury line λ 5461. They speak of it as a diffuse triplet, whereas in fact, as is seen on our plates and as has already been described by Michelson, it consists of nine components, of which the middle three are polarized perpendicularly to the outer six. Kent and Reese, by causing the three interior components to disappear with a nicol, measured the distance of the two exterior groups with their components blended together. Therefore decidedly less weight should be assigned to their measurements than to ours. Whether the scale of the separation increases in direct proportion to the field-strength, or in some other relation,

[1] ASTROPHYSICAL JOURNAL, 13, 294, 1901.

has not been investigated by us, as we have not made any measures of field-strength. Our reduction of all the observations to the same field-strength does not assume direct proportionality, but only that the scale of separation for different lines depends in the same manner on the field-strength.

The details of the reduction are as follows: The differences of wave-length of the nine components of the strong green line λ 5460.97, which is the most widely separated of all the lines of the spectrum by the magnetic field, were measured on five of the best plates. Thirteen unknowns were now determined by the method of least squares, viz., the four factors with which the measures of four of the plates are to be multiplied in order to reduce them to the scale of the fifth, and the nine corrections of wave-length which are to be applied to the measures on the fifth plate. It was at the same time possible to so transform the normal equations that four equations suffice for the four factors. Each of the nine wave-length corrections then comes out as a linear function of the four factors. The other lines of the five plates were now also reduced with the four factors thus obtained, and the separations tabulated below are the means of the values thus found. The field-strengths of the five plates do not differ very greatly from each other. The scales of the separation deviate in the most extreme case by 23 per cent. from each other.

Numerous other plates were, however, also employed for many of the lines, particularly for the fainter ones. In order to reduce these to the same field-strength, the means of the separations of the lines λ 5461, 4359, and 4047, as yielded by the five best plates, were assumed as correct, and for each new plate the reduction factor was determined by the method of least squares, but without the introduction of the wave-length corrections of the separate components as unknowns. Then we have to deal with only one unknown besides the reduction factor sought for, namely, the parallel displacement of the new plate. The parallel displacement is determined, as easily appears from the method of least squares, by the fact that the

center of gravity of the components for the new plate must agree with the center of gravity of the given components.

We did not directly determine the field-strengths to which all the measurements are reduced, but only obtained it as 24,600 c. g. s. units with the aid of measures by Michelson, Reese, Marchand, and Blythswood. The computation is given at the end of the paper.

SECOND SUBORDINATE SERIES I.

Undisturbed Wave-length	Wave-length in magnetic field[1]		Mean Error	Intensity	Δλ	−Δλ/λ²	Remarks
	Parallel	Perpendicular					
		1.605		3	+635	−2.13	On the side toward
		1.454		5	+484	−1.62	shorter wave lengths
		1.289		4[2]	+319	−1.07	can be seen two more
	1.127	.....		3	+157	−0.53	components, and on
5460.97	0.970	.....	0.0024	4[3]	0	0	the side toward longer
	0.807	.....		3	−163	+0.55	wave-lengths, one
		0.654		4[2]	−316	+1.06	more, vibrating per-
		0.483		5	−487	+1.63	pendicular to the
		0.324		3	−646	+2.17	lines of force, which
							probably belong to
		1.929	0.0034	<1	+229	−2.05	satellites of the prin-
		1.876	0.0024	1	+176	−1.58	cipal line, since their
		1.816	0.0024	2	+116	−1.04	distances from the
	1.762	.....		1	+ 62	−0.56	principal line do not
3341.70	1.700	.....	0.0034	2	0	0	vary with the field-
	1.639	.....		1	− 61	+0.55	strength in the same
		1.584	0.0024	2	−116	+1.04	ratio as the other dis-
		1.524	0.0024	1	−176	+1.58	tances. Other faint
		1.462	0.0034	<1	−238	+2.13	lines are also noticed
							between those given
							here.
		5.604	0.006	1	+ 94	−1.10	The components are only
2925.51	5.510	.....	.....	1	......	......	observed as separate
		5.416	0.006	1	− 94	+1.10	when the other compo-
							nents vibrating paral-
							lel to the lines of force
							are suppressed.

[1] The first three figures of the wave-length are omitted.

[2] The components 0.654 and 1.289 were often stronger than 0.483 and 1.454; and often the reverse was true.

[3] The intensity of the innermost components is less than that of their neighbors in tubes with the chamber, the innermost being absorbed in the chamber. We also convinced ourselves of this outside of the magnetic field with an echelon spectroscope. If the pressure in the tube is raised by heating the mercury, the reversal of the lines λ 5461 and 4359 is seen in the tubes with a chamber.

SECOND SUBORDINATE SERIES II.

Undisturbed Wave-length	Wave-length in magnetic field Parallel	Perpendicular	Mean Error	Intensity	Δλ	−Δλ/λ²	Remarks
		8.968		5	+408	−2.15	Two fainter components, vibrating perpendicular to the lines of force, at λ 4359.05 and 4358.07, whose distances from the principal line do not vary with the field-strength proportionally to the other distances, presumably belong to satellites of the principal line. Fainter components, vibrating parallel to the lines of force, are also noticed, which probably belong to the satellites.
		8.867		4	+307	−1.62	
4358.56	8.668	.....	0.0011	4	+108	−0.57	
	8.458	.....		4	−102	+0.54	
		8.249		4	−311	+1.64	
		8.150		5	−410	+2.16	
		3.849	0.0030	2	+179	−2.14	
		3.808	0.0030	2	+138	−1.65	
2893.67	3.713	.....	0.0023	1	+ 43	−0.51	
	3.621	.....	0.0023	1	− 49	+0.58	
		3.534	0.0030	2	−136	+1.62	
		3.496	0.0030	2	−174	+2.08	
		6.419	0.01	..	+109	−1.64	The components are only observed as separate when other components vibrating parallel to the lines of force are suppressed.
2576.31	6.31	.....	....	..	.....	.....	
		6.200	0.01	..	−110	+1.66	

SECOND SUBORDINATE SERIES III.

		7.136		6	+356	−2.17	Three fainter components, vibrating perpendicular to the lines of force, at 7.223, 7.080, and 6.365 presumably belong to satellites, and not to the principal line, as their distances from the principal line do not vary with the field-strength proportionally with the other distances. Fainter components, vibrating parallel to the lines of force, are also noticed, which also probably belong to the satellites.
4046.78	6.780	.....	0.0022	7	0	0	
		6.423		6	−357	+2.18	
		3.086	0.007	1	+176	−2.32	
		.....	0.010	3	0	0	
2752 91	2.910	2.753	0.007	1	−157	+2.07	

FIRST SUBORDINATE SERIES I.

Undisturbed Wave-length	Wave-length in magnetic field: Parallel	Perpendicular	Mean Error	Intensity	$\Delta\lambda$	$-\Delta\lambda/\lambda^2$	Remarks
		3.732		1	+272	−2.06	
		3.672		3	+212	−1.61	
		3.610		2	+150	−1.14	The pairs of lines included in the brace merged into one line when both kinds of vibrations were admitted.
	3.582	.....		5	+122	−0.92	
		3.542		2	+ 82	−0.62	
	3.528	.....		2	+ 68	−0.51	
3663.46 Satellite	3.398	.....	0.0019	2	− 62	+0.47	
		3.370		3	− 90	+0.68	
	3.332	.....		5	−128	+0.97	
		3.318		3	−142	+1.08	
		3.249		4	−211	+1.60	
		3.187		1	−273	+2.07	
		3.428	0.0024	2	+378	−2.81	The two components of greatest wave-length were covered by the components of the line λ 3663.46 at this field-strength. They were observed at less field-strength and then reduced to the larger field-strength.
		3.274	0.0024	1	+224	−1.67	
	3.198	.....	0.0031	2	+148	−1.10	
		3.128	0.0038	<1	+ 78	−0.58	
3663.05 Satellite	3.050	.....	0.0024	4	0	0	
		2.977[1]	0.009	<1	− 73	+0.54	
	2.903	.....	0.0031	3	−147	+1.09	
		2.828	0.0024	1	−222	+1.65	
		2.679	0.0024	3	−371	+2.76	
	3.769	3.769		2	+129	−1.41	Not clearly separated.
3023.64 Satellite	3.734	3.734	0.01	2	+ 94	−1.03	
	3.546	3.546		2	− 94	+1.03	Not clearly separated.
	3.500	3.500		2	−140	+1.53	
		1.792	0.004	3	+112	−1.22	
3021.68 Prin. line	1.705	.....	0.010	3	+ 25	−0.27	
	1.655	.....	0.010	2	− 25	+0.27	
		1.569	0.004	3	−111	+1.21	
2803.69	3.69	3.776	0.014	1	+ 86	−1.10	The components are only observed as separate when the vibrations parallel to lines of force are suppressed.
		3.604	0.014	1	− 86	+1.10	

FIRST SUBORDINATE SERIES II.

Undisturbed Wave-length	Parallel	Perpendicular	Mean Error	Intensity	$\Delta\lambda$	$-\Delta\lambda/\lambda^2$	Remarks
		2.102	0.0037	1	+152	−1.55	The components given as simultaneously vibrating parallel and perpendicular to the lines of force appear not to coincide precisely. Those vibrating parallel presumably have a somewhat greater, those vibrating perpendicular a somewhat less distance than that given.
		2.058	0.0030	3	+108	−1.10	
	2.007	2.007	0.0030	4	+ 57	−0.58	
3131.95 Satellite	1.950	.....	0.0037	<1	0	0	
	1.884	1.884	0.0030	4	− 66	+0.67	
		1.843	0.0030	3	−107	+1.09	
		1.784	0.0053	1	−166	+1.09	

[1] This component is very faint, decidedly more so than λ 3663.128, and hence is determined with only slight accuracy.

Undisturbed Wave-length	Wave-length in magnetic field		Mean Error	Intensity	$\Delta\lambda$	$-\Delta\lambda\ \lambda^2$	Remarks
	Parallel	Perpendicular					
	1.841?	1.841?	0.0026	<1	+181	−1.85	The components marked
		1.815	0.0015	3	+155	−1.58	with a ? were only
	1.764	.....	0.0018	3	+104	−1.06	observed without the
3131.66	.....	1.716	0.0015	3	+ 56	−0.57	calc-spar. Their
Satellite		1.604	0.0015	3	− 56	+0.57	state of polarization
	1.555	.....	0.0018	3	−105	+1.07	therefore cannot be
		1.502	0.0015	3	−158	+1.59	given.
	1.452?	1.452?	0.0026	<1	−208	+2.12	
	5.936?	5.936?	0.0010	1	+156	−1.60	
		5.897	0.0007	2	+117	−1.20	
		5.858	0.0007	3	+ 78	−0.80	
	5.819	.....	0.0010	1	+ 39	−0.40	
3125.78	5.780?	5.780?	0.0010	1	0	0	
Prin. line	5.744	.....	0.0010	1	− 36	+0.37	
		5.705	0.0007	2	− 75	+0.77	
		5.664	0.0007	3	−116	+1.19	
	5.627	5.627	0.0014	1	−153	+1.57	
		5.346	0.0043	1	+ 56	−0.79	The components were
2655.29	5.29	.....	......	1	...	....	seen separately only
Satellite		4.234	0.0043	1	− 56	+0.79	when the components
							vibrating parallel
		3.970	0.012	1	+110	−1.56	were suppressed.
	3.932	.....	0.012	1	+ 72	−1.02	
2653.86	.....	3.906	0.012	1	+ 46	−0.65	
Satellite		3.812	0.012	1	− 48	+0.68	
	3.788	.....	0.012	1	− 72	+1.02	
		3.754	0.012	1	−106	+1.51	
		2.295	0.006	3	+ 75	−1.07	
2652.22	2.220	.....	0.012	3	0	0	
Prin. line		2.145	0.006	2	− 75	+1.07	

FIRST SUBORDINATE SERIES III.

		7.740	0.003	1	+100	−1.14	
2967.64	7.640	.....	0.004	1	0	0	
Satellite		7.541	0.003	2	− 99	+1.12	
	7.423	7.423	0.004	4	+ 53	−0.60	The middle component
2967.37	7.370	.....	0.004	3	0	0	is broad.
	7.318	7.318	0.004	3	− 52	+0.59	
	4.920	4.920	0.001	1	+ 30	−0.47	The components were
2534.89	4.89	.....	.....		...	....	seen separately only
	4.860	4.860	0.001	1	− 30	+0.47	when the components
							vibrating parallel
							were suppressed.

LINES OF THE SPECTRUM OF MERCURY NOT BELONGING TO THE SERIES.

Undisturbed Wave-length	$\Delta\lambda$	$-\Delta\lambda/\lambda^2$	Mean Error	Intensity	Remarks
	+369	−1.10	0.0067	1	For all the lines included here, except the last, the two outer components vibrate perpendicular to the lines of force, but the inner parallel to the lines of force.
5790.49	0	0	0.0082	3	
	−399	+1.19	0.0067	1	
	+414	−1.24	0.0017	2	Another faint line lies at λ 5789.32, whose separation in the magnetic field could not be observed.
5769.45	0	0	0.0021	5	
	−415	+1.25	0.0017	2	
	+271	−1.12	......	1	
4916.41	0	0	[1]	2	
	−259	+1.07	......	1	
	+206	−1.09	0.0018	2	
4347.65	0	0	0.0020	4	
	−206	+1.09	0.0018	2	
	+246	−1.31	0.0048	1	
4339.47	0	0	0.0064	2	
	−252	+1.34	0.0048	1	
	+156	−0.92	......	1	The wave-length was determined by Eder and Valenta,[2] who include the line among the band-lines of mercury.
4108.2	0	0	......	2	
	−180	+1.07	......	1	
	+268	−1.61	0.0034	3	
4078.05	0	0	0.0043	6	
	−274	+1.65	0.0034	3	
3984.08	——	——	——	——	The line λ 3984.08 (Kayser and Runge) in an undisturbed condition consists of three components: 4.196; 4.121; and 4.054. On switching on the field we only got a diffuse band.
	+141	−0.92	......	2	The wave-lengths are those given by Eder and Valenta.
3906.6	0	0	......	4	
	−144	+0.94	......	2	
	+165	−1.08	......	1	
3902.1	0	0	......	2	
	−159	+1.04	......	1	
	+ 94	−1.16	......	1	The components could be observed as separate only when the components vibrating parallel to the lines of force were suppressed.
2847.85	0	0	......	..	
	− 94	+1.16	......	1	

[1] The lines of this list for which no mean error is given were observed only once. The accuracy is much less for these.

[2] "Ueber die verschiedenen Systeme des Quecksilbers." *Abhandlungen der Wiener Akademie*, 1894.

LINES OF THE SPECTRUM OF MERCURY NOT BELONGING TO THE SERIES.

Continued.

Undisturbed Wave-length	$\Delta\lambda$	$-\Delta\lambda/\lambda^2$	Mean Error	Intensity	Remarks
2536.72	+115 −115	−1.79 +1.79	0.0028 0.0028	10 10	This strong line splits into two components, which both vibrate perpendicular and parallel to the lines of force. Alongside of each of these components, in the direction of greater wave-lengths, each shows a faint component at a distance of 0.090. We should expect this to belong to a line at λ 2536.81, but we have not observed any line besides λ 2536.72 in the undisturbed condition.

All of our measurements are collected in the foregoing table. Kayser and Runge's values are used for the wave-lengths of the lines when not under the action of the magnetic field. The relative accuracy of the components is of course considerably greater than the absolute, which is here of secondary importance. The designations "parallel" and "perpendicular" signify that the electrical vibrations are parallel or perpendicular to the lines of force. The mean error given refers only to the relative precision. The intensities of the components are estimated on a scale on which the greatest intensity is 10 and the least is 1 or < 1. The column headed $\Delta\lambda$ contains the differences of wave-length of the components from that of the unaffected line in thousandths of a tenth-meter. The column headed $-\Delta\lambda/\lambda^2$ gives the differences of the vibration numbers ($1/\lambda =$ number of waves to the centimeter), λ being measured in centimeters. The lines are arranged in the order of series and increasing vibration numbers. The subordinate series are properly six in number, but each three of them which are congruent with each other when plotted on the scale of vibration numbers, should probably be also designated as a subordinate series. In the second subordinate series all three consist of simple lines, but in the first subordinate series each is accompanied by satellites. The three series of the second subordinate series are placed first, then the three series

of the first subordinate series with the satellites, and finally the lines which do not belong to the series.

So far as the accuracy of the measures permits, lines of the same series (*i. e.*, lines which are represented by the same empirical formula of Kayser and Runge or of Rydberg) exhibit the same separation due to the magnetic field, in the sense that according to the scale of vibration-numbers the components of all the series-lines have the same distances, and that corresponding components are also polarized in the same way. But in case of the fainter lines as a rule not all the components were observed, and at the shorter wave-lengths the components crowd so closely together as to be no longer separable. Thus, for instance, in the series to which the lines $\lambda 5461$, 3342, and 2926 belong, nine components of the first (aside from the faint components which we ascribe to satellites), nine components of the second, and only three components of the third were observed. It is nevertheless hardly to be doubted that the separation is also the same in case of the third line. It is confirmed by the observed components, while the absence of some of the components is explained by their slight intensities. The precision with which the same vibration-differences repeat themselves corresponds entirely to the precision of the measures. Thus, for instance, we have for $\lambda 5461$ and 3342 the following values of $-\Delta\lambda / \lambda^2$:

$\lambda 5461$	$\lambda 3342$	Differences	Squares
−2.13	−2.05	−0.08	64
−1.62	−1.58	−0.04	16
−1.07	−1.04	−0.03	9
−0.53	−0.56	+0.03	9
0	0	0	..
+0.55	+0.55	0	..
+1.06	+1.04	+0.02	4
+1.63	+1.58	+0.05	25
+2.17	+2.13	+0.04	16
			Sum 143

$$\sqrt{\frac{143}{8}} = 4.2 .$$

The mean error of 0.042 for the difference of the values of $\Delta\lambda/\lambda^2$ agrees sufficiently well with the value which may be computed from the mean errors of the wave-lengths of the components of λ 5461 and 3342. The computed mean error of $\Delta\lambda/\lambda^2$ is 0.011 at λ 5461 and 0.035 and 0.041 at λ 3342. Hence we obtain for the differences of the values of $\Delta\lambda/\lambda^2$ mean errors at λ 5461 and 3342 of 0.037 and 0.042, according as the more accurate or less accurate components of λ 3342 are used.

Only three components can be recognized in case of the third series line λ 2926. But the observed values of $\Delta\lambda/\lambda^2$ are also here in agreement with what was to be expected if we assume that only the strongest component appeared.

The repetition of the type is more difficult to observe in case of the first subordinate series than in the case of the second, because the lines are of shorter wave-lengths, so that the components lie nearer together on the sale of wave-lengths. As far as the accuracy of the measures permits, however, the same separation is also here exhibited by lines of the same series, by both principal lines and satellites. Kent asserts that for lines of the same series the separation is not the same, but that $\Delta\lambda/\lambda^2$, for instance, from *Hg* λ 5461 to *Hg* 3342, increases in ratio from 3 to 4.[1] Inasmuch, however, as he did not resolve the separate components of the lines investigated, the contradiction between our measurements is of small significance.

All of the lines not belonging to the series as far as the strong line λ 2536.72 are separated into three components. The differences of the vibration numbers of the components are nearly the same, but they show deviations considerably in excess of the errors of observation. It cannot be doubted, for instance, that the components of λ 5769 give greater vibration-differences than the components of λ 5790, and similarly those of λ 4339 are greater than those of λ 4348.

A survey of all the vibration-differences occurring is given in the following table. Only the strongest of the series lines is here given for each series; the others would, as remarked above,

[1] ASTROPHYSICAL JOURNAL, 13, 316, 1901.

TABLE OF THE VIBRATION-DIFFERENCES $-\Delta\lambda/\lambda^2$ OF THE UNAFFECTED LINES AS COMPARED WITH THE COMPONENTS IN THE MAGNETIC FIELD (24,600 UNITS).

λ																	
5461	....	−2.13 s	−1.62 s	....	−1.07 s	....	....	−0.53p	op	+0.55p	....	....	+1.06 s	....	+1.63s	+2.17s	....
4359	....	−2.15 s	−1.62 s	....	..	....	....	−0.57p		+0.54p	....	....	....	....	+1.64s	+2.16s	....
4047	....	−2.17 s	....	....	....	....	....	....	op	....	....	....	....	....	....	+2.18s	....
3663.5	....	−2.06 s	−1.61 s	...	−1.14 s	−0.92p	−0.62 s	−0.51p	op	+0.47p	+0.68s	+0.97p	+1.08 s	....	+1.60s	+2.07s	....
3132.0	....	....	−1.55 s	....	−1.10 s	....	−0.58p & s		op	+0.67p & s		....	+1.09 s	....	+1.69s	....	....
2967.6	....	....	....	....	−1.14 s	....	....	....	op	....	....	....	+1.12 s	....	....	....	....
3663.0	−2.81 s	....	−1.67 s	....	−1.10p	....	....	−0.58 s	op	+0.54 s	....	....	+1.09p	....	+1.65s	....	+2.76 s
3131.7	....	−1.85?	−1.58 s	....	−1.06p	....	....	−0.57 s		+0.57 s	....	....	+1.07p	....	+1.59s	+2.12?	...
2967.4	....	....	....	....	....	....	−0.60p & s		op	+0.59p & s		....	....	....	....	....	....
3655.0	....	−1.98 s	−1.56 s	−1.35p	−1.19 s	−0.81p	−0.76 s	−0.37p		+0.38p	+0.77s	+0.82p	+1.15 s	+1.33p	+1.58s	+1.99s	....
3125.8	....	....	−1.60?	....	−1.20 s	....	−0.80 s	−0.40p	o?	+0.37p	+0.77s	....	+1.19 s	....	+1.57?	...	....
3650	....	....	....	−1.28 s	....	....	....	−0.32p		+0.36p	....	....	....	+1.27 s	....	....	....
4078	....	....	−1.61 s	....	....	....	....	....	op	....	....	....	....	....	+1.65s	....	....
5769	....	....	....	−1.24 s	....	....	....	....	op	....	....	....	....	+1.25 s	....	....	....
4339	....	....	....	−1.31 s	....	....	....	....	op	....	....	....	....	+1.34 s	....	....	....
5790	....	....	....	....	−1.10 s	....	....	....	op	....	....	....	+1.19 s	....	....	....	....
4348	....	....	....	....	−1.09 s	...	...	....	op	....	....	....	+1.09 s	....	....	....	....
4916	....	....	....	....	−1.12 s	....	....	....	op	....	....	....	+1.07 s	....	....	....	....
4108	....	....	....	....	−0.92 s	....	....	....	op	....	....	....	+1.07 s	....	....	....	....
3907	....	....	....	....	−0.92 s	....	....	....	op	....	....	....	+0.94 s	....	....	....	....
3902	....	....	....	....	−1.08 s	....	....	....	op	....	....	....	+1.04 s	....	....	....	....
2848	....	....	....	....	−1.16	....	....	....	op	....	....	....	+1.16 s	....	....	....	....
2537	....	....	−1.79 p & s	....	....	....	....	....	op	....	....	....	....	....	+1.79 p & s	....	....

yield the same vibration-differences with complete observation. The letter s or p next to the number denotes that the electrical vibrations are perpendicular or parallel to the lines of force.

The series lines are given at the beginning of the table, and first the representatives of the three series which are included under the designation of the second subordinate series. Next follow the representatives of the first subordinate series, so arranged that the satellites and principal lines are put together whose vibration-numbers yield the same differences as the three series of the second subordinate series. This arrangement accurately corresponds to Rydberg's laws for the compound triplets.[1] The lines which do not belong to series are given last.

The table clearly shows a connection between the vibration-differences of the different lines. Particular differences repeat themselves so often and so precisely that it can hardly be ascribed to chance. In the three related series the line of greatest wave-length has the most components, that of the shortest wave-length the least. But while in the second subordinate series the components dropping out at shorter wave-lengths are taken from the middle, in the first subordinate series the lines at the sides drop out. Of the eleven lines not belonging to series seven are separated into components with the same vibration-differences. The same differences occur for most of the series lines, except that here still other components are added. The vibration-differences occurring in the second subordinate series are very nearly equidistant, being in the mean: -2.15; -1.62; -1.07; -0.55; 0; $+0.54$; $+1.06$; $+1.64$; $+2.17$. The observed values are very slightly different from the multiples of ± 0.54, viz.: ± 0.54; ± 1.08; ± 1.62; ± 2.16. These vibration-differences are also most frequently represented among the remaining lines. In particular ± 1.08 is the range of the last seven triplets not belonging to the series. A connection is thus shown between these triplets and the series, which is perhaps ultimately to be ascribed to a constant charge of the ions.

We have determined the field-strengths obtaining in the case

[1] See RUNGE and PASCHEN, *Annalen der Phys.*, 5, 725, 1901.

of our measures from the measurements of Michelson,[1] Reese,[2] and Blythswood and Marchand,[3] on the assumption that the distances of the components are proportional to the field-strength.

These values resulted:

From Michelson - - -	21367	c.g.s. units, from	4 *Hg* lines
Reese - - - -	26330	" "	" 3 *Hg* lines
Reese - - - -	25020	" "	" 6 *Cd, Zn, Mg* lines[4]
Blythswood and Marchand,	25030	" "	" 4 *Hg* lines

As Michelson and Reese did not fully separate the *Hg* lines, we assign weight 1 to the first two values, and 2 to the last two, and obtain as a mean:

Field-strength = 24633 c.g.s. units (mean error = 1000 units).

A more accurate determination of the field-strength would have been desirable, as the mean error of the field-strength is relatively much greater than that of the vibration-differences of the components.

[NOTE. It is, unfortunately, impossible to reproduce here the excellent illustrations prepared to accompany the original article.—EDS.]

[1] ASTROPHYSICAL JOURNAL, **7**, 136, 1898.

[2] *Ibid.*, **12**, 120–135, 1900.

[3] *Phil. Mag.*, **40**, 397, 1895.

[4] We photographed the *Cd, Zn* and *Mg* lines in the magnetic field simultaneously with the *Hg* lines, the electrodes of those metals having been amalgamated.

AN IMPROVED METHOD OF CALCULATING THE ORBIT OF A SPECTROSCOPIC BINARY.

By HENRY NORRIS RUSSELL.

THE methods of determining the orbit of a spectroscopic binary from its observed velocity-curve may be divided into two general classes :

1. Geometrical, in which the elements of the orbit are determined from the geometrical properties of the curve, especially its maxima and minima.

2. Analytical, in which the observed radial velocity is developed into a trigonometric series, and the elements are found by comparing this series with the corresponding analytical expression for the velocity.

The only method of the second class known to the writer is that of Wilsing.[1] As developed by him, it is available only for orbits of very small eccentricity. The purpose of the present discussion is to extend this method so that it may be generally available.

The theory of the method may be presented as follows: The period U, and the corresponding value μ of the "mean motion," are given at once by the observed velocity-curve. The radial velocity, being then a known periodic function of the time, may be expanded into a Fourier's series of the form

$$\begin{aligned} v = C_0 + C_1 \cos \mu (t - t_0) + C_2 \cos 2\mu (t - t_0) + \cdots \\ + S_1 \sin \mu (t - t_0) + S_2 \sin 2\mu (t - t_0) + \cdots \end{aligned} \quad (1)$$

where t represents the time, and t_0 the initial epoch.

The coefficients of this series may best be obtained as follows:[2] Divide the period into any even number $2n$ of equal parts, beginning at the epoch t_0. Let $v_0, v_1, \text{- - - -} v_{2n-1}$ be the

[1] *A. N.*, 134, 90, 1893.

[2] LEVERRIER, *Annales de l'Observatoire de Paris*, Tome I, pp. 109 ff. WILSING, *loc. cit.*

corresponding values of the velocity (v_0 corresponding to t_0). Then

$$C_0 + C_n = \frac{1}{n}(v_0 + v_2 + \cdots + v_{2n-2})$$

$$C_0 - C_n = \frac{1}{n}(v_1 + v_3 + \cdots + v_{2n-1}) \,.$$

$$C_1 + C_{n-1} = \frac{2}{n}\left(v_0 + v_2 \cos\frac{2\pi}{n} + v_4 \cos\frac{4\pi}{n} + \cdots\right)$$

$$C_1 - C_{n-1} = \frac{2}{n}\left(v_1 \cos\frac{\pi}{n} + v_3 \cos\frac{3\pi}{n} + \cdots\right)$$

$$C_2 + C_{n-2} = \frac{2}{n}\left(v_0 + v_2 \cos\frac{4\pi}{n} + v_4 \cos\frac{8\pi}{n} + \cdots\right)$$

$$C_2 - C_{n-2} = \frac{2}{n}\left(v_1 \cos\frac{2\pi}{n} + v_3 \cos\frac{6\pi}{n} + \cdots\right) \,.$$

$$S_1 + S_{n-1} = \frac{2}{n}\left(v_1 \sin\frac{\pi}{n} + v_3 \sin\frac{3\pi}{n} + \cdots\right)$$

$$S_1 - S_{n-1} = \frac{2}{n}\left(v_2 \sin\frac{2\pi}{n} + v_4 \sin\frac{4\pi}{n} + \cdots\right)$$

$$S_2 + S_{n-2} = \frac{2}{n}\left(v_1 \sin\frac{2\pi}{n} + v_3 \sin\frac{6\pi}{n} + \cdots\right)$$

$$S_2 - S_{n-2} = \frac{2}{n}\left(v_2 \sin\frac{4\pi}{n} + v_4 \sin\frac{8\pi}{n} + \cdots\right) \,.$$

The coefficients C_n, C_{n-1}, etc., are not needed in the later work, but their calculation involves very little additional labor, if any, and furnishes a valuable check on the work, as, since the series (1) is convergent, they must be small.

The number of parts into which the period should be divided, in order to obtain sufficiently accurate values of the coefficients, depends upon the rate of convergence of the series (1), which, in turn, depends upon the eccentricity of the orbit. If this is not more than 0.5 (the largest value that has so far been found in a spectroscopic binary), a division into twelve, or at most sixteen parts, will suffice. The larger number may be used when the form of the velocity-curve is conspicuously different from that of a simple sine-curve.

The numerical values of the coefficients C and S being known, we may transform the series (1) into the form

$$v = A_0 + A_1 \cos \{\mu (t - t_0) + \alpha_1\} + A_2 \cos \{\mu (t - t_0) + \alpha_2\} + \cdots \quad (2)$$

by setting

$$\begin{array}{ll} A_1 \cos \alpha_1 = C_1 & A_2 \cos \alpha_2 = C_2 \\ A_1 \sin \alpha_1 = -S_1 & A_2 \sin \alpha_2 = -S_2 . \end{array} \quad \cdots \quad (3)$$

We have now to find an analytical expression, of the form (2), for the velocity, in terms of the elements. Let

a be the semi-major axis of the orbit;
e the eccentricity,
i the inclination,
ω the longitude of periastron,
measured from the descending node.
r the radius vector of the star,
w its true anomaly,
M its mean anomaly,
M_0 the mean anomaly at the epoch t_0,
z the projection of r on the line of sight,
V the radial velocity of the center of mass,
v that of the bright star.

Then we must have

$$v = V + \frac{dz}{dt} . \quad (4)$$

Now,

$$\begin{aligned} z &= r \sin (w + \omega) \sin i \\ &= r \cos w \sin i \sin \omega + r \sin w \sin i \cos \omega . \end{aligned}$$

$$\begin{aligned} \therefore \quad \frac{dz}{dt} &= \sin i \sin \omega \frac{d}{dt}(r \cos w) \\ &\quad + \sin i \cos \omega \frac{d}{dt}(r \sin w) . \end{aligned} \quad (5)$$

But by well-known formulae for elliptic motion

$$\begin{aligned} r \cos w = -\frac{3}{2} ae &+ a\left(1 - \frac{3}{8} e^2 + \frac{5}{192} e^4 - \cdots\right) \cos M \\ &+ \frac{1}{2} ae\left(1 - \frac{2}{3} e^2 + \frac{1}{8} e^4 - \cdots\right) \cos 2M \\ &+ \cdots . \end{aligned}$$

$$r \sin w = \qquad a\left(1 - \frac{5}{8}e^2 - \frac{11}{192}e^4 - \cdots\cdots\right)\sin M$$
$$+\frac{1}{2}ae\left(1 - \frac{5}{6}e^2 + \frac{1}{12}e^4 - \cdots\cdots\right)\sin 2M$$
$$+\cdots\cdots \quad .$$

Differentiating, remembering that $\frac{dM}{dt} = \mu$, substituting in (5), and, for brevity, setting.

$$\begin{aligned} 1 - \frac{3}{8}e^2 + \frac{5}{192}e^4 - \cdots\cdots &= X_1 \\ 1 - \frac{5}{8}e^2 - \frac{11}{192}e^4 - \cdots\cdots &= Y_1 \\ 1 - \frac{2}{3}e^2 + \frac{1}{8}e^4 - \cdots\cdots &= X_2 \\ 1 - \frac{5}{6}e^2 + \frac{1}{12}e^4 - \cdots\cdots &= Y_2 \text{ , etc.,} \end{aligned} \qquad (6)$$

we obtain

$$\begin{aligned} \frac{dz}{dt} = &\mu\ a \sin i\,(Y_1 \cos \omega \cos M - X_1 \sin \omega \sin M) \\ &+ \mu\, ea \sin i\,(Y_2 \cos \omega \cos 2M - X_2 \sin \omega \sin 2M) \\ &+ \cdots\cdots \quad . \end{aligned} \qquad (7)$$

If in this we set

$$\begin{aligned} X_1 \sin \omega = b_1 \sin \beta_1 \ , &\quad X_2 \sin \omega = b_2 \sin \beta_2 \\ Y_1 \cos \omega = b_1 \cos \beta_1 \ , &\quad Y_2 \cos \omega = b_2 \cos \beta_2 \text{ , etc.,} \end{aligned} \qquad (8)$$

we have

$$\begin{aligned} \frac{dz}{dt} = &b_1 \mu\ a \sin i \cos (\ M + \beta_1) \\ &+ b_2 \mu\, ea \sin i \cos (2M + \beta_2) \\ &+ \cdots\cdots \quad . \end{aligned}$$

Substituting in (4), and remembering that $M = M_0 + \mu\,(t - t_0)$, we obtain

$$\begin{aligned} v = V &+ \mu\ a \sin i \cdot b_1 \cos \ \{\mu\,(t - t_0) + \ M_0 + \beta_1\} \\ &+ \mu\, ea \sin i \cdot b_2 \cos \{2\mu\,(t - t_0) + 2M_0 + \beta_2\} \\ &+ \cdots\cdots \quad . \end{aligned} \qquad (9)$$

This is our desired expression for the velocity in terms of the elements and of the time.

The series (2) and (9), considered as functions of the time, are of the same form. If they are to represent the same quan-

tity, their corresponding coefficients must be equal. That is, we must have

$$V = A_0\,,$$
$$b_1 \mu\ a \sin i = A_1\,, \qquad M_0 + \beta_1 = \alpha_1 \tag{10}$$
$$b_2 \mu\, ea \sin i = A_2\,, \quad 2M_0 + \beta_2 = \alpha_2\,, \text{ etc.}$$

The first of these equations gives V at once. The other four may be used to determine the other elements by a process of approximation. If in (6) we neglect the terms involving e, we have $X_1 = Y_1 = X_2 = Y_2 = 1$, whence, from (8), $b_1 = b_2 = 1$, $\beta_1 = \beta_2 = w$.

The equations (9) become

$$\mu\ a \sin i = A_1\,, \qquad M_0 + \omega = \alpha_1$$
$$\mu\, ea \sin i = A_2\,, \quad 2M_0 + \omega = \alpha_2\,,$$

whence

$$a \sin i = \frac{A_1}{\mu}\,, \qquad M_0 = \alpha_2 - \alpha_1$$
$$e = \frac{A_2}{A_1}\,, \qquad \omega = 2\alpha - \alpha_2\,. \tag{11}$$

These are the final equations of Wilsing's method. In fact, he has limited all his developments to the first term of the series in e, so that the quantities X, Y, b and β do not appear in his discussion.

It is clear that the equations (11) give accurate values of the elements only when e is very small. But, in any case, they give approximate values of e and ω, which, introduced into (6) and (8), give values of b and β, from which, by the aid of (10), much more accurate values of the elements may be deduced.

Limiting ourselves to terms of the second degree in e, we find from (6) and (8):

$$b_1 = 1 - e^2\left(\frac{1}{2} + \frac{1}{8}\cos 2\omega\right)$$
$$b_2 = 1 - e^2\left(\frac{3}{4} + \frac{1}{12}\cos 2\omega\right)$$
$$\beta_1 - \omega = \frac{1}{8}\, e^2 \sin 2\omega$$
$$\beta_2 - \omega = \frac{1}{12}\, e^2 \sin 2\omega\,.$$

If we substitute these values in (10) and represent the approximate values of the elements given by (11) by the letters a', e', ω, we obtain

$$\begin{aligned} a'' &= a' \left\{ 1 + \frac{1}{2} e'^2 \left(1 + \frac{1}{4} \cos 2\omega' \right) \right\} \\ e'' &= e' + \frac{1}{4} e'^3 \left(1 - \frac{1}{6} \cos 2\omega' \right) \\ \omega'' &= \omega' - \frac{1}{6} e'^2 \sin 2\omega' \\ M_0'' &= M_0' + \frac{1}{24} e'^2 \sin 2\omega' , \end{aligned} \tag{12}$$

where a'', e'', etc., are our new and closer approximations to the true values of the elements. The angles in the above formulæ are of course to be expressed in circular measure.

The terms involving e^4 can be similarly calculated. Computation shows that the largest one—which occurs in the expression for a''—is of the order of $\frac{1}{3}e^4$ with reference to the principal term. If e' is less than 0.30, this term will affect the calculated value of a by less than one-third of 1 per cent.—an amount which, in comparison with the ordinary errors of observation, may be neglected.

If, then, e' is less than 0.30, the equations (12) are sufficiently approximate.

If e is greater than 0.30, it is most convenient to compute b_1, b_2, β_1, and β_2 by (6) and (8), using the values of e and ω given by (12), and then to apply the following equations (which may easily be derived from (10)):

$$\begin{aligned} a \sin i &= \frac{A_1}{b_1 \mu} , \quad M_0 = \alpha_2 - \alpha_1 + \beta_1 - \beta_2 , \\ e &= \frac{A_2 b_1}{A_1 b_2} , \quad \omega = 2\alpha_2 - \alpha_1 - 2(\beta_1 - \omega'') + (\beta_2 - \omega'') . \end{aligned} \tag{13}$$

These equations give values of the elements which are within 1 per cent. of the truth, even if $e = 0.70$. If, however, the values of e and ω given by (13) differ much from those taken from (12) for use in the reckoning, it is well to recompute the quantities b and β, using the more accurate values of the elements,

and to solve (13) afresh. But in such a case, and generally when the eccentricity is much greater than 0.4, the method here developed becomes laborious, and the geometrical methods are preferable.

When the final elements have been obtained, the velocity-series may be determined from them, and its agreement with the series derived from observation used to check the computations. In deriving this series, the preceding formulæ can be modified to advantage.

If we set $\frac{1}{2}(Y_1 + X_1) = h_1$, $\frac{1}{2}(Y_1 - X_1) = k_1$, etc., (7) becomes

$$\begin{aligned} \frac{dz}{dt} = & \mu\, a \sin i\, \{h_1 \cos(M + \omega) + k_1 \cos(M - \omega)\} \\ & + \mu e a \sin i\, \{h_2 \cos(2M + \omega) + k_2 \cos(2M - \omega)\} \\ & + \cdots \end{aligned}$$

The resulting expression for the velocity, including all terms which are greater than $\frac{1}{400}$ of the total range of velocity, provided e is less than 0.4, is as follows, where $c = \mu a \sin i$ and $\theta = \mu(t - t_0)$

$$\begin{aligned} v = & \; V \\ & + c\left(1 - \frac{1}{2}e^2\right) \cos(\theta + M_0 + \omega) \\ & - \frac{1}{8} ce^2 \cos(\theta + M_0 - \omega) \\ & + ce\left(1 - \frac{3}{4}e^2\right) \cos(2\theta + 2M_0 + \omega) \\ & - \frac{1}{12} ce^3 \cos(2\theta + 2M_0 - \omega) \\ & + \frac{9}{8} ce^2\left(1 - e^2\right) \cos(3\theta + 3M_0 + \omega) \\ & + \frac{4}{3} ce^3\left(1 - \frac{4}{5}e^2\right) \cos(4\theta + 4M_0 + \omega) \\ & + 1.627\, ce^4\left(1 - \frac{3}{2}e^2\right) \cos(5\theta + 5M_0 + \omega) \\ & + 2.025\, ce^5\left(1 - \frac{7}{4}e^2\right) \cos(6\theta + 6M_0 + \omega) \\ & + 2.56\, ce^6\left(1 - 2e^2\right) \cos(7\theta + 7M_0 + \omega)\,. \end{aligned} \qquad (14)$$

Should the differential coefficients of v with respect to the elements be required, they may be obtained at once by differentiating this expression.

The foregoing method appears upon trial to be somewhat less expeditious in practice than that of Lehmann-Filhès,[1] which is in general use. It should, however, be somewhat more accurate, as the elements are deduced from twelve or more points of the velocity curve, instead of four.

There are some cases, however, where the geometrical methods—depending as they do upon the maxima and minima of velocity—are inapplicable, and in these cases the present method may be found useful.

One such case would occur when a star had a period of about a year, and the maximum or minimum velocity fell in the interval when it was near the Sun. This phase being unobservable, we should be obliged, by the geometrical method, to bridge the gap by a conjectural curve, which might be seriously in error.

In applying the new method, some of the velocities v_0, v_1, etc., would be unknown. But the conditions that the coefficients C_n, C_{n-1}, etc., must be zero (or, at least, very small), give us a set of equations in which the unknown velocities appear, and from which their approximate value may be found, and approximate elements obtained. With these elements, C_n, etc., may be calculated, and accurate values of the unknown velocities obtained, from which satisfactory elements can be computed.

Another case in which the new method is useful is that of a star attended by two dark companions with commensurable periods. In this case the resultant velocity-curve may have several unequal maxima, and the geometrical methods fail altogether. The analytical method, however, enables us to separate the resultant motion into the two component orbital motions, except when the perturbations are large.

If one companion makes m revolutions, and the other n, in a given period U, the radial velocity of the bright star relative to the center of mass (barring secular perturbations) will be a

[1] *A. N.*, 136, 17, 1894.

purely periodic function of the time with period U, and may be expanded into a series of the form (1). But the component of this velocity due to the first companion will (barring perturbations) be of the form

$$v_1 = C_m \cos m\theta + C_{2m} \cos 2m\theta + \cdots$$
$$+ S_m \sin m\theta + S_{2m} \sin 2m\theta + \cdots ,$$

where $\theta = \mu (t - t_0)$ as in (14), while that due to the second companion will be of the form

$$v_2 = C_n \cos n\theta + C_{2n} \cos 2n\theta + \cdots$$
$$+ S_n \sin n\theta + S_{2n} \sin 2n\theta + \cdots .$$

If the motion of the bright star is really due to the action of two dark companions which are not large enough to disturb one another's motions much, the velocity of the bright star will be approximately $v_1 + v_2$, and those terms in the series (1) which do not occur in either v_1 or v_2 will have coefficients that are zero, or very small. The two series can therefore be picked out from the series (1), if present, by inspection, so that the method here described enables us to determine whether any periodic motion in the line of sight can be represented by the action of two dark companions which do not disturb one another's motions much, and, if so, what their respective orbits are.

ζ Geminorum, whose velocity-curve cannot be represented by motion in a single elliptic orbit,[1] may be a system of this sort.

The method above described suggested itself to the writer during an investigation of this star's motion, which is not yet completed, as the stability of the resulting orbits requires investigation. It is hoped that definite results may be reached before long.

PRINCETON, N. J.
February 14, 1902.

[1] CAMPBELL, ASTROPHYSICAL JOURNAL, 13, 94, 1901.

MEASURES OF ABSOLUTE WAVE-LENGTHS IN THE SOLAR SPECTRUM AND IN THE SPECTRUM OF IRON.[1]

By C. FABRY and A. PEROT.

IV. IRON LINES.

THE spectrum of iron is one of those most commonly employed as a comparison spectrum in spectroscopic measurements made by interpolation, on account of the brightness and the number of lines in this spectrum and the ease with which it is obtained; the iron lines almost always appear as impurity lines when the electric arc is produced between carbon poles. Many of these lines are sharp enough to serve as good standards. It was therefore desirable to measure with the highest possible precision the wave-lengths of some of these lines.

The source employed is the electric arc taken between two iron rods 1 cm in diameter, held in a simple hand regulator. The current is produced by a battery of storage cells giving 120 volts. The intensity of the current can be regulated by means of a rheostat, and is ordinarily about 8 amperes.

As the spectrum of this source contains an immense number of lines, it is necessary to isolate successively the various lines which are to be measured by means of an instrument of high dispersion. After various experiments with prisms we have adopted a plane grating by Rowland 8 cm wide, 5 cm high, and with 558 lines to the millimeter, used with a single lens which serves both as a collimator and as a projecting lens.

An image of the arc S (Fig. 3) is projected, by means of the lens L, upon the slit F. The light which passes through this slit is thrown by the total reflection prism P upon the lens L' ($f=$ 70 cm) and the plane grating R. The diffracted light returns through L' and forms a real image of the spectrum upon the the second slit F', which permits the passage of only the line

[1] Concluded from p. 96.

which is to be measured. The grating stands on the table of a goniometer; by rotating it the spectrum, which remains in focus, can be made to pass over the second slit. This motion may be produced from a distance.

In order to identify the line which it is desired to measure, it is necessary to see a certain extent of the spectrum. For this purpose the slit F', which is mounted on the carriage of an

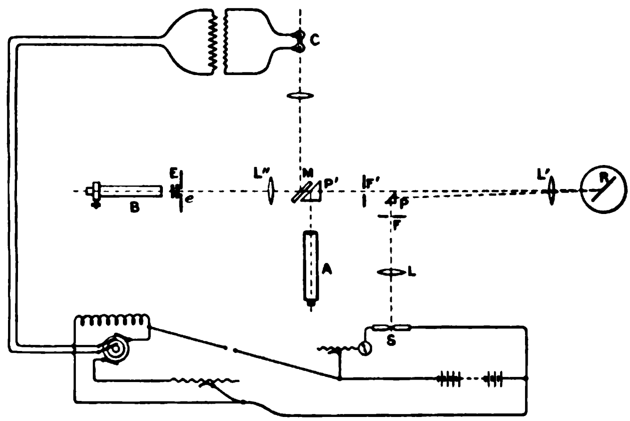

FIG. 3.

optical bench, may be removed, and the spectrum examined by means of a telescope A, used in connection with a total reflection prism P' supported on a platform which can be moved into or out of position by pulling a cord. The cross-wire in the eyepiece of the telescope A is set once for all on the slit F'. To bring a line upon F' the slit is removed, thus permitting a certain extent of the spectrum to be seen, so that the lines can be recognized. The one desired is then set on the cross-hairs and the slit is replaced. In order to identify the lines, we have employed the admirable photographic chart of Kayser and Runge.[1] The interference apparatus (5 mm or 10 mm standard) is placed at E; it is mounted as described above; e is the screen containing a hole 3 mm in diameter through which the light

[1] KAYSER and RUNGE, *Abhandlungen d. K. Akad. d. Wiss. zu Berlin*, 1888.

passes. An image of F' is projected by means of the lens L'' upon this small hole. The image of a cadmium tube C may be projected upon the same diaphragm by inserting the mirror M, which is supported on the same platform with the prism P', with which it moves. The rings are observed and measured with the telescope B focused for parallel rays.

One of the observers measures the diameter of the rings (telescope B), while the other manages the arc and the settings on the spectrum (telescope A); the rod which is used to rotate the grating is within easy reach of his hand. When the platform P' is lowered, observer A can examine the spectrum while B sees the cadmium light; when the carriage is replaced observer B sees the line separated out by F'.

As approximate values of λ' we have used either the values of Kayser and Runge or those of Rowland in the solar spectrum, after having multiplied them by suitable factors which are known approximately, in order to reduce their scale to ours (0.99998 for Kayser and Runge, 0.999963 for Rowland); there is never the slightest doubt about the whole number P'.

The following is an example of the measurement of a line:

Kayser and Runge	561.581
Rowland ☉	561.5877
λ' adopted in computing P'	561.568

	April 2, 1901 5 mm standard		April 16, 1901 10 mm standard	
Observers	Fabry	Perot	Fabry	Perot
P	19484	id.	39462	id.
δ	7.06	7.10	5.79	5.87
$B\delta^2\times 10^6$	50.18	50.75	33.64	34.59
P'	17646	id.	35739	id.
δ'	5.72	5.80	5.21	5.32
$B'\delta'^2\times 10^6$	32.83	33.75	27.20	28.36
$\frac{P\lambda}{P'}$	561.55613	id.	561.56238	id.
λ'	561.5659	561.5657	561.5660	561.5659
Corr. for phase	—0.0002	—0.0002	—0.0001	—0.0001
λ' corrected	561.5657	561.5655	561.5659	561.5658

λ' mean	561.5657
Maximum relative error	3.6×10^{-7}

The 5 mm standard has been used for all the lines measured; only a few are sufficiently fine to permit the use of the 10 mm standard. Finer lines might have been obtained by means of an arc in vacuo;[1] the wave-lengths would have been more accurately determined, but they would have differed slightly from those derived from the arc in air, as used by spectroscopists.

There is no occasion to compare our results with those of other observers: those of Kayser and Runge are not sufficiently accurate, while Rowland's refer in almost all cases to the Sun, and it is well known that there may be a considerable difference between the wave-lengths of the same line in the Sun and in the arc. Our results are given in Table II at the end of this article.

V. SOLAR SPECTRUM.

The innumerable lines of the solar spectrum furnish the most complete spectroscopic scale imaginable; it is for this reason that spectroscopists have always chosen their fundamental standards from this spectrum. In particular, the application of Rowland's method of coincidences, which requires numerous standards, would doubtless have been impossible without this collection of lines. In addition to its very great intrinsic interest, the solar spectrum is of general importance.

Since the time of Rowland's investigations, all spectroscopic measures have been interpolated by means of his wave-lengths, and consequently refer to his scale. Is this scale perfectly correct? Are its relative wave-lengths perfectly exact? Furthermore, what is the precise unit adopted? It was in the hope of answering these questions that we undertook the following investigations. It might have sufficed to measure the wave-lengths of a certain number of bright lines, which could have been referred to Rowland's scale by interpolation with respect to the solar lines. It seemed more certain and at the same time more elegant to compare directly, by an interference method, the wave-lengths of a certain number of solar lines with the green line of cadmium. Fortunately the special properties of

[1] *Journal de Physique* (3), **9**, 369, 1900.

interference phenomena with silvered films rendered this direct interference measurement possible.

The method adopted was as follows: Project a real image of the solar spectrum upon a slit, F', and illuminate the interference apparatus with the light which passes through it. This light, whose homogeneity increases with the fineness of the slit and the purity of the spectrum, will give a system of interference rings, provided the difference of path is not too great. If the slit F' is gradually widened, the light becomes less and less homogeneous, and the fringes become confused and soon disappear. Let us now suppose that in the region of the spectrum occupied by the slit F' there is a solar line of wave-length λ'. In the complex light transmitted by the slit, this radiation is lacking; there will thus be in the field of the telescope a system of dark fringes, having precisely the same properties as the bright fringes given by a line of wave-length λ'. The method of measuring λ', based on a determination of diameters, is thus applicable to this system of rings. It is evident that the slit must not be too wide, in order that the dark fringes may not be lost in a uniform illumination. The same effect results from widening the slit which produces the spectrum. In fact, it is advantageous to widen both slits, and maximum brightness corresponds to the case where the two slits are equal.

Arrangement of the apparatus.— It is necessary that the spectrum shall be sufficiently dispersed to separate out with certainty the line which is to be measured, and this, in certain parts of the spectrum, requires very great dispersion. We have employed the second order spectrum given by a concave Rowland grating of the largest size (14.5 cm by 5 cm ruled surface, 6.50 m radius of curvature, 568 lines to the millimeter). This grating is mounted in a manner different from that ordinarily employed; in Rowland's classic mounting the illuminating slit is fixed, and the diffracted image changes its position in passing from one line to another. It would have been necessary to render movable the slit F', and with it the interference apparatus, the observing

telescope, and the cadmium tube. The preparations required for each measurement would have taken a long time, and we therefore preferred to fix in position all parts of the apparatus employed in the interference work. For this purpose it is necessary that the slit F' be fixed and that the light which traverses it also be fixed in direction, and hence that the grating shall not be moved. In passing from one line to another it is therefore necessary to displace the illuminating slit F without cutting off from it the light of the solar image.

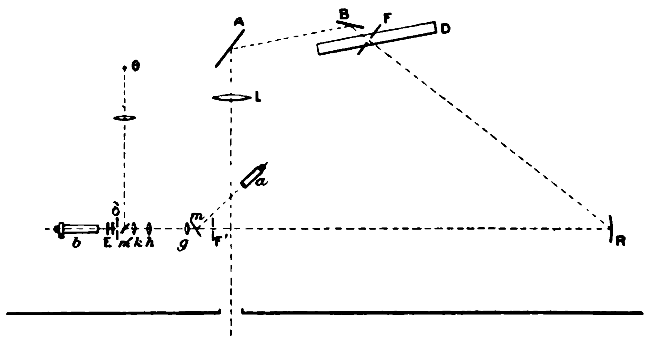

FIG. 4.

The arrangement adopted is shown in Fig. 4. The slit F' is at the center of curvature of the grating R; both are fixed. In order to make the spectrum pass across the slit F' without changing its focus, F must be moved along the circle whose diameter is RF'. In the region of the spectrum which we have employed, from $\lambda 460$ to $\lambda 650$, the displacement of F amounts to 1.90 m, on a circle 6.50 m in diameter. This arc of a circle does not differ greatly from its chord; the slit F was therefore mounted on the carriage of an optical bench D, occupying the position of the chord; this carriage can be given a slow motion by means of a screw controlled from a distance. As the arc of the circle does not coincide with its chord, the slit F must be moved a little in the direction FR in order to keep the spectrum in focus. This motion is effected with a rack and pinion.

Finally, the slit F must continue to receive light from the

solar beam in the direction *FR*. Fig. 4 indicates its path; after being reflected by the mirrors of a polar heliostat and a porte-lumiere, the beam enters the laboratory. At *A* it is reflected in a direction paralled to *D*. The mirror *B*, mounted on the same carriage with *F*, sends the beam toward the grating. In order to pass from one region of the spectrum to another, the carriage on which *B* and *F* are mounted is moved; both of these continue to receive the beam; it is only necessary to turn *D* very slightly (with a tangent screw) in order to keep the beam on *R;* this rotation is negligible when the displacement is small.

A real image of the Sun must be projected upon the illuminating slit *F;* for this purpose a lens *L* of 3.50m focal length, which gives an image about 3cm in diameter, is interposed in the path of the light.

If it is desired to displace the spectrum by a small amount (a few $\mu\mu$) it is only necessary to move the carriage. If the required displacement is great it is also necessary to move *L*, rotate *B*, and readjust the focus by sliding *F* in the direction *FR*. Only the last of these adjustment must be made with precision. There is no difficulty in accomplishing it by moving *F'* slightly in the direction *F'R* with a rack and pinion. In any case, the beam which emerges from *F'* remains fixed in position.

The slit *F'* can be removed from its support and the spectrum examined by means of the telescope *a* and the mirror *m*, which can also be removed. The observer at the telescope *a* has within reach the rod with which the carriage is moved. In order to bring upon *F'* the line which is to be measured we proceed as follows: *F* is made very narrow (about 0.05mm) so as to give a perfectly sharp spectrum. The telescope *a* is focused on *F'*, and the cross-hairs set on the slit. This slit is then removed, permitting the real image of the spectrum reflected in the mirror to be seen. Without changing the adjustments of the telescope the focusing is accomplished by moving *F*. It is then possible, with the aid of Rowland's map, to recognize the lines and to select the one which is to be measured. The slit *F* is then widened (to about 0.5mm). The lines remain visible,

though ill defined. By moving the carriage the one selected is set on the cross-hairs of the telescope; finally F' is replaced and the mirror m removed.

The beam which emerges from F' is to be utilized in the interference apparatus. This beam is comparatively faint and of small angular width, and must therefore be utilized in the best possible manner. This beam is limited by the area of the grating and the narrow opening of the slit. Furthermore, after passing through F', the beam must (1) pass through the small diaphragm which covers the standard; (2) illuminate, in a telescope focused for parallel rays, a surface large enough to show the first two or three rings; a field of from 2° to 3° is sufficient; a larger field would be unsuitable, in view of the faintness of the light.

To realize these conditions, the natural course was to place between F' and the standard an optical system which projects upon the diaphragm an image of the slit as small as possible and which gives in the telescope focused for parallel rays an image of the grating. It is impossible to satisfy both these conditions with a single lens; with two it can be done in an infinite number of ways. The optical system thus constituted must have one of its focal planes coincident with the grating. If Φ is the focal length of the system, a and b the height and width of the grating, the field will be a rectangle of angular extent $\frac{a}{\Phi}$, $\frac{b}{\Phi}$. If r is the radius of curvature of the grating (distance RF'), the image of the slit will be defined thus: $\frac{\text{image}}{\text{object}} = \frac{\Phi}{r}$. Thus, though the problem proposed is soluble in an infinite number of ways, all the solutions are equivalent. Only the focal distance of the system is involved. Φ would be selected as 3.20 m or 1.10 m, depending upon whether the settings on the rings are to be made in the horizontal or the vertical direction. In the first case the image of the slit would be too large, in the second, the angular aperture of the beam in the horizontal direction would be too large and light would be lost.

A much better result may be obtained with a system whose

focal lengths are unequal in two directions, so as to give in the vertical direction a large field and a small image of the slit (Φ small), and an enlarged image of the slit, with a small field, in the horizontal direction (Φ large). This may be accomplished by a combination of cylindrical lenses.

Let us first take two cylindrical lenses, with generatrix horizontal, forming a system which has one focal plane at the grating, a focal length Φ, and giving for a point at F' a horizontal focal line on the diaphragm δ. This system gives horizontal focal lines and consequently limits the images in the vertical direction. Now add two other cylindrical lenses, generatrix vertical, which satisfy the same conditions, but have a focal length Φ'. These limit the images in the horizontal direction without changing the foci of the preceding system. The field will thus be a rectangle whose angular height and width will be $\frac{a}{\Phi}$ and $\frac{b}{\Phi}$; both may thus be chosen by suitably determining Φ and Φ'.

For the angular apertures 150′ was chosen in the vertical direction and 60′ in the horizontal direction, which gives

$$\Phi = 1.10\,\mathrm{m}\ , \quad \Phi' = 8\,\mathrm{m}\ .$$

The dimensions of the image of the slit will be defined thus:

In the vertical direction, $\dfrac{\text{image}}{\text{object}} = \dfrac{1.1}{6.5} = 0.17$.

In the horizontal direction, $\dfrac{\text{image}}{\text{object}} = \dfrac{8}{6.5} = 1.2$.

This optical system, composed of four cylindrical lenses, has the following properties: Every point on the grating has an image at infinity (since the two focal lines corresponding to a point are at infinity), but the image of a circle traced on the grating is an ellipse greatly elongated in the vertical direction (ratio of the axes, 7.3); the image in the telescope of the horizontally elongated rectangle of the grating is a rectangle elongated vertically. Every point on the slit has also an image on the diaphragm, and the image of a circle traced on the slit is an ellipse elongated horizontally (ratio of the axes, 7.3); the

image of the slit thus tends toward a square form. In this way the light is perfectly utilized.

These four lenses, which might be defined in an infinite number of ways, were chosen as follows: Two of the cylindrical lenses, occupying the same point, are replaced by a spherical lens and a cylindrical lens. The two others are then determined.

At g there is placed a spherical lens of + 1 diopter and a cylindrical lens of + 2 diopters, generatrix horizontal.

At h a cylindrical lens of + 2.5 diopters, generatrix vertical.

At k a cylindrical lens of + 7 diopters, generatrix horizontal.

At E is the standard, at δ the diaphragm, at b the telescope used to measure the diameter of the fringes.

The light from the cadmium tube θ may also be thrown on the diaphragm by means of the mirror m', which can be removed; an image of this tube is projected upon δ.

Selection of the solar lines.—In selecting solar lines for measurement, a few of those which give the most perfect fringes were chosen. The strongest lines are too broad, their wave-length is uncertain, and the fringes diffuse. Certain very faint lines give very fine fringes, but these are hardly visible. The lines which seem to us to give the best results are, for the most part, those whose intensities lie between 5 and 7 in Rowland's table.[1] So far as possible isolated lines have been selected in order to avoid confusion.

For the interference apparatus we have employed exclusively the 2.5 mm standard. Many lines give fringes which are clearly visible with greater thicknesses, but even with the 5mm standard these fringes are diffuse and what is gained by the higher order is lost in the precision of the settings.

Order in a measurement.—One of the observers, stationed at telescope a, sets the lines on the slit F'. Without changing his position, he controls the motion of the carriage required to displace the spectrum, moves the slit F' in the direction $F'R$ for

[1] "Preliminary Table of Solar Spectrum Wave-Lengths," ASTROPHYSICAL JOURNAL, 1–5, *passim*. The numbers which denote the intensity of the lines increase with the intensity.

the purpose of focusing, and removes and replaces the slit F' as well as the mirror m. This last change is made when the line is on the slit; the light can then pass to the interference apparatus. The second observer, stationed at telescope b, makes the diameter measurements. He manipulates the mirror m' and the interrupter of the commutator for illuminating the cadmium tube. The diameter measurements are made in the order indicated above, and the calculations are effected in the same manner.

It is highly important to avoid errors due to variations of wave-length arising from the solar rotation. In passing from the center to the limb at the solar equator, the wave-length varies $\frac{1}{100,000}$ of its value; in this way an error seven times as great as the accidental error might be committed. The rings show an easily visible contraction or expansion in passing from one limb to the other at the solar equator. These variations can be avoided by projecting upon the slit an image of the Sun's polar diameter. In the conditions of our experiment this diameter was very nearly vertical; it was only necessary to center the Sun's image on the middle of the slit.

Results.—We have measured the wave-lengths of 33 lines of the solar spectrum distributed between 460 $\mu\mu$ and 650 $\mu\mu$. Table III gives for each line the wave-length λ_R given by Rowland; the wave-length λ found by us; the ratio $\frac{\lambda_R}{\lambda}$ of the first number to the second.

Fig. 5 is plotted from the results in the table; the abscissae are wave-lengths and the ordinates are values of the ratio $\frac{\lambda_R}{\lambda}$.

1. One remark is unavoidable at the outset: the points are not scattered by chance, but are distributed regularly on a curve. The deviation between the points and the ordinates of the curve does not exceed a millionth. This shows that the relative wave-lengths of lines in the same region have been determined by Rowland with an exactness of this order, a precision which we have obtained, if not surpassed, in the comparison of any given line with the green line of cadmium.

2. If Rowland's scale were normal these wave-lengths would

differ from ours only by a constant factor; the ratio $\frac{\lambda_R}{\lambda}$ would be constant, and the above curve would have been a horizontal straight line. It is evident that this is not the case, the systematic variations of the ratio being enormously greater than the accidental deviations. Between wave-lengths 525 $\mu\mu$ and 555 $\mu\mu$ the ratio varies about 8×10^{-6}. On each side of the mean value the deviations amount to 4×10^{-6} or, in terms of the wave-length, 0.002 $\mu\mu$.

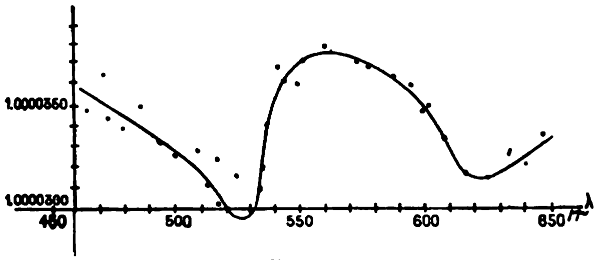

FIG. 5.

The above results confirm what has been said on the properties of gratings: as an apparatus for interpolation the grating gives the millionth, but for widely separated lines the relative wave-lengths measured by Rowland may show deviations eight times as great.

All modern spectroscopic measurements which are based upon Rowland's values contain the same errors. In order to correct them and at the same time to reduce them to Michelson and Benoit's standard, it is only necessary to divide the numbers by the ratio $\frac{\lambda_R}{\lambda}$ read from the curve (Fig. 5).

The following tables give:

Table I, the wave-lengths of certain metallic lines previously determined by us by the method of coincidences.[1]

Table II, the wave-lengths of iron lines in the arc at atmospheric pressure.

Table III, the wave-lengths of solar lines.

[1] *Journal de Physique* (3), **9**, 369, 1900 and *Comptes Rendus*, **130**, 406 and **492**, 1900.

TABLE I.

VARIOUS SOURCES.

Metal	Source	Wave-length
Mercury	arc in vacuo	435.8343
Zinc...........	vibrator in vacuo	468.0138
	"	472.2164
	"	481.0535
Copper	"	510.5543
	"	515.3251
Silver..........	"	520.9081
Copper	"	521.8202
Mercury........	tube	546.07424
Silver..........	vibrator in vacuo	546.5489
Mercury........	tube	576.95984
Copper	vibrator in vacuo	578.2090
	"	578.2159
Mercury........	tube	579.06593
Sodium	flame	588.9965
	"	589.5932
Zinc...........	vibrator in vacuo	636.2345
Lithium........	flame	670.7846

TABLE II.

IRON LINES.

473.6785	523.2954	558.6775	623.0733
485.9763	530.2321	561.5657	649.4992
500.1887	543.4525	576.3023	
508.3345	550.6783	606.5489	

TABLE III.

SOLAR LINES.

λ_R	λ	$\frac{\lambda_R}{\lambda}$	λ_R	λ	$\frac{\lambda_R}{\lambda}$
464.3645	464.3483	1.0000349	549.7735	549.7536	1.0000362
470.5131	470.4960	363	550.7000	550.6794	374
473.6963	473.6800	344	558.6991	558.6778	381
478.3613	478.3449	340	571.5308	571.5095	373
485.9928	485.9758	350	476.3218	576.3004	371
402.4107	492.3943	333	586.2582	586.2368	365
500.2044	500.1881	326	593.4881	593.4666	362
509.0954	509.0787	328	598.7290	598.7081	349
512.3899	512.3739	312	601.6861	601.6650	351
517.1778	517.1622	302	606.5709	606.5506	335
524.7229	524.7063	316	615.1834	615.1639	317
524.7737	524.7587	286	623.0943	523.0746	316
534.0121	533.9956	309	632.2907	632.2700	327
534.5991	534.5820	320	633.5554	633.5346	328
536.7669	536.7485	343	640.8233	640.8027	321
541.0000	540.9800	370	647.1885	647.1666	338
543.4740	543.4544	361			

THE INFLUENCE OF ATMOSPHERES OF NITROGEN AND HYDROGEN ON THE ARC SPECTRA OF IRON, ZINC, MAGNESIUM, AND TIN COMPARED WITH THE INFLUENCE OF AN ATMOSPHERE OF AMMONIA.

By ROYAL A. PORTER.

IT has been frequently assumed that chemical reactions in the electric arc have a considerable influence on the character of its radiations.[1] It seems not unreasonable to expect that the oxidation of the electrodes at high temperature in air would tend to increase the intensity over that obtained when the arc is operated in nitrogen alone. This presumed higher temperature might be sufficient to produce atomic vibrations entirely distinct from the vibrations at a lower temperature. If the atmosphere does have any such influence, the effect might be apparent in the spectrum of the arc.

In the case of a hydrogen atmosphere[2] the most marked effects on the arc spectrum of iron, zinc, magnesium, and tin have been found to be a general diminution of intensity and a change of relative intensity among the lines. The lines relatively enhanced by hydrogen are spark lines.

Liveing and Dewar[3] have noted the effect of atmospheres of hydrogen and nitrogen on a number of lines in the magnesium arc and spark and of these and other atmospheres on the cyanogen[4] bands in the carbon arc, but so far as I am aware no extensive study has been made of the influence of nitrogen and ammonia on the arc spectrum of metals. From a study of the

[1] LIVEING and DEWAR, *Proc. Roy. Soc.*, **30**, 161, 1880; **32**, 192, 1881.

O. H. BASQUIN, ASTROPHYSICAL JOURNAL, **14**, 11, 12, 1901.

A. S. KING, ASTROPHYSICAL JOURNAL, **14**, 329, 330, 1901.

[2] H. CREW, ASTROPHYSICAL JOURNAL, **12**, 167, 1900; LIVEING and DEWAR, *Proc. Roy. Soc.*, **32**, 192, 402, 403, 1881.

[3] *Proc. Roy. Soc.*, **32**, 189–203, 1881.

[4] *Ibid.*, **30**, 152–162.

effects of hydrogen and nitrogen on certain spark lines of magnesium, Liveing and Dewar[1] were led to remark that "it is possible that the atmosphere may, besides the resistance it offers to the discharge, in some degree affect the vibrations of the metallic particles." This conclusion in regard to the spark seems to have been borne out with reference to the arc by the results obtained with a hydrogen atmosphere.[2]

Nitrogen resembling hydrogen in its inability to combine directly with metals, it seemed reasonable to expect that it would have a very similar effect on the iron, zinc, and tin arcs. As magnesium[3] combines directly with nitrogen, a different effect might be expected in the magnesium arc.

METHOD AND APPARATUS.[4]

In order to eliminate the effect of gases other than the one whose effect was being studied, the enclosed rotating metallic arc[5] with "chemically pure" zinc, magnesium, and tin electrodes was used. The iron used, however, was "commercial." The speed of the rotating electrode was approximately 1100 revolutions per minute. Excepting in two instances noted later, the arc was operated by a 104-volt alternating current.

The spectrum was photographed with a Rowland ten-foot concave grating, four exposures being made on each plate as shown in the diagram.

Long Exposure, Arc in Air
Long Exposure, Arc in Gas
Short Exposure, Arc in Gas
Short Exposure, Arc in Air

[1] *Proc. Roy Soc.*, **32**, 203, 1881. [2] ASTROPHYSICAL JOURNAL, **12**, 167–175, 1900.

[3] LIVEING and DEWAR, *Proc. Roy. Soc.*, **32**, 161, 1881.

[4] The funds to meet the expense of this experiment were kindly appropriated by the committee in charge of the Rumford fund of the American Academy of Arts and Sciences.

[5] CREW and TATNALL, *Phil. Mag.*, **38**, 379–386, 1894.

Near the top of the plate was photographed the spectrum of the arc in air; just below this was photographed the same arc operated in an atmosphere of the gas being studied; below this, again, was made a short exposure with the arc in the same gas; and at the bottom, a short exposure with the arc in air. The exposures were so timed as to make the intensities of the two inner photographs intermediate between the intensities of the two outer ones. By comparison one can readily determine from such a plate whether any change in intensity is due to length of exposure or to change of atmosphere. This plan of making four exposures on each plate has some additional advantages: (1) it affords a test of uniformity of results; (2) the two short exposures allow an easier comparison of lines so strong as to be ordinarily overexposed; (3) the second exposure in the gas being made immediately after the first, and without opening the "hood" of the arc, a photograph is obtained of the spectrum of the arc in its final atmosphere, that is, in an atmosphere which includes the gaseous products of the arc, if there be any.

The effect of hydrogen was taken from the results published by Professor Crew and from the original plates obtained by him. These negatives were made under the same conditions as here described, except in a different atmosphere and with a direct current. The change from a direct to an alternating current, however, produces no effect on the spectrum. In the case of the tin arc, less dust is produced when a direct current is used, and I have therefore employed such a current in photographing the spectrum of tin in ammonia.

The hydrogen atmosphere was obtained by the electrolysis of acidulated water. The ammonia used was taken from a drum of compressed ammonia gas, such as is used in refrigeration. Nitrogen was generated by the reaction of ammonium sulphate and sodium nitrite solutions. Professor J. H. Long kindly suggested an arrangement of the nitrogen generator by which air was excluded from the entire system throughout the work.

Traces of oxygen were removed by pyrogallic acid, while

the water vapor was taken out by passage through concentrated sulphuric acid and over phosphorus pentoxide.

Before commencing an exposure a stream of nitrogen was kept flowing through the hood for at least twenty minutes. The same plan was used in filling the hood with ammonia. During an exposure a stream of the gas was kept flowing through the hood, both for the purpose of keeping a fresh supply of the gas about the arc and to drive out the dust which formed.

The spectra of the following four metals have been examined photographically in the region lying between λ 2300 and λ 5300.

MAGNESIUM.

The ordinary arc lines of magnesium seem to be almost wholly unaffected by substituting nitrogen for air. The heavy spark line at λ 4481, which appears also in the spectrum of the rotating arc, is reduced by nitrogen to about one-fifth its intensity in air. Naturally, the magnesium oxide fluting at λ 5007 is practically blotted out in nitrogen and greatly intensified in oxygen. On the other hand, no new lines make their appearance in nitrogen.

The intensity and reversal of the characteristic line at λ 2852, which are so strongly affected by hydrogen and ammonia, are unaffected by nitrogen, while the sharp line at λ 4571 is slightly reduced by hydrogen and ammonia, although not changed by nitrogen.

The magnesium-hydrogen fluting beginning at λ 5210 discovered by Liveing and Dewar[1] appears in ammonia, as does also the F line of hydrogen. Fairly intense hazy lines at approximately λ 4580, 4434, 4430, and 4390 also appear in ammonia. These lines are apparently intensified by oxygen also. By nitrogen they are unaffected. A very faint trace of these lines can be seen in air. I have not yet succeeded in identifying them.

TIN.

Of the four metals studied, tin is the one whose spectrum is most modified by changes of atmosphere. In air, nitrogen, and

[1] *Proc. Roy. Soc.*, **27**, 494–496; **30**, 93–99.

oxygen the tin arc works well. But in hydrogen and ammonia the arc is very short and is maintained with difficulty.

The intensity of the tin arc in nitrogen is estimated at one-third of its intensity in air; while in ammonia its intensity is not more than one-twentieth of its intensity in air, and in pure hydrogen it is even less.

Not only is the average intensity of the tin spectrum strongly affected, but also the character and relative intensity of its individual lines. In Kayser and Runge's table of wave-lengths of the tin arc more than forty lines are described as reversed. Many of these reversals show very clearly on my plates. When the arc is surrounded with nitrogen, however, some of these lines appear to be doubly reversed, the rest not reversed at all. On the other hand many of the lines that are reversed in air appear to have their reversal widened by ammonia. These reversals are not affected by an atmosphere of oxygen.

The two very strong spark lines at $\lambda 3351$ and $\lambda 3282$, appearing also in the spectrum of the rotating arc, have their relative intensities decreased by nitrogen. But by ammonia they are relatively enhanced at least twenty times. In the arc in air, these two lines are barely visible; but in ammonia they become two of the most prominent lines of the spectrum. These two lines are similarly affected by hydrogen. The spark line at $\lambda 2368$ is also intensified in ammonia, but less than the two preceding. The spark lines of wave-lengths 2449, 3471, 3539, 3574, appear plainly in ammonia and hydrogen but not at all in air or nitrogen. They are more enhanced by hydrogen than by ammonia. At $\lambda 3360$ and $\lambda 3370$ appear two unidentified lines of intensity 6 and 3 (on a scale increasing from 1 to 10), respectively, in ammonia, and not quite so intense in nitrogen, which can scarcely be detected in air and hydrogen. These same lines appear also in the magnesium and zinc arcs in air, nitrogen, and ammonia; but oxygen has the effect of immensely weakening both of them in the tin and magnesium arcs. $\lambda 3370$ appears in the tin spark, but $\lambda 3360$ does not.

ZINC.

The average intensity of the lines of the zinc arc spectrum is reduced approximately one-half by nitrogen. The width of reversed lines is not affected.

The only zinc lines that suffer a disproportionate reduction by an atmosphere of nitrogen are $\lambda 2558$ and $\lambda 2502$. These, with $\lambda 5182$, are the lines which are enhanced by hydrogen. They are strong spark lines. In ammonia their intensity is two or three times as great as it is in air, although the average intensity of the zinc spectrum is diminished perhaps five times by ammonia.

IRON.

The substitution of nitrogen for air about the iron arc produces very little change in the spectrum. The general intensity is not altered. Compared with the number of lines relatively affected by hydrogen the number of iron lines affected by nitrogen is small, as has been found to be the case with other metals. Many of the lines that are affected by nitrogen are impurity lines. Of fifty iron lines between $\lambda 3660$ and $\lambda 4060$ that are markedly enhanced by hydrogen, six are distinctly reduced by nitrogen.

On examination of the region between $\lambda 3600$ and $\lambda 4600$ I found twenty-six lines that are particularly affected by nitrogen. A few others are slightly changed. Of these twenty-six lines seventeen are from two to ten times as strong in nitrogen as in air. But these seventeen lines were all found to be due to impurities, fifteen belonging to manganese, one to chromium, and one to cobalt. The remaining nine lines are *reduced* by nitrogen to from one-half to one-tenth their intensity in air. Six of these nine lines belong to the spectrum of the iron spark.

EFFECT OF EXCLUDING NITROGEN.

The effect of pure nitrogen being so slight, it seemed possible that the presence of so large a percentage of nitrogen in the air about the ordinary arc might account for the smallness of the change. I therefore attempted to determine the effect of nitro-

gen by a process of exclusion. This was done by substituting for air an atmosphere of commercial oxygen taken from an ordinary stereopticon gas-cylinder. A stream of this oxygen was kept flowing through the hood of the arc.

If the chemical affinity of the electrodes for the atmosphere has any effect on the spectrum, one might certainly expect this effect to be exhibited when such easily oxidizable metals as iron, magnesium, and tin are employed in the atmosphere of oxygen.

Mr. A. S. King[1] found that the metallic lines were intensified by increasing the supply of oxygen about the *carbon* arc. But just the contrary seems to be true with reference to the *metallic* arc. A current as large as ten amperes was tried with chemically pure magnesium electrodes, but the oxygen had little effect on the working of the arc or the appearance of the so-called "flame." To the eye the "flame," especially with iron electrodes, appears to be less blue and more yellow.

Of eighteen exposures made with the iron, tin, and magnesium arcs in oxygen, all but one show a greater average intensity for the same length of exposure in air than in oxygen. This is not the only respect in which the action of oxygen on the metallic arc resembles that of hydrogen and ammonia. The metallic lines that have been noted as being relatively enhanced or reduced by hydrogen are precisely the ones which are so affected by oxygen. The changes produced by oxygen are not so great as those produced by hydrogen, but they are in the same direction.

SUMMARY.

The results of these experiments may therefore be summarized as follows:

1. The average intensity of the iron and magnesium arcs is not changed by substituting pure nitrogen for air as an atmosphere. The average intensity of the zinc and tin arcs is reduced two or three times by nitrogen. By hydrogen the average intensities are reduced from five to twenty times as much as by nitrogen. Ammonia apparently does not produce quite so great a reduction as does hydrogen.

[1] ASTROPHYSICAL JOURNAL, 14, 329, 1901.

2. The relative intensities of many lines depend upon the atmosphere. The lines that are relatively reduced by nitrogen are spark lines. As a rule these lines are relatively enhanced by hydrogen or ammonia.

3. The influence of ammonia on intensities and reversals is intermediate between that of nitrogen and hydrogen, and in general it seems true that *the effect of ammonia is approximately equal to the sum of the effects of its constituents.* This, in fact, is the particular point which I had in mind to determine when I began these experiments.

4. *The influence of oxygen is similar to that of hydrogen.*

5. Nitrogen affects the reversed lines of tin by either destroying the reversal or producing faint double reversals.

These results seem sufficient to show that *the readiness of an atmosphere to form chemical union with the electrodes under ordinary conditions is a very small, probably insignificant, factor in determining the intensity of the arc. The intensity appears to be due to electrical causes rather than to chemical reactions.*

Some experiments have been performed by Professor Basquin[1] which seem to confirm the theory that the intensification of spark lines in hydrogen is caused by the increased resistance due to the hydrogen atmosphere about the arc. This greater resistance has been attributed to the absence of chemical reaction in the hydrogen arc. Such a change of resistance in the products of the arc may explain the phenomena which occur in hydrogen. But if the resistance of the arc depends on the reactions in it this fact makes it difficult to see how the spark lines can be intensified by an atmosphere of oxygen.

This work was done under the direction of Professor Crew, who, in fact, himself began it, and to whom I am indebted for continued advice and assistance.

NORTHWESTERN UNIVERSITY,
Evanston, Ill.,
April 22, 1902.

[1]ASTROPHYSICAL JOURNAL, **14**, 14–17, 1901.

ON A NUMERICAL RELATION BETWEEN LIGHT AND GRAVITATION.

By Victor Wellmann.

Since Helmholtz, in his epoch-making treatise on "Die Erhaltung der Kraft," demonstrated the unity and the mutual capacity for transformation of all nature-forces, and pointed out the task of natural science in the words, "The phenomena of nature must be referred back to motions of matter with unchangeable energy of motion, depending on spacial relations alone," attempts have been made in the spheres of physics and chemistry to explain the phenomena through the collision of masses in motion. In astronomy, also, various attempts have been made to explain the universal attractive power of masses, acting according to Newton's law, through the gas-pressure of the supposedly gaseous interstellar medium, in order to escape from the hypothesis of an "*actio in distans*," in itself inexplicable and impossible.

As a matter of fact, universal gravitation allows of a simple, unconstrained explanation by means of the pressure of the interstellar medium under the application of the laws of the kinetic theory of gases. Yet, at the same time, we see that the law formulated by Newton, $K=\frac{k}{r^2}$, is only strictly accurate under the limiting condition that the two reciprocally attracting bodies do not change their reciprocal position; in this case the law of Newton must be modified to suit the condition that the force depends not only on the reciprocal distance, but also on the motions of the bodies in relation to one another. This of course also alters the laws of motion which are valid for the heavenly bodies, so that an examination of them is of great interest. Thus Professor Laves[1] has subjected the motional

[1] "The Ten Integrals of the Problem of *n*-Bodies," *Astronomical Journal*, **19**, 97, 1898.

equations of the n-bodies problem to a very interesting examination, in which he applies especially the electro-dynamical laws of Weber, Riemann, and Clausius.

In the following lines I intend—on the basis of an analogous law of attraction, according to which the causes of gravitation are likewise referred back to the pressure of the interstellar medium—to point out a numerical relation between light and gravitation, which ought to show the way to an explanation of the connection of these two forces.

Let δ_0 be the symbol of the "relative density" of the interstellar medium, that is to say, of the mass of ether, which in the unit of time passes the unit of surface, and let V be the velocity of the single particles of this mass; then Newton's law of gravitation will be thus expressed:[1]

$$K = \frac{\delta_0 V^2}{r^2}, \tag{1}$$

from which is derived Gauss's constant, $k = \delta_0 V^2$.

Suppose that the attracted body is not at rest, but moves in relation to the attracting body with the radial component of velocity $\frac{dr}{dt}$, then the number of the interstellar particles which meet the attracted body in the unit of time will be:

$$\delta = \delta_0 \left(\frac{V - \frac{dr}{dt}}{V} \right).$$

At the same time in (1) we must put $\left(V - \frac{dr}{dt}\right)$ for V. Thus we have for the attractive force of masses in motion the equation:

$$K = \frac{\delta_0 V^2}{r^2} \left(1 - \frac{1}{V}\frac{dr}{dt}\right)^3;$$

or, as we can, on account of the smallness of the factor $\frac{1}{V}$, sub-

[1] V. WELLMANN, "Ueber die Ursachen der Gravitation," *A. N.*, **144**, 121, 1897; "Ueber das Newton'sche Gravitations-Gesetz," *A. N.*, **148**, 169, 1899; "Ueber den Einfluss des widerstehenden Mittels auf die Planetenbahnen," *A. N.*, **148**, 297, 1899.

stitute for $\frac{dr}{dt}$ the value which results from the Newtonian law of attraction, $\frac{dr}{dt}=\frac{k}{r}$, we have the result:

$$K=\frac{k}{r^2}\left(1-\frac{k}{rV}\right)^3. \qquad (2)$$

That this divergence from the law of Newton has not been demonstrable by observation is not surprising, considering the smallness of the factor $\frac{k}{V}$.

Suppose we take the imaginary case $\frac{dr}{dt}=1$ (of course with all magnitudes reckoned in Gauss's units), *i. e.*, suppose we consider an imaginary planet at the distance $r=k$, and take for the velocity of light in vacuo $V=173{,}492$,[1] we have the "factor of gravitation:"

$$F=\left(1-\frac{k}{rV}\right)^3_{r=k}=0.98280755\ .$$

Hence the complementary magnitude:

$$K'=1-\left(1-\frac{k}{rV}\right)^3_{r=k}=0.01719245\ ,$$

while the Gaussian constant is:

$$k=0.01720210\ .$$

To the difference $k-K'=\Delta k=0.00000965$, if it were actually to exist, would correspond an inaccuracy of 0."00497 in the solar parallax, or an inaccuracy of 169 kilometers in the velocity of light, while the corresponding errors given by Harkness amount to 0."00567 and 80 kilometers.

We see that our "factor of gravitation," derived from the velocity of light, agrees with the constant of Gauss within the required limits of accuracy, whence the theorem:

"At the distance k the 'factor of gravitation' is equal to the complement of Gauss's constant k."

It might be objected to this calculation, that the dimensions of the equations for K' and k are different, and hence that these two equations cannot be compared with each other.

[1] WILLIAM HARKNESS, "The Solar Parallax and its Related Constants," *Washington Observations* for 1885.

This objection at first seems certainly to be to some extent justified; but it is, however, purely of a formal nature. Equations of different dimensions can very well stand in causal connection with one another, in spite of this difference. Thus, for example, the linear velocity of the Earth, $w=\frac{2\pi r}{T}=0.01720213$, stands unquestionably in causal connection with the Gaussian constant k and agrees with it within the decimals above stated, although its dimensions are entirely different from those of the Gaussian constant, $k=\frac{2\pi a^{\frac{3}{2}}}{T\sqrt{M+m}}$. The demand that our constant K' shall agree with the dimensions of k' is in general absolutely unwarranted. The dimensions of k are derived from Kepler's law, which is itself based on the strict validity of Newton's law, whereas our considerations rest on the basis of another law.

If we grant that gravitation depends not only on the mass and the reciprocal distance of the bodies, but also on their relative velocity, we must of course also assign to it other dimensions, as in this case Kepler's law is no longer strictly valid. Besides this, it is more correct to consider the constant of Gauss as a so-called "factor of proportionality,"[1] *i. e.*, as without dimensions, just as our factor of gravitation is without dimensions.

Another more lucid formula, which at the same time shows that our relation can be expressed as universally valid, independently of the limiting condition $r=k$, can be given to our relation by introducing, instead of the radial component of velocity, the linear velocity $\frac{ds}{dt}$.

Hence from $\frac{ds}{dt}=\frac{k}{\sqrt{r}}$ follows:

$$\left(1-\frac{ds}{dt}\frac{\sqrt{r}}{kV}\right)^3=1-k\,, \tag{3}$$

or

$$V=\frac{1}{1-\sqrt[3]{1-k}}\,. \tag{4}$$

[1] WEINSTEIN, *Handbuch der physikalischen Maassbestimmungen*, Bd. II.

If we calculate by this formula the velocity of light from the Gaussian constant of attraction, the result is $V = 173{,}394$ in Gauss's units, or a value in so close agreement with that given by Harkness that a merely accidental coincidence of the two values is out of the question, and a causal connection between light and gravitation seems to be expressed in these figures.

SPECTROGRAPHIC MEASURES OF THE VELOCITIES OF GASEOUS NEBULÆ.[1]

By J. Hartmann.

Although the bright line spectra of the nebulæ have repeatedly been photographed, so far as I know no attempt has yet been made to secure radial velocity determinations from the plates obtained. The cause for this may be that the spectrograms did not have a comparison spectrum, or that the small scale of the photographs precluded accurate measurement. The epoch-making work of Keeler[2] did, indeed, furnish relatively accurate velocities for fourteen of the brighter nebulæ; but anyone acquainted with the extreme difficulty of making such visual measures, must admit that in this case, as in others, a very considerable increase of accuracy is to be obtained by the application of modern spectrographic methods. Such a result I regard as of extreme importance. For, if it becomes possible to secure velocity determinations for the nebulæ with an error not exceeding a few tenths of a kilometer, we may certainly expect to find relative motion within every one of these objects; and the detailed study of such motions will prove of fundamental importance not only for our knowledge of these systems, but for our ideas of cosmogony in general.

A somewhat casual photograph of the planetary nebula *G.C.* 4390, which I made with the 80 cm photographic refractor, gave a very strong image of this object after an exposure of but fifteen minutes, a result which led me to the conclusion that it must be possible to obtain photographs of the spectra of the brightest of the nebulæ with the spectrographs at hand.

In my investigations I have made use of Spectrographs I and

[1] Translated from the *Sitzungsberichte der k. Acad. der Wiss. zu Berlin.* Session of February 27, 1902.

[2] "Spectroscopic Observations of Nebulæ." *Publications of the Lick Observatory*, 3, 1894.

III, which were constructed for the 80 cm refractor. Spectrograph I has a single flint glass prism of 60° angle, and a collimator of 530 mm and camera of 720 mm focal length. This instrument, on account of its low dispersion and long camera, cannot be regarded as exactly suitable for the spectra of nebulæ. It has, however, the advantage of giving a sharp image of the entire extent of spectrum from λ3600 to λ5900, and so has enabled me to photograph $H\gamma$ at the same time with the green nebular line. Spectrograph III, in the form in which I have used it here, is better adapted for the spectra of nebulæ. It has a collimator of 480 mm focal length, and three flint glass prisms of 63° each. In place of the camera of 560 mm focal length, in ordinary use for stellar spectra, a shorter one of 410 mm may be attached, giving a corresponding increase of intensity at the focus. This camera, whose objective gives but a short extent of the spectrum in sharp focus, I adjusted in such a way that the excellent group of iron lines extending from λ4860 to λ5006 fell in good focus at the center of the plate, and so furnished a very convenient comparison spectrum for the three brightest nebular lines. To avoid disturbing the excellent adjustment of the instrument, the prisms were left at their setting for minimum deviation on $H\gamma$.

With these two instruments, and the assistance of Dr. Ludendorff, I secured the following plates:

TABLE I.

Instrument and No. of Plate	1901	Central European Time	Nebula	Length of exposure
I 120.......	September 23	8^h 35^m	*G.C.* 4390	100^m
I 123.......	" 24	8 25	*G.C.* 4390	90
I 127.......	" 25	9 4	*G.C.* 4373	120
I 144.......	October 31	8 45	*N.G.C.* 7027	120
III 389.......	September 30	10 30	*G.C.* 4373	270
III 390.......	October 1	8 20	*G.C.* 4390	180
III 392.......	" 3	8 25	*G.C.* 4390	210

The following should be noted of the individual plates:

I 120. The slit was set at position angle 90°, and the edge

of the nebula kept on the middle of the slit. The lines consequently do not occupy the full length of the slit, but end at its center. They are very weak, and only the chief nebular line (λ 5007), which I will denote by N_1, admits of at all accurate measurement. Traces of the second nebular line N_2 (λ 4959), and of $H\beta$ are present.

I 123. A very good plate taken at the center of the nebula. N_1 and N_2 strong; $H\beta$ also well measurable. $H\gamma$ very weak and difficult to measure.

I 127. A rather weak plate. N_1 alone well measurable; N_2, $H\beta$, and $H\gamma$ very difficult and uncertain.

I 144. Only N_1 and N_2 measurable. Faint traces of $H\beta$ and $H\gamma$.

III 389. A very weak negative. N_1 alone measurable, and that with difficulty.

III 390. N_1 strong, N_2 also well measurable. A very weak trace of $H\beta$, which appears so faint under the microscope as to be measurable only with great difficulty.

III 392. N_1 extremely strong, N_2 strong, $H\beta$ rather weak, but still well measurable.

The arc spectrum of iron was used as the comparison spectrum, and a glass plate was interposed between the arc and slit to diffuse the light. I adopted the following wave-lengths from Rowland's table of the solar spectrum:

4294.30	4376.11	4903.50
4315.26	4736.96	4920.68
4337.22	4859.93	4957.67[1]
4352.91	4878.41	5006.12

I have measured each plate four times, namely, in both directions (violet right, and violet left) from two positions of the micrometer screw differing from each other by 0.5 revolution. The progressive screw errors have been accurately determined, and are allowed for in the reduction. A double thread was used for the settings upon the lines, and the measures could be made with extreme accuracy whenever the lines were sufficiently dense. As the emission lines of the nebulæ have exactly

[1] A double line (see remarks below).

the same appearance as the lines of the comparison spectrum, the measures are much less liable to systematic errors of setting than in the case of stellar spectra with absorption lines.

In order to secure as accurate a value as possible for the best plate III 392, and to give it suitable weight in the series, I have made two sets of measures upon it, or eight measures in all. The two sets of measures are indicated in the table below by III 392a, and III 392b. The following wave-lengths were found for the lines measured upon the different plates:

TABLE II.

Plate	N_1	N_2	$H\beta$	$H\gamma$
I 120	5007.36	....	....	....
I 123	5007.25	4959.34	4861.71	4340.86
I 127	5006.10	4958.26	4860.58	4339.65
I 144	5007.44	4959.58	....	....
III 389	5005.89	....	....	....
III 390	5007.30	4959.46	4861.79	....
III 392 a	5007.31	4959.42	4861.79	....
III 392 b.....	5007.27	4959.43	4861.76	....

I have used these wave-lengths in the following way: From the measures of the hydrogen lines $H\beta$ and $H\gamma$, made on plates I 123, III 390, III 392a, and III 392b, the velocity of the nebula *G. C.* 4390 was first determined (Table III). With this velocity the wave-lengths of the lines N_1 and N_2 were next derived (Table IV); and finally with the use of these wave-lengths the velocities in Table V were obtained from all the values of Table II.

The hydrogen lines give the following determinations of the radial velocity of *G. C.* 4390.

TABLE III.

Plate	Line	Wave-length in Nebula	$d\lambda$	V_1 (In ref. to Earth)	Reduction to Sun	V (In ref. to Sun)
I 123...	$H\beta$	4861.71	+0.18	+11.1 km	−25.8 km	−14.7 km (½)
	$H\gamma$	4340.86	+0.23	+15.9	−25.8	− 9.9 (½)
III 390 ..	$H\beta$	4861.79	+0.26	+16.0	−25.7	− 9.7
III 392 a .	$H\beta$	4861.79	+0.26	+16.0	−25.6	− 9.6
III 392 b .	$H\beta$	4861.76	+0.23	+14.2	−25.6	−11.4

The weight of ½ was assigned to the values given by plate

I 123, on account of the lower degree of accuracy permitted by the weak dispersion of the instrument. As we see, the values of the velocity agree very well. The final result $V=10.75$ km has a probable error of ± 0.56 km, while the final result for the *Orion* nebula, which Keeler found from 13 nights' visual measures of the $H\beta$ line, has a probable error of ± 1.29 km. The conclusion may well be drawn that the accuracy of Keeler's values has been surpassed, in spite of the rather unsuitable character of the apparatus which I have made use of in these preliminary investigations. To fully appreciate this photographic result we should bear in mind that the dispersion used by Keeler in the third and fourth orders of his grating would correspond to that given by 14 and 24 prisms, respectively. It is evident, therefore, that the photographic method, when used with an instrument especially constructed for this purpose, would be capable of giving considerably more accurate results. For the motion of the nebula in reference to the observer we find:

	I 123	III 390	III 392
Motion of nebula referred to Sun.........	−10.75 km	−10.75 km	−10.75 km
Orbital motion of Earth	+25.66	+25.53	+25.43
Rotation of Earth......................	+ 0.16	+ 0.18	+ 0.20
V_1	+15.07	+14.96	+14.88

These three velocities give a correction to the values of the wave-lengths of the two nebular lines N_1 and N_2 of −0.25 tenth-meters. Applying this we find from the apparent wave-lengths given in Table II the following true wave-lengths, freed from the influence of motion.

TABLE IV.

Plate	N_1	N_2
I 123	5007.00 (½)	4959.09 (½)
III 390	5007.05	4959.21
III 392 a	5007.06	4959.17
III 392 b	5007.02	4959.18
Mean........	5007.04	4959.17

The agreement of the independent values is, in this case as before, so excellent that their mean deserves confidence in spite of the small amount of observational material.

The value found by me for the wave-length of the chief nebular line N_1 agrees almost exactly with that of $\lambda = 5007.05 \pm 0.03$ derived by Keeler from his observations of the *Orion* nebula. But I find a considerably larger value for the wave-lengths of the second line, for which Keeler gives $\lambda = 4959.02 \pm 0.04$. The latter determination is based upon five comparisons of the nebular line with the double line $\lambda 4957.480$ and $\lambda 4957.785$ of the iron spectrum. Keeler, in his observations, did not see this line double, and so used as the wave-length of his comparison line the arithmetical mean 4957.63. In Rowland's solar spectrum table the two lines have the intensities 5 and 8. If we form the weighted mean with these intensities we find as the wave-length of the blend formed by the two lines 4957.67, which is the value I adopted in my reductions. Using this result we find that Keeler's wave-length for the second nebular line becomes 4939.06, a value which still differs by 0.11 tenth-meters from my determination.

With a view to explaining this difference, which is rather large considering the accuracy of both results, I undertook the following investigations with the large Bamberg spectrometer of the Astrophysical Observatory, and a Rowland plane grating.

1. The relative intensities of the two components of the double line in the arc spectrum of iron were determined and found to be in the ratio of 1 : 2 in the spectrum of the fourth order. Forming the weighted mean with these numbers we find the wave-length 4957.683 for the optical center of gravity of the pair.

2. As Keeler must have had to use a rather wide slit in making his measures, I tried by opening the slit to secure complete merging of the two components. The lines, however, never united into a single uniformly illuminated line, but the weaker component was always visible as a narrow border on the more refrangible side of the principal line. In spite of this I endeav-

ored to measure the wave-length of the pair through comparison with neighboring lines, and found $\lambda = 4957.685$. With the much lower dispersion of the spectrographs which I used, however, the pair of lines merged upon the photographic plate into a single sharp line. These two experiments completely confirm the value adopted by me for this line.

3. In view of the fact that Keeler did not use the arc spectrum but that of the spark, we might be led to suspect that the relative intensities of the two lines were quite different in the latter. This has proved to be the case. In a spark produced by a large induction coil and two Leyden jars the line 4957.480 was so weak as to be hardly visible by the side of the principal line. I estimated the ratio of intensities at 1 : 5. Hence I regard it as very probable that in the less brilliant spark which Keeler used for his comparison spectrum only the line λ 4957.785 was seen, and that the nebular line was referred to this. If we take the wave-length 4957.78 instead of 4957.63 for Keeler's comparison line, we find from his measures the value 4959.17 for the second nebular line. This agrees exactly with my determination.

The individual values of V determined with Spectrograph III show a very good agreement. It is, however, to be noted that in the case of *G. C.* 4390 the mean of the velocities derived from N_1 and N_2 must agree closely with that derived from $H\beta$ and $H\gamma$, since the wave-lengths of N_1 and N_2 have been obtained from the velocity determined originally from $H\beta$ and $H\gamma$. The results obtained with Spectograph I vary rather more widely. On a negative taken with this instrument a distance of 0.01 mm at the position of the nebular lines corresponds to a difference of wave-length of 0.46 tenth-meters, or a velocity of 28 km. As it has been found impossible on the most sensitive plates to measure a single line to a greater degree of accuracy than 0.002 mm, the individual velocity determinations made with this instrument will be uncertain by at least $\pm$ 5 km.

Using the wave-lengths I have derived for N_1 and N_2 we find from Table II the following determinations of velocity :

TABLE V.

Nebula	Plate	Line	$d\lambda$	V	Red. to Sun	V
G.C. 4390 West Edge (h 2000)	I 120	N_1	+ 0.32 t-m.	+19.2 km	−25.9 km	−6.7 km
Center............	I 123	N_1	+0.21	+12.6	−25.8	−13.2
		N_2	+0.17	+10.3	−25.8	−15.5
		$H\beta$	+0.18	+11.1	−25.8	−14.7
		$H\gamma$	+0.23	+15.9	−25.8	− 9.9
	III 390	N_1	+0.26	+15.6	−25.7	−10.1
		N_2	+0.29	+17.5	−25.7	− 8.2
		$H\beta$	+0.26	+16.0	−25.7	− 9.7
	III 392a	N_1	+0.27	+16.2	−25.6	− 9.4
		N_2	+0.25	+15.1	−25.6	−10.5
		$H\beta$	+0.26	+16.0	−25.6	− 9.6
	III 392b	N_1	+0.23	+13.8	−25.6	−11.8
		N_2	+0.26	+15.7	−25.6	− 9.9
		$H\beta$	+0.23	+14.2	−25.6	−11.4
G. C. 4373 Center.. (IV 37)	I 127	N_1	− 0.94	−56.3	− 0.1	−56.4
		N_2	− 0.91	−55.1	− 0.1	−55.2
		$H\beta$	− 0.95	−58.6	− 0.1	−58.7
		$H\gamma$	− 0.98	−67.7	− 0.1	−67.8
North Edge.......	III 389	N_1	− 1.15	−68.9	− 0.1	−69.0
N. G. C. 7027 Center (Webb)	I 144	N_1	+0.40	+24.2	−19.6	+ 4.6
		N_2	+0.41	+24.8	−19 6	+ 5.2

The results of Table V give the following mean values for the separate plates:

TABLE VI.

Nebula	Plate	V Hartmann	V Keeler
G. C. 4390 Edge (h 2000)...............	I 120	− 6.7 km	
Center..............................	I 123	−13.3 (½)	
	III 390	− 9.3	
	III 392a	− 9.8	
	III 392b	−11.0	
	Mean	−10.5	−9.7 km
G. C. 4373 Center (IV 37).............	I 127	−59.5 (½)	
Edge..............................	III 389	−69.0	
	Mean	−65.8	−64.7
N. G. C. 7027 Center (Webb)............	I 144	+ 4.9	+10.1

In view of what I have said above in regard to the limits of accuracy for the negatives made with Spectograph I, I should not wish to affirm that the slight difference of velocity between the edge and center of *G. C.* 4390 and *G. C.* 4373 is due to relative motion within the nebula. A more important fact, as it seems to me, is that on almost all of the photographs of the nebula *G. C.* 4390 the lines have a slight curvature as well as a slight inclination to the direction of the comparison lines. This makes the existence of relative motion within the nebula very probable. I have been unable to settle the question on account of the disappearance of the nebula in the western twilight, but I hope to be able to continue successfully the work outlined with the aid of a spectograph especially designed for the purpose. At the request of Director Vogel, who considers the smaller photographic refractor of the Observatory better adapted for the investigation of the spectra of extended nebulæ on account of its greater ratio of aperture to focal length, photography of the spectrum of the *Orion* nebula was begun by Dr. Eberhard in November of last year, and these plates have given more certain evidence as bearing on the existence of relative motion within the nebula.

ASTROPHYSIKALISCHES OBSERVATORIUM,
Potsdam.

Minor Contributions and Notes

BANDS IN THE BUNSEN FLAME SPECTRUM OF SODIUM.

Recently, while examining a Bunsen flame saturated with sodium chloride, using a pocket spectroscope which gives a very bright spectrum, the writer observed faint indications of several bands in the red. On introducing fresh salt into the flame, these bands flashed out quite distinctly. Three could be clearly seen, with some indications of a fourth further in the red. At times a fluted structure was suggested, with sharp edges toward the violet, but the details were too weak to admit of certainty on this point.

Several ordinary laboratory spectroscopes failed to show any trace of these bands, and it was believed that they must have been optical illusions, due to internal reflections; but finally, when using a spectrometer with larger lenses, of shorter focus, two of these bands could be faintly distinguished when a very intense sodium flame was used. Others beside the writer saw them. Settings were very uncertain, but the wave-lengths of these two bands were roughly determined to be about 6000 and 6100. The third band, seen only with the pocket spectroscope, must have been of wave-length about 6200.

It is, of course, possible that these bands were due to impurities; but they were given by six different kinds of sodium salts. Moreover, their positions are very nearly the same as those of three bands observed by Hartley[1] in the oxyhydrogen flame spectrum of sodium, of wave-lengths 6026, 6138, and 6233, so it seems probable that they were really due to sodium. So far as known to the writer, however, they have never before been seen in the Bunsen flame spectrum of sodium.

Eder and Valenta[2] have published a reproduction of their photograph of the Bunsen flame spectrum of sodium, taken with twenty-four hours' exposure. A very decided maximum of about wave-length

[1] *Phil. Trans.*, **185**, 177, 1899.

[2] *Denkschriften d. Wiener Ak.*, **60**, opp. p. 476, 1893.

6100 is very clearly shown on this plate, but no comment was made upon it by them.

This band is the brightest of those seen by the writer. It is interesting to note that Scheiner, in his *Astronomical Spectroscopy* (Frost's translation, p. 216), in referring to the outburst of sodium vapor in Comet 1882 I as it approached the Sun, states that the hydrocarbon bands previously seen were extinguished, while the red hydrocarbon band at $\lambda 6130$, previously unseen, was observed by Vogel.[1] Such an effect is obviously improbable. If a sodium band can appear in this neighborhood at comparatively low temperatures, it seems more reasonable to assume that this was a sodium band. It is also possible that the sodium vapor became fluorescent under the action of solar radiation, giving rise to the red band observed by Wiedemann and Schmidt[2] in the spectrum of fluorescing sodium vapor.

PERCIVAL LEWIS.

UNIVERSITY OF CALIFORNIA,
April 1902.

GRANT BY THE SMITHSONIAN INSTITUTION TO THE ASTROPHYSICAL JOURNAL.

THE readers of the ASTROPHYSICAL JOURNAL are doubtless aware that the expenses of a journal of this kind must greatly exceed the receipts, in spite of the fact that the number of subscribers is comparatively large and is rapidly increasing. Through the aid of a gift from Miss Catherine Wolfe Bruce in 1895, supplemented by a gift from Mrs. William Thaw, it has been possible to illustrate the JOURNAL freely, thus materially increasing its value. These special funds are now exhausted, after providing all the illustrations used in the JOURNAL since its foundation. It is therefore with special satisfaction that the editors announce the award by the Secretary of the Smithsonian Institution of a grant of two hundred dollars annually for the four years 1899–1903. This substantial assistance on the part of Secretary Langley, who has contributed so much to the development and the furtherance of astrophysical science, will be appreciated by all who are interested in the welfare of the JOURNAL.

[1] See also *A. N.*, **102**, No. 2437.

[2] ASTROPHYSICAL JOURNAL, **3**, 207, 1896.

MARIE-ALFRED CORNU.

In the untimely death on April 12 of Marie-Alfred Cornu, France loses one of her ablest physicists, the École Polytechnique one of its most beloved teachers, and science one of its most eminent votaries.

M. Cornu has been an Associate Editor of this Journal since its foundation, and his loss will be keenly felt.

An appropriate sketch of his life will appear in an early number of the Journal.

THE

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MARIE-ALFRED CORNU.

By J. S. AMES.

NEWS of the death of Professor Alfred Cornu has brought to every student of Physics and Astronomy, and in particular to every spectroscopist, feelings of surprise and deep regret. His work seemed by no means over; and everyone who saw him preside over the meetings of the International Congress of Physics, at Paris, in 1900, felt that he was in the fulness of his powers.

Cornu was born at Châteauneux, March 6, 1841, and died at La Chansonnerie, near Romorantin, April 12, 1902. His professional career was most brilliant. He entered the École Polytechnique at the age of nineteen, and the École des Mines two years later. He left this as "engineer" in 1866; and the next year, at the early age of twenty-six, he was appointed professor of physics at the École Polytechnique, a position which he held until his death. He was elected a member of the Academy of Sciences in 1878, and later a member of the Bureau of Longitudes. He received honors from many scientific societies of many countries. It may be sufficient to mention a few of these: he was foreign member of the Royal Society of London, of the

Academies of Vienna and of St. Petersburg, and had received honorary degrees from both Oxford and Cambridge. At the last named university he delivered, in 1899, the Rede lecture in connection with the jubilee celebration of Sir George Gabriel Stokes, his subject being the "Wave Theory of Light." He had prepared to lecture in English, but being requested by some of his friends to give the address in French, he rewrote it entirely on a few hours' notice. At other times, too, Cornu came to England to deliver public lectures, and always charmed his audiences by his clearness of thought and expression, and by his simplicity of manner.

As Poincaré has well said of Cornu: "He has left his impress upon all portions of Physics; but his preference was specially for Optics. I think that which attracted him in the study of Light was the relative perfection of this branch of science, which, since Fresnel, seems to share at once in the rigorous exactness and the severe elegance of Geometry itself. It was there, better than elsewhere, moreover, where he could fully satisfy the natural longings of his mind for logical order and clearness." He was in a way the successor of Fresnel, Arago, Biot, and Jamin. His first physical papers were on reflection and refraction; and these were soon followed by his famous work on crystalline reflection. He invented a most ingenious and useful method for determining the optical constants of lenses; and the method of studying problems of diffraction by the use of "Cornu's Spirals" is familiar to everyone. His paper on the focal lines of gratings is complete and elegant; and his discussions of all kinds of optical instruments were exhaustive and always interesting.

But the two great researches in Light for which Cornu will always be renowned are those on the velocity of light and on Ultra-violet Spectra. He perfected the method of Fizeau for the measurement of the velocity of light—that of the toothed-wheel—and made a most exact determination of this great constant of nature. In a paper presented at the International Congress in 1900, he gave a critical discussion of the various methods used for the measurement of this quantity, and at his

death he was engaged in superintending a repetition of his research.

His interest in all questions pertaining to spectroscopy was always keen, as is shown by his having been on the Board of Editors of this JOURNAL from its foundation; and his examination of the spectra of hydrogen, of the Sun, and of many metals was epoch making. These researches were carried out mainly at his country home, where he in general spent his holidays.

For a complete list of Cornu's memoirs one must refer to some catalogues such as that of the Royal Society, but no summary of his scientific activity, however brief, would be complete without reference to his repetition of the Cavendish experiment on the density of the earth, in conjunction with Baille; his acoustic investigations; his influence as a member of several electrical commissions; and his work as president of the International Committee of Weights and Measures.

By one who knew Cornu personally, the impression most definitely remembered is that of a broad-minded, modest, clear-thinking scholar; one who must have been an ideal teacher. He was easily the leading physicist of France, and one of the great men of the world. His loss will be felt not alone by his immediate associates, but by all students of physics.

RADIAL VELOCITY OF THE *ORION* NEBULA.[1]

By H. C. Vogel.

Plates of the spectra of several planetary nebulæ with high dispersion have recently been obtained by Dr. Hartmann with the large twin refractor of the Astrophysical Observatory at Potsdam.[2] Interest having been aroused by these observations, attempts were made by Dr. Eberhard to obtain the spectrum of the *Orion* nebula with our photographic refractor of 32.5 cm aperture and 343 cm focus, which has been assigned to him for more than a year for spectrographic observations of radial velocities of stars. In consequence of the larger angular aperture of this instrument and the consequent greater intensity of the unit of area in the focal image, this instrument is superior to the great refractor for extended objects like the *Orion* nebula. Since the objective of this instrument is achromatized for the chemical rays, but has no correcting lens, as does the objective of the great 80 cm refractor by which that is converted into an objective uniting the optical rays properly, it was necessary at the start to give up the use of the two strongest lines of the nebular spectrum in the green (λ 5007 and λ 4959), as well as the hydrogen line $H\beta$; indeed, only the fourth line of the nebular spectrum, $H\gamma$, could be impressed upon the plate.

Dr. Eberhard's first attempt succeeded upon the 23d of last November. An exposure of 180 minutes very clearly brought out the $H\gamma$ line of the nebular spectrum, with Spectrograph IV with three prisms (the linear dispersion of which at $H\gamma$ is 0.25 mm = $0.424\mu\mu$). I may remark incidentally that the unavoidable temperature changes during such long exposures can have no injurious effect, inasmuch as the electrical heating arrangement of the spectrograph makes it easily possible to maintain a con-

[1] Translated from advance proofs, sent by the author, of a paper read before the *Kgl. Akademie der Wissenschaften zu Berlin.*

[2] See Astrophysical Journal, 15, 287, 1902.

stant temperature within a tenth of a degree inside the prism-box for several hours at a time.

The unfavorable weather, unfortunately, made it impossible to obtain a second plate until January 31, 1902, and hitherto observations could be made in all on only nine evenings, of which six may be regarded as successful.[1] In consequence of weather changes during the necessary exposure of about three hours, one of the remaining plates, taken on February 13, was a failure, while the two others, taken on January 31 and February 6, are not wholly useless.

I now give the results of the measurements independently made by myself and Dr. Eberhard with different measuring machines. They refer to a portion of the $H\gamma$ line, due to light coming from a portion of the nebula near the well-known trapezium. This is nearly the same place as that at which the direct spectroscopic observations of velocity of the *Orion* nebula were made by Professor J. E. Keeler[2] in 1890 and 1891 with the great refractor at the Lick Observatory.

Date	Central European time	Exposure time	Plate No.	Velocity			
				Relative to Earth		Relative to Sun	
				Vogel	Eberhard	Vogel	Eberhard
1901				km	km	km	km
Nov. 23	$12^h 26^m$	180^m	879	+ 8.9	+ 5.0	+ 18.0	+ 14.1
1902							
Jan. 31	6 55	50	924	+ 37.8::	—	(+ 17.8)	—
Feb. 6	9 40	40	928	+ 37.6::	—	(+ 15.9)	—
" 11	8 45	150	934	+ 39.9	+ 40.7	+ 17.0	+ 17.8
" 15	7 40	180	945	+ 41.8	+ 42.6	+ 18.1	+ 18.9
" 16	8 10	180	947	+ 42.3	+ 43.7	+ 18.4	+ 19.8
" 21	7 45	180	950	+ 41.0	+ 40.5	+ 16.2	+ 15.7
						+ 17.5	+ 17.3

The observations were arranged as in the case of stellar spectra, so that before and after the exposure of the nebula two

[1] Dr. Scholz assisted at five exposures, Dr. Ludendorff at three.

[2] *Publications of the Lick Observatory*, 3, 1894.

comparison spectra of iron, $\frac{1}{3}$ mm wide, and at a distance from each other of $\frac{1}{3}$ mm, were impressed upon the plate. By removing the diaphragm in front of the slit which limits the comparison spectra, the nebular line $H\gamma$ was obtained in its full length. It therefore appears not only in the space of $\frac{1}{3}$ mm between the comparison spectra, but crosses it and extends out on both sides.

Dr. Eberhard made this arrangement in order to obtain at the same time any irregularities of the $H\gamma$ line in respect to position or intensity. On good plates the line was from 2 mm to 2.5 mm long, corresponding to 2′. I must not omit to state that during the exposure the guiding on the nebula was so accurate that a superposition of different portions was excluded as far as possible, and that the spectra of stars in the nebula occurring on two plates appear linear.

The measurements of the observations given above refer, however, only to the small portion of the line between the comparison spectra; this corresponds to a place in the nebula slightly preceding the star θ^1 *Orionis*. On plates Nos. 924 and 928, for which the exposure had to be cut off too early, only a small portion of the $H\gamma$ line appears in the lower comparison spectrum (violet to right on the microscope stage). The observations are therefore unreliable, and especially so on account of the extreme faintness of this portion of the line, and were excluded from the mean.

On the plate of February 21 the iron lines were also given full length, since there was no danger of a disturbance of the nebular line, and since it made possible the determination of the position of other parts of the nebular line as compared with the lines of the comparison spectrum; or, in other words, it rendered possible the measurement of the velocity at different portions of the nebular line. The result of the measurement of this plate given above again refers, however, only to a portion of the $H\gamma$ line near θ^1.

All precautions were taken during the measurements. In order to avoid physiological errors of setting, which, indeed,

were less to be feared in view of the similarity of the comparison and nebular lines than in case of a star spectrum with the emission lines appearing dark and the absorption lines bright on the negative, the measurements were made with the plate in two different positions on the microscope stage (red end to right and red to left);[1] different magnifying powers were also employed during the measures. On account of the faintness of the $H\gamma$ line on some plates, measurement was difficult. The nebular line was measured with respect to several of the lines of the iron spectrum, and the amount of the displacement was obtained with the use of Hartmann's interpolation formula.

The observations given above beautifully confirm Keeler's measurements for the *Orion* nebula, and are of particular importance because it was the measurements on the *Orion* nebula which furnished the foundation for Keeler's classical researches on the motion of fourteen of the brighter nebulæ. Keeler's observations of $H\beta$ in the *Orion* nebula put him in a position to determine very accurately the wave-lengths of the brightest nebular lines (λ 5007 and λ 4959), which were until then only approximately known, and then to base all further measurements upon these, particularly upon the first.

I give for comparison Keeler's observations[2] of the motion of the *Orion* nebula, based upon the displacement of the $H\beta$ line.

On our plates Nos. 879, 934, 945, and 947, the nebular $H\gamma$ was not of uniform intensity. Where it crossed the comparison spectrum it seemed somewhat stronger and broader than in the space between the comparison spectra, due to the slight preliminary exposure of the film to the traces of continuous spectrum in the comparison spectrum. The line, however, was distinctly more intense in the lower than in the upper spectrum, and also extended further down beyond the comparison spectrum (violet to right). The greatest intensity of the hydrogen

[1] Differences in measurement in the different positions of the plate actually did appear in the investigations given below, which refer to very faint portions of the $H\gamma$ line.

[2] *Loc. cit.*, p. 198.

VELOCITY RELATIVE TO THE SUN (KEELER).

	Miles	Kilometers
1890 October 16	+10.9	+17.5
23	+12.0	+19.3
23	+ 9.7	+15.6
30	+ 4.0	+ 6.4
1891 January 23	+ 8.4	+13.5
23	+14.8	+23.8
26	+ 5.3	+ 8.5
28	+21.5	+34.6
February 12	+12.4	+20.0
12	+11.8	+19.0
March 6	+ 8.7	+14.0
18	+13.5	+21.7
20	+10.2	+16.4
	+11.0±0.8	+17.7±1.28

line, and hence probably also of the portion of nebula entering the slit, was 0.'6 from the star θ^1, preceding it on the parallel, or in position angle 270°.

On these plates, $H\gamma$ gives the impression of being oblique to the length of the comparison spectra, or as if its vertex (the lines are very appreciably curved) did not lie between the comparison spectra, but in the upper spectrum. The numerous measures of the displacement independently made by myself and Dr. Eberhard differ between the upper and lower comparison spectrum by an amount corresponding to a difference of velocity of about 6 km. It might be concluded from the observations that the nebulous matter at the brightest point preceding by 0.'6 the star θ^1 was moving toward the observer with a velocity of 5 or 6 km relatively to the nebulous matter in the immediate neighborhood of the nebulous star.

It has not hitherto been possible for us to explain the oblique position of the $H\gamma$ line in any other way than by the presumption, which, indeed, is in general a very probable one, of a different motion of the different parts of the nebula, which, in its projection on the celestial sphere, radiates outward in all directions. The possibility seemed to be excluded of finding the explanation in any erroneous adjustment of the excellent spec-

trograph, which had been most thoroughly investigated in every respect.

The lines of the iron spectrum were of full length at the observation on February 21, and moreover the star θ^1 was kept on the slit, so that the narrow linear star spectrum crossed the $H\gamma$ line; the setting on the nebula was also so made that the $H\gamma$ line scarcely penetrated the lower comparison spectrum, and extended about as far above the upper spectrum as it did above the lower spectrum on the previous plates. (Two narrow iron spectra were impressed on the plate at the middle of the exposure time.)

The measurements confirm anew the result of those on the four previous plates. My very numerous and accordant measures on plate No. 950, as well as the measures of Dr. Eberhard, yielded the following result:

	Velocity			
	Relative to Earth		Relative to Sun	
	Vogel	Eberhard	Vogel	Eberhard
Position angle 90° from θ^1 *Orionis*; $\Delta=0'.8$; beginning of $H\gamma$ line.......	+41 km	+42 km	+16 km	+17 km
At θ^1...............................	+41	+41	+16	+16
Position angle 270 : $\Delta=0'.6$; strongest part of $H\gamma$ line.....................	+37	+36	+12	+11
Position angle 270°, $\Delta=1'.2$ to $1'.4$; nearly at the end of $H\gamma$.............	+33	+37	+ 8	+12

It must not be left unmentioned that the $H\gamma$ line does not run along regularly, but shows a nick on some plates and a small bend at another place on other plates. Fig. 1 gives a representation of the $H\gamma$ line and the neighboring lines of the *Fe* spectrum from plate No. 945. The drawing is of course only schematic and lacks the gradations of brightness. $H\gamma$ should be very weak in comparison with the iron lines and the fine iron lines between λ 4337 and λ 4353 should be more delicate. The oblique position of the $H\gamma$ line, as well as the nick between the two comparison spectra is, however, clearly evident.

On February 26, at $9^h\ 0^m$ G. M. T., Dr. Eberhard made another plate (No. 955) in a wholly different region of the nebula—namely, in the parallel of the star Bond No. 685—which also confirms the observations just described. The exposure time was 210^m, but the atmosphere was not very transparent so that the $H\gamma$ line is in general very weak. It shows a peculiar bending at *c*, Fig. 2, and is very narrow in the neighborhood of the star Bond No. 685. At the bend it is broader and weaker, beyond the bend more intense and then thins out to a delicate line. The figure gives a rough representation, the principal object of which is a more precise designation of the places measured.

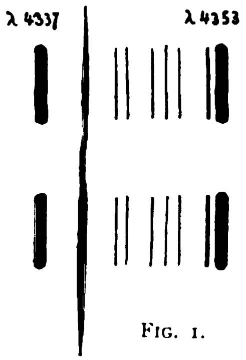


FIG. 1.

Our observations on this plate are as follows:

	Velocity			
	Relative to Earth		Relative to Sun	
	Vogel	Eberhard	Vogel	Eberhard
At *b*	+31	+31	+6	+6
c	+53:	+66:	+28:	+41:
a	+36	+53	+11	+28
End of $H\gamma$	—	+42	—	+17

b is at pos. angle 270° from Bond 685, $\Delta = 0'.4$.
c is the bend, $\Delta = 0'.9$.
a is the most intense part of $H\gamma$; $\Delta = 1'.3$.
End of $H\gamma$, $\Delta = 2'.3$.

The length of the $H\gamma$ line from the star amounts to about $2'.3$ in position angle 270° and $0'.2$ to $0'.3$ in position angle 90°.

It should be further remarked that on plates 950 and 955, on which the $H\gamma$ line crosses the spectra respectively of the stars θ^1 and Bond 685, it shows no broadening, strengthening, or bending at the point of intersection with the star spectrum.

The measurements are difficult and of less accuracy on account of the faintness of the $H\gamma$ line, but a difference in the displacement of the line at *b* relatively to *a* seems to me to be well established. It is unfortunate that the continuously bad weather did not permit a plate of this portion of the *Orion* nebula to be repeated, so that it must now be postponed until the beginning of next year.

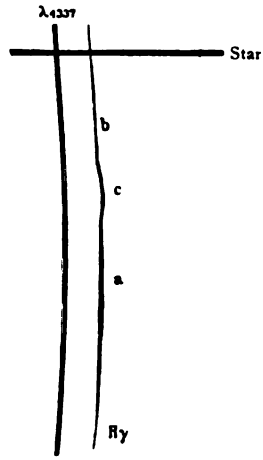


FIG. 2.

In his memoir on the motions of the nebulæ in the line of sight above cited, Keeler also attempted to discover relative motions in different parts of the *Orion* nebula (*loc. cit.* p. 199). He reaches the conclusion that displacements by relative motion in the nebula corresponding to 21 km per second should have been clearly recognizable and that in the brighter parts of the nebula displacements of one-third of this amount would probably have been discovered. He elsewhere attempted (p. 203) to discover a rotation of the nebula *G. C.* 2102 (IV 27). But he considers it doubtful that a velocity of less than 11 to 13 km could have been found with his appliances, quite aside from the improbability that a nebula should have so large a rotary motion.

The explanation of the above described deformations and anomalies observed here in the $H\gamma$ line in the spectrum of the *Orion* nebula as a consequence of the relative motion of the nebulous matter, which to my mind is very probable, is not excluded by the remarks of Keeler, inasmuch as the changes of velocity found are for the most part within the limits of detection considered by Keeler as probable.

NEW HEADS TO CYANOGEN BANDS.

By C. C. HUTCHINS.

THOSE who have had to do with photographing metallic spectra with the carbon arc have had all too frequent occasion to observe the carbon and cyanogen bands, which, by their very numerous lines, often cover and obscure the metallic lines sought. Rowland sets the number of carbon lines at 10,000.

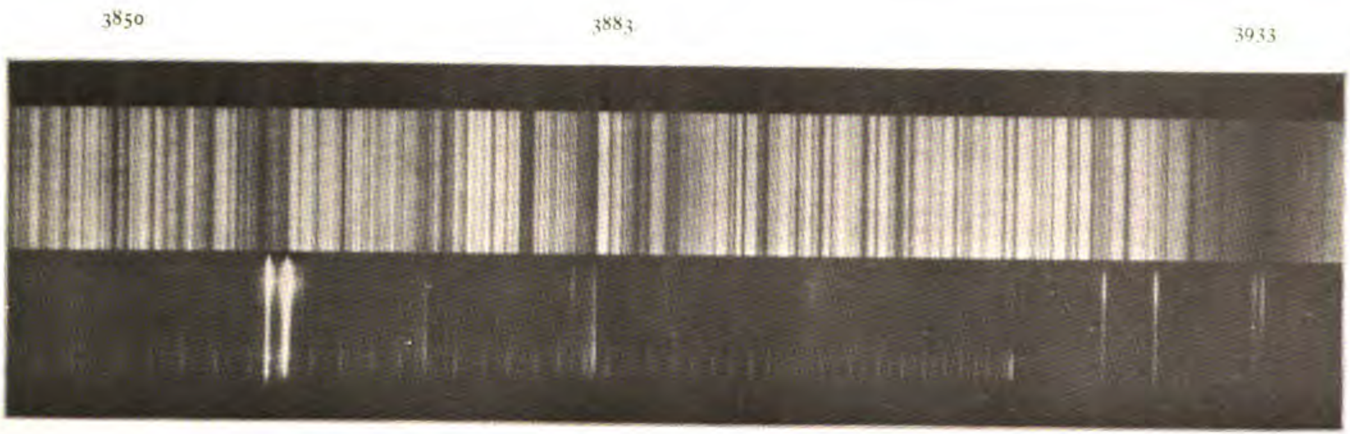
These bands have frequently been the object of special research. It will be sufficient here to mention the work of Kayser and Runge[1] and the late contribution of King.[2]

The cyanogen band whose first head, according to Kayser and Runge, is at λ 3883.55 is of extraordinary intensity, a single second's exposure sufficing, with the apparatus I am using, to fill the whole region, as far as λ 3720, with a multitude of lines, the source of light being the ordinary carbon arc.

This band is commonly given as containing five heads, of which three are plainly marked. Considering the frequency with which it has been examined, I was surprised to find a new head at λ 3914.47, which, from its character and arrangement of lines, seems unmistakably to belong to the cyanogen spectrum and to be the real first head of the λ 3883 band. The circumstances of its appearance were as follows: Two rods of commercial copper, 1.2 cm in diameter, were used in the arc lamp for obtaining the copper spectrum. The arc burned badly, jumping from side to side, and the rods were then pointed and the current reduced to about 10 amperes. When first started the arc hisses and the negative pole is tipped with a very bright glow. Upon photographing the spectrum of this hissing arc, in addition to the copper lines and those of other impurities contained in the copper, the cyanogen band at λ 3883 appeared, and with a new head with its system of lines. The lines were faint, and a

[1] *Wied. Ann.*, 38, 80, 1889. [2] ASTROPHYSICAL JOURNAL, 14, 323, 1901.

PLATE XIV.



New Heads to Cyanogen Bands (C. C. Hutchins).

prolonged exposure seemed to have but little intensifying effect; but by burning the arc for a few seconds, stopping it and allowing the rods to cool for a short time, re-starting, and so on for half an hour they were brought up to rather weak printing density.

Examination of such negatives as happen to be at hand made without carbon poles shows the new head in several cases. With nickel rods the arc spectrum shows no cyanogen bands, but the spark of an alternating current transformer between poles of the same metal shows the principal lines faintly.

Cobalt under like circumstances shows the new head in the spark and the $\lambda 3883$ head in the arc. Two or three other cases like the above might be mentioned. It is worthy of remark that whenever the new head appears in the spark spectrum, the remaining heads are absent.

Examination of the region of the second cyanogen band at $\lambda 4216$, shows also a new head located at $\lambda 4278.50$.

This head has a long attendant train of lines covering the whole space usually covered by the six heads of this band and their attendant lines, but in which the usual heads after the first are wanting, or only the first and second faintly marked; just as the spark spectrum of nickel contains the 3914 head but none thereafter.

There is also evidence of a new head to the 4606 band, but so far the photographs have been so unsatisfactory that I hesitate to give its position.

Examination of the 3590 band has so far yielded negative results; which, however, may well be due to the lack of transparency of the large prisms of the spectrograph for such short waves.

If one were disposed to speculate on the cause of modification of these bands in the copper arc the high heat conductivity of copper at once suggests that a cooling of the arc takes place, and that in the arc whose temperature is so lowered the cyanogen spectrum is modified. The greater wave-length of the new heads supports this view. On the other hand it is by no means certain that the metallic arc is cooler than the carbon arc. It is

well known that a thin filament of carbon glows with great brilliancy and wastes away when held in the metallic arc. If one of the copper poles be moistened with the smallest quantity of oil, the 3883 head appears at once with enormously increased intensity, the 3914 head remaining unaffected. Again, the unmistakable appearance of the 3914 head in several instances in the strongly condensed spark, speaks against a low temperature modification. In absence of other suggestions it may be looked upon as a change wrought in the spectrum of one body by the presence of another.

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SPECTROSCOPIC RESULTS OBTAINED DURING THE SOLAR ECLIPSE OF MAY 18, 1901.[1]

By W. J. HUMPHREYS.

THE spectrograms described in this paper were obtained at Fort de Kock, Sumatra, under the auspices of the eclipse expedition sent out by the United States Naval Observatory. This station is about three thousand feet above sea level, was near the northern edge of the path of totality, and during the eclipse had an ideally clear and perfect sky.

APPARATUS AND ADJUSTMENTS.

The apparatus with which I worked consisted of a large cœlostat and a concave grating of thirty feet focal length used without slit or lens. The cœlostat, made to do double service by having a large silvered mirror on either end of a heavy axis, was made by Brashear to meet a rush order of the Naval Observatory. As we used it one of these mirrors sent a beam of light horizontally through a lens of forty feet (12.2 m) focal length and gave the excellent structural photographs which Mr. Peters secured of the corona, while the other sent a beam of light in the opposite direction to the grating and produced the spectrograms presently to be described.

While the running of the clock was in every way satisfactory enough, there was unfortunately a periodic error, ultimately traced to the worm, which we could not correct, that every two or three minutes produced most aggravating, even if not very great, vertical motions in the solar image. It is not likely, however, that the spectrograms were materially affected by this disturbance, since, besides being always slight, it was in such direction as to cause the lines to shift parallel to the rulings on the grating and therefore in the main along their own length.

[1] Published in advance by permission of the superintendent of the Naval Observatory.

The grating, which belongs to Mr. Jewell, has a diffractive surface eight inches (20 cm) long and five inches (12.7 cm) wide, though owing to some accident to the diamond point during the process of ruling a portion of this surface is not good, so that in actual use about two-fifths of it was covered up. It was placed in a large light-tight box fitted with a suitable plate-holder, and an opening with a shutter so arranged that light from the cœlostat could be admitted or cut off at pleasure. The spectra obtained were rendered normal by assembling the apparatus, according to the method described by Poor and Mitchell in the ASTROPHYSICAL JOURNAL for March 1898, so that a line drawn from the center of the plate-holder to the middle of the grating would be normal to the latter.

The final focusing was left to Mr. Jewell as general director of the spectroscopic work, who did it with an ingenious device of his own design, consisting of a slit properly placed between a condensing mirror and a parabolic reflector that approximately face each other. Light from the cœlostat is intercepted by the condensing mirror and brought to a focus on the slit, and that part which gets through is reflected by the parabolic surface in parallel rays to the grating, there diffracted and each wave-length brought to its own line focus on the focal curve. By placing the camera so that the photographic film or plate will fit this curve the final adjustment is completed. It is evidently necessary that the light from the parabolic mirror to the grating be strictly parallel, and this condition Mr. Jewell secured by focusing the small telescope of a surveyor's transit on some distant hills and then moving the slit till its image in the parabolic mirror, as viewed by the telescope thus set for parallel light, became clear and distinct.

The cœlostat mirror, grating, and plate-holder were in a horizontal plane about four feet above the ground. The box containing the grating and plate-holder was laid flat on the top of eight heavy posts planted about three feet deep and braced in every direction. This horizontal arrangement did not produce parallelism between the spectrum crescents and the lines of the

grating at either flash, but it did secure about the best possible compromise between the two.

The focal curve was a little too steep to admit of being fitted by glass plates; consequently heavy celluloid films two and a half inches (6.3 cm) wide and thirty-six inches (91 cm) long were used. These the M. A. Seed Dry Plate Co. had kindly coated with their "gilt edge" emulsion, which is exceedingly sensitive to the blue, violet, and ultra-violet parts of the spectrum.

EXPOSURES.

During totality time was kept by Dr. Odell, surgeon in the U. S. Navy, and by Professor W. S. Eichelberger, who used a telescope and gave the signals that announced the instants of second and third contacts. The exposures were begun as promptly as possible after second contact, and the prearranged plan carried out except in regard to the last film, which we had counted on exposing two to three seconds, beginning just before third contact and extending through the second flash. The duration of totality, however, was several seconds longer than the computed time, and it was thought best to shut off the light at the end of eight seconds, which proved to be fully three seconds before third contact, lest the film be ruined, and this was probably well enough, as development showed the film to be fully exposed. Only about two seconds were consumed each time a fresh film was put in place, and in all six spectrograms were obtained in approximate accordance with Table I.

TABLE I.

SPECTROGRAMS OBTAINED.

Number of film	Duration of exposure	When exposed, counted from second contact
	Seconds	Seconds
I	2	0–2
II	5	5–9
III	15	12–26
IV	120	29–148
V	15	151–165
VI	8	168–175

It was decided as best to leave the development of all spectrum films to one person, and this part of the work was therefore given to Mr. Jewell. Lines were found on all these films, those on I and II being best defined, and VI having the greatest number. Each film showed that unfortunately it had not been quite in the focal plane, still the dispersion was so great that many of the lines evidently could very easily be identified.

MEASUREMENTS.

The films have been measured on a very excellent dividing engine, constructed by the *Société Genevoise*, fitted with a suitable microscope made by Bausch and Lomb. The engine reads to 0.001 mm, and the dispersion in the first spectrum, the one used, was such as to give approximately 3.66 Ångström units to the millimeter. As already stated, the focusing was not perfect, but the grating gave lines that split up into triplets as they went out of focus instead of producing a single broad blur, and consequently by always selecting the same fragment, the middle one, of each line, it was possible to secure fairly satisfactory and consistent measurements.

Each film had more or less pronounced light and dark streaks, due chiefly to irregularities in the Moon's outline, and these aided greatly in properly aligning it on the engine, and the microscope was so set that one of its cross-hairs was tangent to the crescents along that part of the film on which they were best defined.

EXPLANATION OF TABLE II.

The first column of Table II gives the relative wave-lengths as found by measurement and interpolation. The next three columns are taken from Rowland's Table of Solar Spectrum Wave-lengths, and give the probable corresponding solar lines. The remaining columns give the intensities and lengths of the lines found on the respective films. These intensities are only relative, and, though the lines were gone over carefully for the purpose of comparing them, not much accuracy is claimed for the values assigned. A line marked 0 is one that just can be

seen, a line marked 1 is somewhat more distinct, and so on with increasing numbers to the broadest and heaviest lines. Every one knows that, even with a constant source of light, the relative energies of the several spectrum lines are not accurately proportional to their several photographic effects, and this want of agreement is still more pronounced when the luminous source undergoes such rapid and marked changes as those which take place during a solar eclipse. Still, imperfect as they are, these estimates may be of some service and are therefore given.

A line marked *s s* is a very short one, covering an arc of not more than 10°; one marked *s* extends roughly 20°; *l* signifies that the line is 30° to 40° in extent, and *l l* that it is of still greater length. These values, like those of intensity, are only roughly approximate, but give some idea of the heights to which the several substances appear, at least to a marked extent, in the solar atmosphere. Long arcs signify great elevations of the producing elements, while the short ones indicate but small heights.

That Table II contains occasional errors I am quite ready to believe, though every precaution has been taken to reduce them to the smallest number possible. A better focus would have given more accurate measurements, with less chances for errors of any kind, and probably would have furnished many more lines. In a number of instances the spectrum arcs seemed to consist of two or more so overlapping, owing to want of sharp definition, that it was impossible to clearly separate them. In all such cases the solar wave-lengths of the probable constituents are indicated in the table by being bracketed together.

TABLE II.

Observed wave-length	From Rowland's Table			Intensity and Character of Lines					
	Wave-length	Substance	Intens. and charac.	I	II	III	IV	V	VI
3118.5	3118.498	*V*	3	....	....	....	....	....	0 ss
3121.3	3121.270	*V*	4	....	....	....	....	....	0 ss
3130.3	3130.380	*V*	3	....	....	....	....	....	1 ss
3161.9	3161.317 3161.887 3162.683	*Ti* *Ti* *Ti*	3 3 4	....	....	....	....	....	0 ss
3168.5	3168.640	*Ti*	4	....	....	....	....	....	0 ss
3191.2	3191.011	*Ti*	3	....	....	....	....	....	0 ss
3202.5	3202.651	*Ti, Fe*	2	....	....	....	....	....	0 ss
3218.0	3217.183 3218.389	*Ti* *Ti*	2 2	....	....	....	....	....	0 ss
3228.8	3228.735	*Ti ?*	2	....	....	....	....	....	0 ss
3234.6	3234.635	*Ti*	3	....	0 ss	....	....	....	2 ss
3236.8	3236.703	*Ti*	7N	....	0 ss	....	....	....	2 ss
3239.3	3239.170	*Ti*	7	....	0 ss	....	....	....	2 ss
3242.3	3242.125	*Ti*, —	8	....	....	....	....	....	0 ss
3248.8	3248.841	*Ti*	2	....	0 ss	....	....	....	2 ss
3252.0	3251.977	*Ti*	3	....	....	....	....	....	0 ss
3253.1	3253.012	*Ti, Fe*	5	....	....	....	....	....	0 ss
3254.2	3254.314	*Ti*	3	....	....	....	....	....	1 ss
3258.3	3258.542	*Mn, Co*	3	....	....	....	....	....	0 ss
3261.8	3261.705	*Ti*	3	....	....	....	....	....	0 ss
3263.8	3263.813	*Ti*	4	....	....	....	....	....	2 ss
3267.8	3267.834	*V*	6	....	....	....	....	....	2 ss
3272.0	3271.791 3272.217	*Ti, Fe* *Ti*	6d ? 5	....	....	....	....	....	1 ss
3278.3	3278.420	*Ti*	5	....	....	....	....	....	4 ss
3279.0	3279.060	*Ti*	4	....	....	....	....	....	4 ss
3282.4	3282.459	*Ti, Zn*	5	....	....	....	....	....	3 ss
3286.9	3286.898	*Fe*	7N	....	....	....	....	....	1 ss
3287.9	3287.793	*Ti*	5	....	....	....	....	....	4 ss
3292.2	3292.151 3292.206	*Fe* *Ti, Co*	4 3	....	....	....	....	....	0 ss
3295.8	3295.951	*Fe, Mn*	6	....	....	....	....	....	2 ss
3299.0	3298.869	*V*	3	....	....	....	....	....	0 ss
3302.8	3302.510 3303.109	*Na* *Na*	6 5	....	....	....	....	....	3 ss
3308.5	3308.888 3308.947	*Mn* *Co, Ti*	2 5	....	....	....	....	....	0 ss
3315.6	3315.457	*Ti*	3	....	....	....	....	....	0 ss
3318.3	3318.160	*Ti*	6	....	....	....	....	....	1 ss
3322.9	3323.056 3323.116	*Ti*	5 3	....	2 l	....	0 ll	....	10 l
3327.0	3326.907 3326.998	*Ti*	5 3	....	......	....	0 ll	....	1 ss
3329.6	3329.568 3329.648	*Ti, Co*	5 3	....	2 l	....	....	....	10 l
3332.3	3332.240	*Ti*	3	....	....	....	....	....	2 ss
3335.3	3335.299	*Ti*	4	....	3 l	....	0 ll	....	8 l
3340.5	3340.464	*Ti*	3	2 l	3 l	....	1 ll	....	5 l
3342.1	3341.967	*Ti*	4	....	5 l	2 ll	....	0 l	10 l

TABLE II.—*Continued.*

Observed wave-length	From Rowland's Table			Intensity and Character of Lines					
	Wave-length	Substance	Intens. and charac.	I	II	III	IV	V	VI
3349.5	3349.597	*Ti*	7	4 l	15 l	5 ll	2 ll	3 l	20 l
3354.0	3353.875	*Sc*,—	4	....	....	....	....	....	3 ss
3361.5	3361.327	*Ti*	8	3 l	10 l	3 ll	2 ll	1 l	15 l
3368.2	3368.193	*Cr*,–	5 d ?	1 s	2 l	0 ll	0 ll	....	5 l
3372.9	3372.901 } 3372.994 } d?	*Ti–Pd*	10	3 l	10 l	3 ll	2 ll	1 l	15 l
3380.2	3380.397	*Ti*	3	....	1 l	....	....	....	3 l
3383.9	3383.951	*Ti*	3	1 l	10 l	3 ll	2 ll	1 l	15 l
3388.0	3387.988[1]	*Ti–Zr*	5 d ?	0 s	2 l	0 l	0 ll	....	5 l
3392.2	3392.109	*Zr*	2	....	....	....	....	....	0 ss
3394.8	3394.685	*Ti*	3	0 ss	2 l	....	0 ll	....	5 l
3403.4	3403.404	*Cr*	2	....	1 s	....	....	....	3 l
3408.9	3408.911	*Cr*	3	....	1 s	0 l	0 ll	....	4 l
3414.9	3414.911	*Ni*	15	....	....	....	....	....	3 s
3421.4	3421.353	*Ni Cr Pd*	4	....	1 l	0 l	0 ll	....	3 l
3422.9	3422.892	*Cr*	4	....	1 l	0 l	0 ll	....	3 l
3426.5	3426.466 } 3426.535 } 3426.769 }	*Fe* *Fe* *Fe*	3 3 3	....	....	....	....	....	1 ss
3433.6	3433.453	*Cr*	3	0 ss	2 l	0 l	0 ll	....	2 l
3438.5	3438.376	*Zr*	2	....	....	....	....	....	2 ss
3442.0	3442.118	*Mn*	6	....	3 l	1 l	1 ll	0 l	6 l
3444.4	3444.467	*Ti*	4	....	....	....	....	....	3 ss
3452.9	3453.039	*Ni*	6 d ?	....	....	....	....	....	2 ss
3456.6	3456.528[1]	*Ti*	3	....	....	0 ll	3 ll	....	1 ss
3460.6	3460.460	*Mn*,—	4 d ?	0 ss	1 l	0 l	0 ll	....	5 l
3466.0	3465.900 } 3466.015 }	*Co* *Fe*	4 6	....	....	....	....	....	2 ss
3474.3	3474.287	*Mn*	2	....	1 s	0 l	0 ll	....	4 l
3477.4	3477.323	*Ti*	5	0 ss	1 s	....	....	....	2 l
3482.9	3483.047	*Mn* —	5 d ?	....	1 s	....	0 ll	....	3 l
3488.9	3488.817	*Mn*	4	....	1 s	0 l	....	....	2 s
3491.2	3491.195	*Ti*	5	....	1 s	....	....	....	2 s
3496.4	3496.348	*Zr*	2	0 ss	1 s	....	....	....	5 s
3500.6	3500.474	*Ti*	3	....	....	....	....	....	1 ss
3505.0	{ 3505.015 } { 3505.056 }	*Fe* *Ti*	3 2	0 ss	1 s	....	....	....	5 s
3510.9	3510.985	*Ti*	5	....	1 s	....	....	....	5 s
3514.0	{ 3513.965 } { 3514.082 }	*Fe*	7 4	....	....	....	....	....	3 s
3520.3	3520.397	*Ti*	2	....	....	....	....	....	2 s
3524.5	3524.677	*Ni*	20	....	....	....	....	....	4 s
3530.8	3530.919[2]	....	3	....	....	....	....	....	2 ss
3535.4	3535.554	*Ti*	4	....	....	....	....	....	5 ss
3541.3	3541.237	*Fe*	7	....	....	....	....	....	0 ss
3545.2	3545.336	—, *V*	4	....	....	....	....	....	1 ss
3548.5	3548.332	*Mn, Ni*	5	....	....	....	....	....	1 ss
3552.2	3552.098	*Zr*	1	....	....	....	....	....	1 ss

[1] Coincident with a hazy line of great extent.

[2] Exner and Haschek give a heavy vanadium line at λ 3530.96.

TABLE II.—*Continued.*

Observed wave-length.	From Rowland's Table			Intensity and Character of Lines					
	Wave-length	Substance	Intens. and charac.	I	II	III	IV	V	VI
3557.0	3556.738 3556.830 3556.944[1] 3557.036	*Zr* *Fe* *Fe*	2 2 4 3	....	....	....	....	....	4 ss
3561.8	3561.898	*Ni*	3	....	....	....	....	....	2 ss
3565.6	3565.535	*Fe*	12	....	1 s	....	....	....	5 ss
3568.0	3567.835	....	4	....	....	....	....	....	1 ss
3570.3	3570.273	*Fe*	20	....	2 s	....	....	....	4 ss
3572.7	3572.617 3572.712	.... —, *Sc*	4 6	o ss	2 s	....	....	....	5 ss
3575.2	3575.106	*Cr—Co*	5	....	....	....	....	....	o ss
3576.5	3576.469 3576.527	.... —, *Sc?*	4 3	o ss	1 s	....	....	....	5 s
3579.0	3578.832	*Cr*	10	....	....	....	....	....	2 ss
3581.3	3581.349	*Fe*	30	o ss	2 s	....	....	....	5 s
3585.4	3585.479	*Fe*	7	o ss	1 s	....	....	....	8 s
3590.0	3589.773 3589.908[2]	....	5 5d?	....	....	....	....	....	6 s
3593.4	3593.636	*Cr*	9	....	....	....	....	....	4 s
3596.1	3596.195	*Ti*	4	....	....	....	....	....	3 s
3600.7	3600.880	*Y*	3	....	....	....	....	....	5 s
3604.0	3603.922 3603.972	*Ti* *Fe*	3 4	....	....	....	....	....	2 s
3609.2	3609.008	*Fe*	20	....	....	....	....	....	3 s
3613.8	3613.947	—, *Sc*	4	o ss	3 l	....	....	....	5 l
3618.9	3618.919	*Fe*	20	o ss	1 s	....	....	....	5 s
3624.9	3624.979	*Ti, Fe*	5	....	....	....	....	....	4 s
3630.8	3630.876	....	4	1 s	3 l	o l	....	....	8 l
3636.3	3636.305	*Fe*	3	....	....	....	....	....	1 ss
3642.7	3642.820	*Ti*	7	o ss	1 s	2 ll	2 ll	....	8 l
3645.5	3645.429 3645.475	.... *Sc?*, —	3 3	....	....	....	....	....	3 ss
3648.0	3647.988	*Fe*	12	o ss	....	....	....	....	4 ss
3651.5	3651.614	*Fe*	7	o ss	....	....	....	....	3 ss
3652.0	3651.940	—, *Sc*	4	....	....	....	....	....	5 ss
3656.0	3655.801	....	3	....	....	....	....	....	1 ss
3660.0	3659.901	*Fe—Ti*	5	....	....	....	....	....	4 ss
3662.4	3662.378	*Ti*	5	....	....	....	....	....	2 s
3663.8	3663.5[3]	*H29*	....	....	Not very well defined, and	probably still further confused	by the strong iron, nickel, and	other lines of this region.	1 s
3664.5	3664.7[3]	*H28*	....	....					1 s
3666.0	3666.1[3]	*H27*	....	....					2 s
3667.3	3667.4[3]	*Hω*	....	....					2 s
3668.8	3669.0[3]	*Hψ*	....	....					2 s
3671.0	3670.8[3]	*Hχ*	....	o ss					3 s
3672.5	3672.7[3]	*Hφ*	....	....					3 s
3674.3	3674.2[4]	*Hυ*	....	1 s	1 s	....	....	....	3 s
3676.4	3676.457	*Fe, Cr*	6	....	....	....	....	....	3 s

[1] Exner and Haschek give a heavy vanadium line at λ 3556.93.

[2] Exner and Haschek give a heavy vanadium line at λ 3589.90.

[3] Computed from Balmer's formula $\frac{1}{\lambda} = a\left(1 - \frac{4}{m^2}\right)$, using Ames's value of a, 27418.3.

[4] Hale's value.

TABLE II.—*Continued.*

Observed wave-length	From Rowland's Table: Wave-length	Substance	Intens. and charac.	Intensity and Character of Lines: I	II	III	IV	V	VI
3679.6	3679.5[1]	*Hτ*	....	1 s	1 s	....	....	....	3 s
3683.5	3683.5[2]	*Hσ*	....	1 s	1 s	....	....	....	3 s
3685.3	3685.339	*Ti*	10d?	10 l	15 l	6 ll	5 ll	5 l	20 l
3686.8	3686.7[3]	*Hρ*	....	2 l	2 l	....	....	....	6 s
3691.6	3691.5[3]	*Hπ*	....	2 l	3 l	1 l	0 ll	....	8 l
3697.4	3697.4[3]	*Ho*	....	2 l	4 l	1 l	0 ll	0 l	8 l
3704.1	3704.0[3]	*Hξ*	....	3 l	5 l	1 l	1 ll	0 l	10 l
3706.3	3706.175	*Ca, Mn*	6d?	0 s	....	....	....	....	5 s
3709.4	3709.389	*Fe*	8	....	....	....	....	....	2 s
3711.8	3711.8[3]	*Hν*	....	3 l	6 l	2 l	1 ll	0 l	12 l
3715.7	3715.615[4]	*Mn?*	4	....	....	....	....	....	1 ss
3720.0	3720.084	*Fe*	40	0 s	1 l	....	....	....	4 ss
3721.8	3721.9[3]	*Hμ*	....	4 l	8 l	3 l	1 ll	0 l	14 l
3727.8	3727.778	*Fe*	4	....	....	....	....	....	2 ss
3734.3	3734.2[2]	*Hλ*	....	4 l	10 l	3 ll	2 ll	1 l	14 l
3737.3	3737.281	*Fe*	30	1 s	3 l	0 l	....	....	5 l
3741.8	3741.791	*Ti*	4	....	1 l	....	....	....	4 l
3745.8	3745.717	*Fe*	8	0 s	1 l	....	....	....	5 l
3750.3	3750.2[2]	*Hκ*	....	6 l	15 l	8 ll	2 ll	1 l	15 l
3759.5	3759.447	*Ti*	12d?	6 l	20 ll	10 ll	4 l	2 l	15 l
3761.5	3761.464	*Ti*	7	6 l	20 ll	10 ll	4 ll	2 l	15 l
3764.1	3763.945	*Fe*	10	....	....	....	....	....	1 s
3767.5	3767.341	*Fe*	8	....	....	....	....	....	0 ss
3770.8	3770.8[1]	*Hι*	....	8 l	20 ll	10 ll	3 ll	1 l	15 l
3774.6	3774.473	*Y*	3	....	....	....	....	....	3 ss
3783.6	3783.674	*Ni*	6	....	....	....	....	....	0 ss
3788.9	3788.839	*Y*	2	....	....	....	....	....	3 ss
3798.2	3798.1[1]	*Hθ*	....	10 l	25 ll	12 ll	5 ll	3 l	20 l
3806.9	3806.865	*Mn-Fe*	8d?	....	....	....	....	....	1 ss
3814.6	3814.698		8d	....	....	....	....	....	4 ss
3816.1	3815.987	*Fe*	15	....	....	....	....	....	4 ss
3820.7	3820.586	*Fe-C*	25	1 s	2 l	1 ll	2 ll	....	5 l
3824.7	3824.591	*Fe*	6	1 s	0 s	....	....	....	3 s
3825.8	3826.027	*Fe*	20	1 s	0 s	....	....	....	3 s
3829.5	3829.501	*Mg*	10	3 s	3 s	....	....	....	5 l
3832.5	3832.450	*Mg*	15	5 l	5 l	1 ll	....	....	8 l
3835.5	3835.54[1]	*Hη*	....	20 l	30 ll	15 ll	6 ll	5 l	20 l
3838.4	3838.435	*Mg-C*	25	5 l	10 l	5 ll	3 ll	....	8 l
3841.3	3841.195	*Fe-Mn*	10	....	....	....	....	....	2 ss
3850.1	3850.118	*Fe-Cr*	10	0 s	....	....	....	....	3 ss
3856.6	3856.524	*Fe*	8	1 s	1 s	....	....	....	4 ss
3860.2	3860.055	*Fe-C*	20	2 s	3 s	....	0 ll	....	5 l
3872.5	3872.639	*Fe*	6	....	....	....	....	....	3 ss
3878.8	3878.720	*Fe*	7 Nd?	0 s	1 s	....	....	....	5 s
3883.3	3883.426	*C-Fe*	2		....	....	....	....	3 ss
3889.0	{ 3888.789[5] 3889.14[6]	*He* *Hζ*		30 ll	80 ll	30 ll	20 ll	20 ll	40 ll

[1] Hale's value. [2] Deslandres' value. [3] Deslandres' and Hale's value. [4] Exner and Haschek give a heavy vanadium line at λ 3715.70 [5] Runge and Paschen's value. [6] Young's value.

TABLE II.—*Continued.*

Observed wave-length	From Rowland's Table			Intensity and Character of Lines					
	Wave-length	Substance	Intens. and charac.	I	II	III	IV	V	VI
3895.9	3895.803	*Fe*	7	....	....	....	....	....	5 ss
3900.7	3900.681	*Ti-Fe*	5	o s	5 l	o ll	1 ll	....	10 l
3905.7	3905.660	*Si*	12	....	....	....	1 ll	....	2 ss
3906.5	3906.628	*Fe*	10	....	....	....	....	....	3 ss
3913.5	3913.609	*Ti-*	5 d ?	....	....	....	....	....	6 ss
3920.2	3920.410	*Fe*	10	....	....	....	....	....	o ss
3923.0	3923.054	*Fe*	12 d ?	....	....	....	....	....	3 ss
3928.2	3928.075	*Fe*	8	....	....	....	....	....	2 ss
3933.8	3933.825	*Ca*	1000	100 ll	200 ll	150 ll	100 ll	125 ll	200 ll
3944.0	3944.160	*Al*	15	....	1 s	....	....	....	4 l
3950.3	3950.102	*Fe*	5	Cen	ter of	con	fused	group	3 ss
	3956.476	*Ce,Co-Ti*	4						
3956.6	3956.603	*Fe*	4	....	1 s	....	....	....	3 ss
	3956.819	*Fe*	6						
3961.6	3961.674	*Al*	20	o s	1 l	....	....	....	3 l
3968.6	3968.625	*Ca*	700	75 ll	150 ll	130 ll	75 ll	100 ll	150 ll
3970.3	3970.177	*He*	5	30 l	50 ll	20 ll	10 ll	?	30 ll
3977.8	3977.891	*Fe*	6	....	....	....	....	....	1 ss
3982.7	3982.630	*Ti-Mn*	2	....	....	....	....	....	3 ss
	3982.742	*Y*	3						
3989.8	3989.912	*Ti*	4	....	....	....	....	....	2 ss
3998.8	3998.790	*Ti*	4	....	....	....	....	....	3 ss
4005.6	4005.408	*Fe*	7	....	....	....	....	....	3 ss
4012.7	4012.541	*Ti, Ce*	4	....	....	....	....	....	3 ss
4026.3	4026.342[1]	*He*	..	1 l	10 ll	5 ll	5 ll	5 l	8 ll
	4030.878								
4031.0	(4030.918)	*Mn*	10 d ?	....	....	....	....	....	5 ss
	4030.947								
4033.3	4033.224	*Mn*	8 d ?	....	....	....	....	....	5 ss
4034.7	4034.644	*Mn*	6 d	....	....	....	....	....	2 ss
4041.5	4041.431	*Fe*	3	....	....	....	....	....	1 ss
	4041.525	*Mn*	5						
4046.0	4045.975	*Fe*	30	o s	1 l	....	....	....	8 s
	4048.818	*Zr*	1						
4048.9	(4048.883)	....	..	....	....	....	....	....	1 ss
	4048.910	*Mn-Cr*	5						
4055.1	4055.023	*Fe*	3	....	....	....	....	....	1 ss
	4055.189	*	3						
4055.6	4055.701	*Mn*	6	....	....	....	....	....	1 ss
4059.0	4058.915	*Fe-Cr*	3	....	....	....	....	....	1 ss
4063.7	4063.759	*Fe*	20	....	1 l	....	....	....	5 ss
4067.3	4067.139	*Cr-Fe*	5	....	....	....	....	....	2 ss
4071.9	4071.908	*Fe*	15	....	o s	....	....	....	4 ss
4077.9	4077.885	*Sr*	8	10 l	20 ll	15 ll	8 ll	5 l	20 l
	4084.647	*Fe*	5						
4085.0	4085.161	*Fe*	4	Cen	ter of	con	fused	group	2 ss
	4085.467	*Fe*	4						
4101.9	4102.000	*Hδ, In*	40 N	40 ll	90 ll	50 ll	20 ll	15 ll	60 ll
4109.9	4109.905	*V*	2	....	....	....	....	....	2 ss
	4109.953	*Fe*	3						

[1] Runge and Paschen's value.

* *Ce-Ti-Fe, Zr.*

TABLE II — *Continued.*

Observed wave-length	From Rowland's Table			Intensity and Character of Lines					
	Wave-length	Substance	Intens. and charac.	I	II	III	IV	V	VI
4118.8	4118.708 4118.934	*Fe* *Co*	5 4	....	....	....	....	....	2 ss
4123.2	4123.384	*La*	1	....	0 s	0 l	0 ll	....	3 ss
4128.3	4128.251	*Ce–V,—*	6d	....	....	....	....	....	2 ss
4132.1	4132.235	*Fe–Co*	10	....	....	....	....	....	2 ss
4137.3	4137.156	*Fe*	6	....	....	....	....	....	0 ss
4144.0	4144.038	*Fe*	15	....	0 s	....	....	....	5 ss
4149.5	4149.360 4149.533	*C, Zr* *Fe*	2 4	....	....	....	....	....	4 ss
4156.3	4156.391	*Zr*	1	....	....	....	....	....	2 ss
4163.9	4163.818	*Cr–Ti,–*	4	....	....	....	....	....	2 ss
4167.6	4167.438	....	8	....	....	....	....	....	0 ss
4171.9	4171.854	*C, Fe?*	2	....	....	....	....	....	3 ss
4173.7	4173.624 4173.710	....	3 3	....	....	....	....	....	3 ss
4177.8	4177.698	....	3	...	....	....	....	....	3 ss
4179.0	4179.025	....	3	....	....	....	....	....	2 ss
4184.3	4184.158 4184.472	....	4 2	....	....	....	....	....	1 ss
4188.0	4187.943	*Fe*	5	....	....	....	....	....	2 ss
4191.7	4191.595	*Fe*	6	....	....	....	....	....	1 ss
4199.2	4199.267	*Zr–Fe*	5	....	....	....	....	....	2 ss
4205.4	4205.545	*Fe*	1N	....	....	....	....	....	2 ss
4210.4	4210.494	*Fe*	4	....	....	....	....	....	2 ss
4215.7	4215.703	*Sr*	5d?	8 l	10 l	2 ll	1 ll	3 l	15 l
4222.2	4222.382	*Fe*	5	....	....	....	....	....	1 ss
4226.9	4226.904	*Ca*	20d?	5 s	3 l	....	....	....	8 l
4233.4	4233.328	*Mn*	4	0 ss	....	....	....	....	4 l
4247.0	4246.996	*Sc*	5	5 s	2 l	0 l	....	....	8 l
4250.8	4250.945	*Fe*	8	....	....	....	....	....	3 ss
4254.5	4254.505	*Cr*	8	2 s	0 s	....	....	....	4 l
4260.5	4260.640	*Fe*	10	....	....	....	....	....	4 ss
4267.8	4267.985	*Fe*	3	....	....	....	....	....	0 ss
4271.8	4271.934	*Fe*	15	....	....	....	....	....	4 ss
4274.9	4274.958	*Cr*	7d?	0 ss	....	....	....	....	5 ss
4282.4	4282.565	*Fe*	5	....	....	....	....	....	1 ss
4283.1	4283.169	*Ca*	4	....	....	....	....	....	2 ss
4289.6	4289.525	*Ca*	4	....	0 s	....	....	....	4 s
4289.9	4289.885	*Cr*	5	1 s	0 s	....	....	....	4 s
4294.3	4294.301	*Fe*	5	1 s	0 s	....	....	....	6 s
4300.2	4300.211	*Ti*	3	....	1 s	0 ll	....	....	8 s
4302.3	4302.353	*Fe*	2	....	....	....	....	....	2 ss
4302.8	4302.692	*Ca*	4	....	....	....	....	....	2 ss
4307.9	4307.907 4308.081	*Ca* *Fe*	3 6	1 s	1 s	....	....	....	6 ss
4313.2	4313.034	*Ti*	3	....	1 s	....	....	....	3 s
4319.0	4318.817	*Ca, Mn?*	4	0 s	1 s	. ..	....	....	3 s
4321.0	4320.907	*Sc*	3	....	1 s	....	....	....	5 s
4326.0	4325.939	*Fe*	8	0 s	1 s	....	....	....	8 s
4330.9	4330.866	*Ti, Ni*	2	....	....	....	....	....	1 ss
4340.6	4340.634	*Hγ*	20N	60 ll	100 ll	60 ll	40 ll	30 ll	100 ll

TABLE II — *Continued.*

Observed wave-length	From Rowland's Table: Wave-length	Substance	Intens. and charac.	Intensity and Character of Lines: I	II	III	IV	V	VI
4352.8	4352.908	*Fe*	4	0 s	....	....	....	....	6 ss
4359.9	4359.784	*Cr*	3	....	Film Spoiled.	....	....	....	2 ss
4363.3	4363.267	*Cr*	1 N	....		....	....	....	2 l
4367.8	4367.749	*Fe*	5	....		....	....	....	2 ss
4370.0	4369.941	*Fe*	4	....		....	....	....	2 ss
4374.7	4374.628	*Sc, Fe?*	3	....		....	....	....	3 ss
4376.0	4376.107	*Fe*	6	0 s		0 l	....	....	5 l
4383.6	4383.720	*Fe*	15	1 s		0 l	....	....	6 l
4395.1	4395.201	*Ti*	3	3 s		1 ll	....	....	8 l
4399.9	4399.935	*Ti, Cr*	3	0 s		....	....	....	3 l
4400.5	4400.555	*Sc*	3	0 s		....	....	....	3 l
4404.9	4404.927	*Fe*	10	0 s		....	....	....	5 l
4408.7	4408.582	*Fe*	3	....		....	....	....	4 ss
4415.2	4415.293	*Fe*	8	....		..	....	....	5 s
4417.7	4417.884	*Ti*—	3	....		....	....	....	6 s
4422.7	4422.741	*Fe, Y*	3	....		....	....	....	1 ss
4427.5	4427.482	*Fe*	5	....		....	....	....	3 ss
4430.9	4430.785	*Fe*	3	....		....	....	....	1 ss
4435.3	4435.129	*Ca*	5	....		....	....	....	4 ss
4443.8	4443.976	*Ti*	5	3 s		0 l	....	....	8 l
4450.6	4450.654	*Ti?*	2	....		....	....	....	4 s
4455.0	4454.953	*Ca, Zr*	5	....		....	....	....	4 s
4461.8	4461.818	*Fe*	4	....	....	....	....	....	4 s
4468.5	4468.663	*Ti*—	5	2 s	0 s	....	....	....	6 l
4471.6	4471.646[1]	*He*	..	20 ll	40 ll	25 ll	10 ll	15 ll	30 ll
4490.2	4490.253	*Mn-Fe*	3 N	....	....	....	....	....	4 ss
4494.8	4494.738	*Fe*	6	....	....	....	....	....	1 ss
4501.4	4501.445	*Ti*,—	5	2 s	1 s	0 l	....	....	8 l
4508.5	4508.455	*Fe?*,—	4	....	....	....	....	....	3 ss
4515.6	4515.508	....	3	....	....	....	....	....	3 ss
4520.3	4520.397	*Fe?*,—	3	....	....	....	....	....	3 ss
4522.9	4522.974	*Ti*	2	....	....	....	....	....	5 ss
4528.9	4528.798	*Fe*	8	....	....	....	....	....	2 ss
4534.1	4534.139	*Ti-Co*	6	2 s	2 s	1 l	....	....	8 l
4541.6	4541.690	*Cr*	2	....	....	....	....	....	1 ss
4549.8	4549.808	*Ti-Co*	6d?	3 s	2 s	1 l	....	....	8 l
4554.3	4554.211	*Ba*	8	3 s	2 s	....	....	....	10 l
4558.8	4558.827	*Cr?*	3	....	....	....	....	....	2 ss
4564.0	4563.939	*Ti*	4	1 s	2 s	1 l	....	....	8 l
4572.1	4572.156	*Ti*—	6	2 s	2 s	1 l	....	....	8 l
4584.0	4584.018	*Fe*—	4	0 ss	0 s	....	....	....	6 s
4588.5	4588.381	....	3	....	....	....	....	....	2 s
4601.0	4600.932	*Cr*	3	....	....	....	....	....	0 ss
4611.3	4611.469	*Fe*	5	....	....	....	....	....	0 ss
4619.0	4618.971	*Fe*—	4d?	....	....	....	....	....	2 ss
4629.4	4629.521	*Ti-Co*	6	....	....	....	....	....	4 ss
4634.4	4634.254	....	2	....	....	....	....	....	0 ss
4646.5	4646.347	*Cr*	5	....	....	....	....	....	3 ss
4651.5	4651.461	*Cr*	4	....	....	....	....	....	2 ss
4656.5	4656.644	*Ti*	3	....	....	....	....	....	3 ss

[1] Runge and Paschen's value.

TABLE II.—*Continued.*

Observed wave-length	From Rowland's Table			Intensity and Character of Lines					
	Wave-length	Substance	Intens. and charac.	I	II	III	IV	V	VI
4668.4	4668.331	*Fe*	4	....	....	....	....	....	3 ss
4685.4	4685.452	*Ca*	2 N	....	0 s	1 l	1 ll	2 ll	5 l
4704.0	4703.994	*Ni*	3	....	....	....	....	....	1 ss
4713.3	4713.252[1]	*He*	....	....	3 l	2 ll	1 ll	2 ll	5 l
4731.6	4731.651	*Fe ?*	4	....	....	....	....	....	0 ss
4754.1	4754.225	*Mn*	7	....	....	....	....	....	0 ss
4763.9	4764.108	*Ti–Ni*	4 d	....	....	....	....	....	0 ss
4780.3	4780.169	*Co*	2	....	....	....	....	....	0 ss
4799.4	4799.598	*Fe*	1	....	....	....	....	....	0 ss
4805.3	4805.285	....	3	....	....	....	....	....	0 ss
4823.8	4823.697	*Mn*	5	....	....	....	....	....	3 ss
4827.9	4827.804	*Ti*	00	....	....	....	....	....	1 ss
4841.0	4841.074	*Ti*	3	....	....	....	....	....	0 ss
4847.6	4847.497	*Ca*	0	....	....	....	....	....	1 ss
4861.5	4861.527	*Hβ*	30	80 ll	120 ll	80 ll	40 ll	60 ll	150 ll
4871.6	4871.512	*Fe*	5	....	....	....	....	....	3 ss
4877.6	4877.772	....	0	....	....	....	....	....	1 ss
4883.8	4883.867	*Yt earth*	2	....	....	....	....	....	2 ss
4891.5	4891.683	*Fe*	8	....	....	....	....	....	3 ss
4900.1	4900.095	*Ti, La*	2	....	....	....	....	....	2 ss
4911.2	4911.374	....	1	....	....	....	....	....	1 ss
4920.5	4920.685	*Fe*	10	....	....	....	....	....	3 ss
4924.2	4924.107	*Fe*	5	....	....	....	....	....	10 s
4934.3	4934.214 } 4934.277 }	*Ba, Fe ?*	6	....	....	....	....	....	6 s
4939.9	4939.868	*Fe*	3	....	....	....	....	....	1 ss
4957.8	4957.785	*Fe*	8	....	....	....	....	....	3 ss
4994.1	4994.316	*Fe*	3	....	....	....	....	....	0 ss
4999.8	4999.689	*Ti, La*	3	....	....	....	....	....	0 ss
5006.4	5006.306	*Fe*	5	....	....	....	....	....	0 ss
5014.5	5014.369 } 5014.457 }	*Ti* *Fe*	2 3	....	....	....	....	....	0 ss
5018.7	5018.629	*Fe*	4	....	....	....	....	....	6 ss
5031.3	5031.199	....	3	....	....	....	....	....	0 ss
5041.9	5041.795	*Ca*	2	....	....	....	....	....	0 ss
5167.5	5167.497	*Mg*	15	....	....	....	....	....	4 ss
5172.8	5172.856	*Mg*	20	....	....	....	....	....	4 ss
5183.8	5183.791	*Mg*	30	....	....	....	....	....	4 ss
5204.7	{ 5204.680 } { 5204.768 }	*Cr* *Fe*	5 3	....	....	....	....	....	0 ss

DESCRIPTION OF TABLE III.

Table III gives a list of the elements and the number of long and short lines due to each found on the films. The symbols *ss*, *s*, *l*, and *ll* have the same meanings here that are given

[1] Runge and Paschen's value.

to them in Table II ; that is, they designate respectively very short, short, long, and very long arcs. The long arcs, as already explained, are due to appreciable amounts of light of these particular wave-lengths coming from great elevations in the chromosphere, while the short ones denote wave-lengths that are given off from lower parts of the Sun's atmosphere.

A double column is given under the several lengths. The first in each pair, marked *single*, refers to lines which in Rowland's table are attributed to single substances, while the column

TABLE III.

LENGTHS OF THE CRESCENT ARCS.

Substance	ss		s		l		ll	
	Single	Mixed	Single	Mixed	Single	Mixed	Single	Mixed
Al	..	..	..	..	2	..	..	..
Ba	..	..	..	1	1	..	..	..
Ca	5	..	2	3	1	..	3	..
C	..	3	..	..	..	..	..	3
Ce	..	3	..	1	..	..	..	..
Co	3	5	..	2	..	2	..	2
Cr	8	3	3	3	3	1	3	2
He	..	..	..	..	..	..	4	..
H	..	..	10	..	1	..	14	1
In	..	..	..	..	..	..	..	1
Fe	65	19	25	5	8	..	..	3
La	..	..	..	..	..	..	1	..
Mg	3	..	..	..	1	..	1	1
Mn	9	8	..	2	2	..	2	2
Ni	6	3	2	..	..	..	..	1
Pd	..	..	..	..	..	..	..	3
Sc	..	4	1	2	2	1	..	..
Si	..	..	..	..	..	..	1	..
Na	2	..	..	..	..	..	..	..
Sr	..	..	..	..	..	..	2	..
Ti	34	18	10	3	4	6	15	6
V	6	2	..	..	..	..	..	..
Y	4	1	1	..	..	..	..	..
Zn	..	1	..	.	..	..	..	..
Zr	5	5	1	1	..	..	..	1
Unknown	20	11	6	4	1	4	1	3

designated *mixed* has reference to lines which are assigned to two or more elements. It is impossible, of course, to determine just what substance or substances any of these lines, as they occur on my films, are due to, though in many cases one would

naturally suspect them to be produced by iron or titanium rather than by other elements that give lines of nearly the same wave-lengths. But such a separation would be entirely arbitrary, and I shall not attempt it.

Table IV gives the approximate heights, as determined from the spectrum arcs on film VI, reached by appreciable amounts of several substances in the solar atmosphere. In making these estimates I took the semidiameter of the Sun as 948″.4, and that of the Moon as 1013″.8, and used that part of each crescent that extends from the point of contact across the film in a direction roughly parallel to the rulings on the grating.

The great majority of the arcs are so short that they indicate an elevation of less than one second, and many even less than half a second; but any substance must extend at least as high as the outermost source of its longest lines, and hence only the longest arcs are given in Table IV. These estimates of extreme heights are not in close agreement with those of certain other observers; but so much depends on the lines selected, on the

TABLE IV.

Approximate height of substance	Substance	Wave-length
0″.25	*Al*	3944.160, 3961.674
0.25	*Co*	4780.169
0.25	*La*	4123.384
0.50	*Na*	3302.510, 3303.109
0.50	*V*	3118.498, 3121.270, 3130.380
1	*Ni*	3414.911, 3453.039
1	*Y*	3600.880
1	*Zr*	3496.348
3	*Cr*	3422.892, 3433.453
4	*Ba*	4554.211
4	*Fe*	3737.281, 4376.107, 4383.720
5	*Sc*	4246.996
5	*Si*	3905.660
6	*Mg*	3829.501, 3832.450
7	*Mn*	3442.118, 3460.460
9	*Sr*	4077.885, 4215.703
9	*Ti*	3349.597, 3383.951, 3685.339, 3759.447, 3761.464
34	*Ca*	(K) 3933.825, (H) 3968.625
34	*He*	4471.646
34	*H*	(γ) 4340.634, (β) 4861.527

light-gathering power of the spectroscope, and on the time of exposure, that, with differences in these particulars, agreement could not be expected.

DISCUSSION OF THE RESULTS.

It will be seen that most of the elements that produce a considerable number of lines give both long and short ones, which show that the elements themselves are more or less similarly and pretty generally distributed throughout the solar atmosphere. It seems, too, from the photographs that the light intensity, and presumably the density of each substance, grows rapidly more pronounced at greater depths.

Table II shows that a number of lines which are short on the films that had only a few seconds exposure came out as long ones on those which were exposed a greater length of time, and this, too, indicates a very extensive distribution of the elements in the chromosphere, with no narrow separating boundaries producing distinct layers of any kind.

The streaks of continuous spectra are most marked about prominences, and at those places which, owing to irregularities in the Moon's outline, were exposed to the deeper portions of the Sun's atmosphere. But as film IV, which was exposed two minutes during mid-totality, has distinct bands of faintly continuous spectra extending approximately from λ 3200 to λ 5200—that is, from near the limit of the ultra-violet up as far as the film was especially sensitive—it would seem that, whatever this is due to, whether to great quantities of gas or to more concentrated masses, it must extend, without very abrupt changes, pretty much throughout the solar atmosphere, but in rapidly decreasing amounts at higher levels.

There are two lines in the ultra-violet that perhaps deserve special mention, since they are quite distinct in appearance from any of the others, being very long, broad, and hazy, seemingly true coronal lines. Their approximate wave-lengths are respectively λ 3388 and λ 3456.5, so nearly coincident with the titanium lines λ 3387.988 and λ 3456.528 that I could not distinguish

between them by measurement; and, in fact, it is not their positions on the films, but their general appearance, especially as produced by the long exposure, that makes me doubt their titanium origin.

I have tried to find what relations, if any, exist between the lines given in Table II and the spectra of the elements as produced under various other conditions; but so far no very close relation has been discovered, except the natural one that in general the heavier, and only the heavier, Fraunhofer lines appear as bright lines in the flash, and that the relative intensities in the two cases are roughly comparable.

A line that comes out strong, both in the arc and the spark spectrum, is likely to appear also in the flash spectrum. Probably, too, the chances are more than even in favor of the appearance of the "enhanced" lines, but the exceptions are too many and too decided to justify speculations based on this important phenomenon.

It is evident from the tables that, with the exception of those produced by hydrogen and helium, the great bulk of the lines are due to elements belonging to that Mendelejeff series which terminates with the iron, nickel, and cobalt groups; but there can be no surprise in this, since the same is true of all the identified solar lines, and it simply shows that the upper and lower levels of the Sun's atmosphere contain, in the main, the same elements, and that the Fraunhofer lines are true reversals of gaseous bright lines.

SHADOW BANDS.

It was no part of my programme to observe the shadow bands, but I happened to see them very favorably several seconds before the eclipse became total. They were quite distinct, though the edges were not sharply marked, and only some eight or nine inches from center to center.

I was aware of the great variety in the estimates that have been made of the velocity with which these queer shadows move, and I was surprised to find that I should have to add another, distinctly different from any with which I was familiar, for, with

the exception of a slight tremulous motion, they seemed to me to be nearly or altogether stationary. But by this time the spectroscope was claiming my attention, and I had no opportunity to watch them further. However, a Dutch official, who at the time of the eclipse was only a short distance from our station, told me that he watched the bands carefully, and that at first they were narrow and practically stationary, but subsequently seemed to grow wider apart and to move faster and faster till they were wholly lost to view.

SUGGESTIONS.

It seems to me that it would be very desirable, in connection with the work at future eclipses, to obtain photographs of as many of the flash and coronal lines as possible, and for this purpose both prism and direct grating spectroscopes may be used, but in either case it would probably be well to make at least some very long exposures on carefully backed plates. The strong lines would of course be greatly overexposed, but these are already known, and the long exposures would bring out many of the fainter ones. A few exposures could be made with advantage both just before and just after totality, and to allow for some judgment on the part of the operator, the plate or film magazine should be amply stocked so as to meet any such contingencies as unexpected duration that might arise. Perhaps it would even be well to have one exposure covering the whole of the eclipse, except the two flashes, for the sake of detecting the faintest of the coronal lines.

I would suggest that the grating be put in approximate focus, either by careful measurement based on a knowledge of the grating itself, or by some such method as that devised by Mr. Jewell. But since it is so essential to have exact definition I would strongly urge final focusing, just before totality, on the narrow crescent of the Sun.

Besides the spectroscopes already mentioned I should particularly like to see a concave grating used with a slit and mounted according to the Rowland method. In this case mirrors alone

should be used in focusing the light on the slit, so as to avoid all chromatic aberration; and it should be done so as to get as much as possible of the light on the ruled surface of the grating, for the only serious danger is that there may not be light enough. A suitable portion of the spectrum could be observed visually by the operator, who should be able to so shift the image as to keep its brightest portion on the slit. It would be very desirable in this case to have the disappearing and the reappearing crescents tangent to the slit, and this could be obtained either by an image rotator, or preferably, for the sake of economizing the light, by suitably rotating the spectroscope itself between the exposures. I would suggest an exposure at each flash and a single long one between these with the instrument set to catch the brighter parts of the polar streamers, and further that each of these be followed by a comparison arc spectrum on the same plate. I am quite sure that taking a comparison spectrum need not consume much time, for I have obtained very fair iron spectra, using a grating of twenty-one feet focal length, with an exposure of one second. With such plates as these, even if the lines were not very numerous, it would be possible to detect any slight changes in wave-length due to velocity of the polar rays or to any other cause, since these changes would be the residuals, if any, left over after allowing for the ordinary line of sight motion due to such known causes as the rotations of the Sun and Earth on their axes, and the motion of the Earth in its orbit.

There is just one other suggestion I should like to make. The photographic plates evidently ought to be as sensitive as can be obtained through the whole range of the spectrum to which they are exposed. But those commercial plates which are most sensitive to the ultra-violet, are not the best for the green and yellow, nor, on the other hand, are orthochromatic plates the quickest in the region of short wave-lengths. This trouble, I fancy, may be overcome in several ways. The makers may be persuaded to coat the two ends of the plates or films with different emulsions, one for the violet, the other for the green of the spectrum; or it would be quite practicable to take, say, a "gilt

edge" plate and suitably stain one end of it, thus rendering that part orthochromatic while leaving the other end particularly sensitive to the violet.

I am aware that these suggestions may either be old, or on actual trial found somewhat wanting; but I do not remember having seen them, and they are offered because I believe they are practicable and that they would give some desirable results.

UNIVERSITY OF VIRGINIA,
April 1902.

ON THE SEPARATION OF CORRESPONDING SERIES LINES IN THE MAGNETIC FIELD.[1]

By C. RUNGE and F. PASCHEN.

PRESTON observed that lines of different elements which correspond in respect to the laws of series are separated in the same manner in the magnetic field, viz., so that on the scale of vibration numbers the components of corresponding lines have the same distances in the same strength of field.

For testing this law as accurately as possible we selected the lines of the second subordinate series in the spectra of mercury, magnesium, zinc, cadmium, and strontium. The lines of the first subordinate series are not as well adapted for this purpose, because they exhibit a smaller separation and also because the satellites that occur there disturb an accurate determination.

The spark spectra of *Hg* and *Zn* were simultaneously produced by passing the spark between two pieces of sheet zinc, one of which had been amalgamated. Spark spectra of *Zn* and *Mg*, of *Zn* and *Cd*, of *Mg* and *Cd*, and of *Zn* and *Sr* were similarly produced simultaneously by passing the spark from one metal to the other. A solution of a strontium salt was placed by drops on the zinc electrode for *Zn* and *Sr*. These electrodes, which were cut into small strips from the sheet metal, were placed between the poles of a Dubois electro-magnet. One of them was stretched across over one pole-piece and attached to it with pins, while the other was placed opposite to it and insulated from the other pole-piece by a sheet of mica. The spark thus had the direction of the lines of force and was situated in the most intense part of the field without being crowded out by the field. In this way the poles could be brought up within a few millimeters of each other. The field strength varied between 28000 and 33000 c.g.s. units.

[1] Translated, from advance proofs, sent by the authors, of a paper to appear in the *Sitzungsberichte der Kgl. Akad. zu Berlin.*

By producing the two spectra at the same time we avoid the determination of the field-strength, which cannot be made with the same relative accuracy as the measurement of the distances of the components. The only essential is to preserve the field-strength constant during an exposure, while the field-strength may be different for different exposures. We therefore gave no particular care to securing accurately the same separation of the poles after the pole pieces had been moved apart in order to attach the metals. But attention was given to maintaining the constancy of the current of the electromagnet during the exposure, by throwing out a corresponding resistance from the circuit with an increasing temperature of the resistance coil, so that an ammeter indicated the same current within a few per cent.

Only five of the nine components of the *Hg* line at λ 5461 were measured, the outer ones being too faint to come into question in the confirmation of the law. Similarly, only seven of the nine components of the *Zn* line at λ 4811 were measured.

The distances of the components are given in the following table on the scale of the vibration numbers, where by the vibration number we mean the number expressing the vibrations the light makes in passing over a distance of 1cm. The distances are computed from the center of gravity of the components, and are designated by the sign + when the components lie on the side of larger vibration numbers.

These four plates show that as far as the accuracy of the measurement goes, the distances of corresponding components on the scale of the vibration numbers are the same in the same field. If the lines were normal triplets, which, according to the views of H. A. Lorentz, correspond to the vibrations of an electrically charged particle vibrating freely about its position of equilibrium, then it would follow from the equality of the distances that the ratio of the charge to the mass of the particle was the same in the spectra of the different elements. We should then be inclined to assume that we were dealing with the same particles in the different spectra, which represent an inter-

	Zn	*Hg*	Difference		*Mg*	*Cd* 1 Order	*Cd* 2 Order	Difference
Zn λ 4811 *Hg* 5461				*Mg* 5184 *Cd* 5086	−2.76	−2.83		+0.07
	−1.90				−2.15	−2.14	−2.06	−0.05
	−1.30	−1.29	−0.01		−1.39	−1.43	−1.39	+0.02
	−0.66	−0.67	+0.01		−0.74	−0.68	−0.68	−0.06
	0.00	0.00	0.00		−0.01	+0.04	+0.02	−0.04
	+0.62	+0.64	−0.02		+0.69	+0.70	+0.70	−0.01
	+1.31	+1.33	−0.02		+1.43	+1.43	+1.39	+0.02
	+1.92				+2.12	+2.13	+2.02	+0.04
					+2.80	+2.78		+0.02
Zn 4722 *Hg* 4359	−2.59	−2.59	0.00	*Mg* 5173 *Cd* 4800	−2.73	−2.79		+0.06
	−2.01	−2.03	+0.02		−2.16	−2.14		−0.02
	−0.68	−0.67	−0.01		−0.71	−0.71		0.00
	+0.65	+0.70	−0.05		+0.68	+0.70		−0.02
	+1.98	+1.96	+0.02		+2.07	+2.15		−0.08
	+2.64	+2.63	+0.01		+2.86	+2.79		+0.07
Zn 4680 *Hg* 4047	−2.65	−2.67	+0.02	*Mg* 5168 *Cd* 4678	−2.82	−2.82		0.00
	+0.01	0.00	+0.01		−0.02	0.00		−0.02
	+2.64	+2.66	−0.02		+2.84	+2.82		+0.02
Mg 5184 *Zn* 4811	−1.83	−1.94	+0.11	*Zn* 4811 *Cd* 5086	−2.02	−2.03		+0.01
	−1.26	−1.24	−0.02		−1.34	−1.34		0.00
	−0.67	−0.65	−0.02		−0.68	−0.69		+0.01
	−0.04	0.00	−0.04		+0.02	+0.01		+0.01
	+0.65	+0.60	+0.05		+0.67	+0.68		−0.01
	+1.22	+1.31	−0.09		+1.36	+1.37		−0.01
	+1.92	+1.91	+0.01		+1.98	+2.00		−0.02
Mg 5173 *Zn* 4722	−2.45	−2.51	+0.06	*Zn* 4722 *Cd* 4800	−2.65	−2.68		+0.03
	−1.90	−1.88	−0.02		−2.06	−2.04		−0.02
	−0.66	−0.61	−0.05		−0.67	−0.67		0.00
	+0.60	+0.66	−0.06		+0.67	+0.69		−0.02
	+1.90	+1.85	+0.05		+2.04	+2.04		0.00
	+2.51	+2.48	+0.03		+2.66	+2.66		0.00
Mg 5168 *Zn* 4680	−2.51	−2.50	−0.01	*Zn* 4680 *Cd* 4678	−2.72	−2.71		−0.01
	−0.01	0.00	−0.01		0.00	0.00		0.00
	+2.52	+2.50	+0.02		+2.72	+2.71		+0.01

molecular matter in contrast to the chemical elements. This view can also be maintained when the vibrations of the charged particles are not free, but if systems of particles are linked together and vibrate about their position of equilibrium. We then obtain more complicated separations of the spectral lines in the magnetic field, as Lorentz has shown. The equality of

the distances of the components measured on the scale of vibration-numbers, should, however, again be regarded as indicating that in the spectra of different elements similar electrically charged particles vibrate about their position of equilibrium, while the chemical molecules only determine the forces with which the particles are drawn back into the position of equilibrium without the magnetic field. Thus the spectrum is always different for the different elements, but the separation of corresponding lines in the magnetic field is the same in the different spectra.

The second subordinate series was observed in the spectrum of *Ca* and *Sr* in addition to *Hg*, *Mg*, *Zn*, and *Cd*. But here the first members of the series lie so far to the red as to make photography difficult. The second members of the series are already distinctly fainter in the spark spectra. However, the strontium lines at λ 4362 and λ 4327 were taken simultaneously in the first order with the zinc lines at λ 4722 and λ 4680. The dispersion of a large Rowland grating in the first order was insufficient for separating the components of the *Sr* line at λ 4436. We got the impression that the type of the separation is the same as for *Hg* λ 5461, *Zn* λ 4811, etc., but we did not succeed in measuring the distances of the components. Both of the other two lines confirmed the law, however, when we consider that the accuracy is less than for the lines cited above.

	Sr	*Zn*	*Difference*
	−2.69	−2.86	+0.17
	−2.23	−2.16	−0.07
Sr λ4362	−0.78	−0.72	−0.06
Zn 4722	+0.68	+0.70	−0.02
	+2.26	+2.18	+0.08
	+2.76	+2.85	−0.09
Sr 4327	−2.76	−2.87	+0.11
Zn 4680	−0.08	−0.02	−0.06
	+2.84	+2.89	−0.05

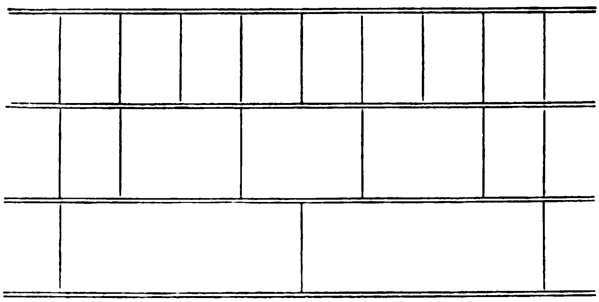
The three types of the second subordinate series possess a relationship, as Runge and Paschen have shown in the case of

mercury.[1] Referred to the scale of vibration-numbers they may be represented in the following diagram. The first line, of greatest wave-length, is separated into nine equidistant components in the magnetic field. In case of the second line the central component and the second following ones on both sides disappear. In case of the third line only the interior components and the two exterior ones remain. Accordingly, all three types

Type *Hg* λ 5461

Type *Hg* λ 4359

Type *Hg* λ 4047



may be characterized by a single constant, by the distance of the neighboring components of the first line, or by the eightfold of this—the distance of the two outside components. This distance is a function of the field-strength.

We have shown here that the corresponding lines for *Zn*, *Mg*, *Cd*, and *Sr*, show the same separation as the lines of mercury. In order now to investigate how accurately equidistant the components are distributed we have computed the distance by the method of least squares in the five cases on the assumption of equidistant distribution, and then have deduced the mean error of the deviation from an equidistant distribution. We have for

	Distance of the outer components from the center	Mean error of the deviation from an equidistant distribution
Zn-Hg	2.63	0.030
Mg-Zn	2.51	0.030
Mg-Cd	2.81	0.033
Zn-Cd	2.70	0.021
Sr-Zn	2.84	0.072

[1] Astrophysical Journal, **15**, 235, 1902.

simplicity here given equal weight to all the vibration numbers. The mean error so found is so small that we may assert that the components are distributed in the manner described, so far as the accuracy of the observations goes.

The fact that the same distance was not observed each time on the different plates is explained, as already remarked, by the fact that the field-strength was not exactly the same at the different exposures.

There is still another way by which we can investigate whether an equidistant distribution of the components applies. We can examine for the different elements the deviations of any one component from the position which it must have with an equidistant distribution, and investigate whether any systematic deviation occurs towards the one side or the other. The mean of the deviations should not be further away from the equidistant position than would correspond to its accuracy.

	Index[1] of the components	Deviations of the mean of the single components from the equidistant position	No. of observations
First type	−4	+0.013	2
	−3	+0.003	8
	−2	+0.007	9
	−1	−0.011	9
	0	+0.004	9
	+1	−0.008	9
	+1	+0.010	9
	+3	−0.012	8
	+4	−0.018	2
Second type	−4	+0.043	10
	−3	−0.038	10
	−1	−0.014	10
	+1	−0.001	10
	+3	+0.020	10
	+4	−0.013	10
Third type	−4	−0.006	10
	0	−0.012	10
	+4	+0.017	10

[1] The distance of the equidistant position from the middle, in units of the interval, is called the index. Negative numbers denote smaller vibration numbers.

According to the number of observations the mean should be about three times as accurate as the single observations, aside from the two outside components of the first type, which were only observed twice, and whose mean, therefore, has only about once and a half the accuracy.

The mean does not have the expected accuracy in case of the four outside components of the second line (index: $-4, -3, +3, +4$). These show a systematic deviation. The two pairs (index: $-4, -3$) and (index: $+3, +4$) are closer than they should be. I believe, however, that this may be explained by a slight running together of the components. If the ordinates of two curves which each have a maximum are superposed, the two maxima of the resulting curve lie nearer together than those of the separate curves. Aside from the extreme components which show a similar displacement, the components of the first type are not affected in the same way, because here each component lies between two equidistant components, whose effect is mutually canceled.

If we except these components the mean error of the deviations is 0.010, corresponding in fact to the expected accuracy of the mean.

The experimental portion of this investigation was carried out by the authors together, but on account of his removal to Tübingen, F. Paschen was unable to participate in the measurement of the photographic plates and in the discussion of the results of the measures.

THE SPECTROSCOPIC BINARY β *CEPHEI.*

By Edwin B. Frost.

Six spectrograms of this star were obtained during the winter by Mr. W. S. Adams with the Bruce spectrograph. His measures of the third plate suggested that the radial velocity of the star varied, and the subsequent plates have confirmed this.

The spectrum is, in general, of the *Orion* type, Vogels' Ib, and in particular of Miss Maury's IIIa, of which group the typical star is β *Canis Majoris.*

The principal lines in the region of spectrum in good focus on our plates are due to oxygen, silicon, helium, and magnesium. The lines are, of course, rather broad and difficult to set upon, although much better than in many of the sub-groups of the *Orion* type. A statement of the accordance of the measures made on spectra of this character by different observers will, therefore, perhaps be of interest.

I accordingly give the results of the measures of Mr. Adams and myself on the five plates which we have both measured, together with those I have made on five other plates I have recently taken to determine the period.

RADIAL VELOCITY OF β *CEPHEI.*

Plate No.	Date	Taken by	Radial Velocity			
			Adams	No. of lines	Frost	No. of lines
B 255	1901 Dec. 18	Adams	− 5.2 km	6	− 3.9 km	15
A 304	1902 Jan. 8	"	− 9.1	5	......	..
B 302	Mar. 13	"	+ 4.3	11	+ 4.7	12
A 338	Mar. 26	"	− 15.1	8	− 9.0	10
B 307	April 2	"	− 19.3	12	− 20.3	10
B 325	April 16	"	+ 10.1	11	+ 11.3	12
B 336	April 23	Frost	......	..	− 0.8	11
B 345	May 14	"	......	..	− 2.2	12
B 349	May 14	"	......	..	− 16.3	11
B 350	May 16	"	......	..	+ 6.4	11
B 353	May 23	"	......	..	− 16.8	11

These values are only provisional, as they may be appreciably changed when we obtain more accurate determinations of the wave-lengths of the oxygen lines, a piece of work upon which we are at present engaged.

It should be remarked that the observer assigns a weight to each star line, according to its sharpness, as he measures it; of course he is wholly unconscious of the effect this weighting will have on the result.

The agreement between the measures by the two observers is better than had been expected, the difference averaging 1 km for the four plates taken with camera B, but the decimal of the kilometer has no significance. Plate A 338, taken with the short camera, is poor, and the comparison lines were overexposed, which accounts for the large difference of 6 km. As plate A 304 was a broken one I did not attempt to measure it.

We had assumed from the first plates that the period would be rather long, but a suspicion to the contrary led me to take two plates on the night of May 14, and during the interval of five and one-half hours the velocity changed by 14 km, or nearly half of the whole range so far observed.

It has not so far been possible to obtain other observations of the star at short intervals, so that the period cannot now be stated with certainty. This will be settled as soon as the weather permits observations on two consecutive nights.

At certain phases of the star's variation of velocity there are suggestions of additional complexities in some of the lines, which may prove of interest, but the data are at present insufficient for drawing further conclusions.

YERKES OBSERVATORY,
May 29, 1902.

MINOR CONTRIBUTIONS AND NOTES.

REMARKS ON PROFESSOR KAYSER'S ARTICLE, "SPECTRAL PHENOMENA CONNECTED WITH THE COOLING OF VERY HOT STARS."[1]

IT has happened that I have only just learned of the criticism of my paper on the spectra of stars with both bright and dark hydrogen lines, contained in Professor Kayser's article. In this case I wish to depart from my practice of ignoring his criticisms, on account of their inappropriate tone, in order to illustrate by a striking example the mode of his procedure.

Referring to my statement as to the simultaneous occurrence of bright and dark lines in the spectrum of one and the same element Professor Kayser says: "This expression of opinion shows that Scheiner is not acquainted with the numerous experiments which have been made on the reversal of spectral lines . . . ."

In my paper the following sentence directly follows the statement referred to:[2] "The phenomena of reversal which may affect some lines while other lines of the same element remain single do not come into consideration here, since, in addition to the peculiarity here to be explained, they appear to arise as further complications in case of some of the stars of this class."

How Mr. Kayser reaches this criticism in spite of this sentence can only be understood on the assumption that he has again committed his former error of great superficiality, and has overlooked my sentence in respect to the reversal of lines.

J. SCHEINER.

POTSDAM, May 1902.

THE COLLECTED PHYSICAL PAPERS OF HENRY A. ROWLAND.

A VOLUME containing the physical papers of Professor Henry A. Rowland, for twenty-five years professor of physics in the Johns Hopkins University, is now in preparation. It will be issued under the

[1] ASTROPHYSICAL JOURNAL, 14, 313, 1901. [2] *A. N.*, 156, 198, 1901.

editorial direction of a committee appointed for that purpose, consisting of President Remsen, Professor Welch, and Professor Ames.

The book will contain Professor Rowland's articles and memoirs on physical subjects, together with his popular writings and addresses, numbering sixty in all. These have been collected from over twenty different magazines and journals. The subjects treated in these papers cover a wide range. In heat there is the great memoir on the "Mechanical Equivalent of Heat," with several shorter articles on thermometers. In electricity and magnetism there are the fundamental researches on magnetization, on the "Magnetic Effect of Electrical Convection," on the "Value of the Ohm," on the "Theory and Use of Alternating Currents," etc. In light there are the renowned discovery and theory of the concave grating and the long series of investigations made in the field of spectroscopy. Lists of wave-lengths will not be reprinted in this volume, as they are readily accessible elsewhere; and any subscriber to this volume may obtain by application to the Johns Hopkins Press, Baltimore, a copy of Rowland's "Preliminary Table of Solar Wave-Lengths." There will be, further, a description of Rowland's ruling engine, used for the making of gratings, details of which have never before been published. The memorial address of Professor Mendenhall and a portrait of Professor Rowland will also be included.

The volume will be printed in royal octavo, bound in cloth, and will contain between six and seven hundred pages.

The price set is five dollars net per copy ($5.00; £1. 1.; francs 26; marks 21) for orders sent in advance of publication.

Orders may be sent to Professor Joseph S. Ames, secretary of the Committee of Publication, Johns Hopkins University, Baltimore, Md.

A NEW *ALGOL* VARIABLE. $+43^\circ$ 4101.[1]

A STRIKING illustration of the value of the library of glass photographs collected at this Observatory during the past seventeen years has been shown within the last few days. Comet a 1902 was discovered by Dr. Brooks on April 14, and it was found that a photograph had been taken here on April 3 with the 8-inch Draper Telescope, approximately in the direction from which the comet came. An examination of this plate was accordingly made by Mrs. Fleming, superposing it upon

[1] *Harvard College Observatory Circular* No. 65.

another plate of the same region taken with the same instrument on March 7, 1900. No trace of the comet was found, and in fact the elements now indicate that it was a little beyond the region covered by the photograph. One star, however, in the constellation *Lacerta*, according to Heis, but in *Cygnus* according to the *Uranometria Nova*, appeared faint on the early plate and bright on that taken later. A further examination showed that this object was the north preceding component of $+43° 4101$. Its position for 1900 is R. A. $= 21^h 55^m.2$, Dec. $= +43° 52'.$ The difference in right ascension of the two components is about $2^s.0$, the difference in declination, $0'.3$. A further examination showed that the star was generally bright and constant in light, so that it must be a variable of the *Algol* type. It is not very distant from the remarkable variable star SS *Cygni*, which precedes it 16^m, and is $44'$ south. This last star was discovered at this Observatory in 1896 and is ordinarily faint, becoming suddenly bright at intervals which appear to be irregular. Only one other star, U *Geminorum*, is known to undergo similar changes. The star SS *Cygni* has been carefully studied here, several hundred photographs having been taken of it. It has also been observed visually on several hundred nights both here and elsewhere, but as yet the law regulating its outbursts of light has not been discovered. Again the advantage of the photographic method is indicated, since each plate taken for SS *Cygni* can be used for studying the new variable, or any others that may be discovered in this part of the sky, as well as if taken for each alone, while of course the visual observations of SS *Cygni* can be used for no other star. The total number of plates showing the new variable at full brightness is 388, of which 1 was taken in 1889, 10 in 1890, 12 in 1891, 8 in 1892, 9 in 1893, 3 in 1894, 10 in 1895, 37 in 1896, 184 in 1897, 30 in 1898, 37 in 1899, 28 in 1900, 7 in 1901, and 12 in 1902. Besides these, the star appears on 54 photographs taken with the 2.5 inch Anastigmat, but they have not been included in the present discussion, since the proximity of the other component in some cases renders it difficult to decide whether the variable is at its full brightness, or not.

Besides these plates, 19 were found on which the variable, which ordinarily has the magnitude 8.9, was of the magnitude 9.3 or fainter. They are enumerated, together with six others taken later, in Table I, which gives in the first three columns the year, month and day, the Greenwich Mean Time of the middle of the exposure in hours and

minutes, and the Julian Day and thousandths. The fourth column gives the photographic magnitude, and the fifth, the correction to the minimum corresponding to this magnitude. A positive sign denotes that the photograph was taken before minimum, a negative sign, after. When the star was not seen, the plus and minus sign is used to indicate that the correction may have any value within the limits given. Thus, the star was of the magnitude 10.5 for about 0^d71 before and after minimum, and accordingly any correction between $+0^d.71$ and $-0^d.71$ may be applied to the observed time of the second plate without indicating an error in the observation. All of these values are derived from the light curve found as described below. At first the period was thought to be $1^d.498$, but this was found to be an error. The true period appears to be about $31^d.304$, and the times of minimum are represented by the formula, $2,410,015.05 + 31.304$ E. The value of E, and the residual found by subtracting the computed time of minimum from the time of the photograph, are given in the sixth and seventh columns. The eighth column gives the corrected residual found by taking the algebraic sum of the fifth and seventh columns. The letter F is inserted when the variable was invisible, and the phase is less numerically than the correction given in the fifth column. In these cases, no correction to the period is indicated. The letter M is inserted when the phase is less than ± 0.45 and the variable is of the magnitude 11.4, or fainter, since it is then near minimum and is varying so slowly that an accurate correction cannot be obtained.

It will be seen, from the last column of the table, that all the magnitudes are nearly represented by the formula, except those derived from photographs taken on J. D. 1962, and 4431. On these dates, the star was nearly at its full brightness, and the deviations, which amount to 0.4 of a magnitude, are probably due to errors of observation.

The form of light curve may be determined by plotting the magnitudes and phases given in the fourth and seventh columns. The star retains its full brightness for 28 days, its photographic magnitude at maximum being 8.9. About a day before the minimum it begins to diminish, attaining the magnitude of 9.0 at $1^d.05$ before minimum, 9.5 at $0^d.94$, 10.0 at $0^d.84$, 10.5 at $0^d.71$, 11.0 at $0^d.58$, and 11.5 at $0^d.43$. The light remains nearly constant for more than half a day, with the minimum magnitude 11.6. The time of increase is more uncertain, but apparently is nearly the same as that of decrease. The period of this *Algol* star, 31.4 days, is more than three times that of any other

TABLE I.

OBSERVED MINIMA OF $+43°$ 4101.

Y.	M.	D.	G. M. T.	J. D.	Magn.	Corr.	E	Phase	Cor. phase
			h m	d					
1891	8	17	19 21	1962.806	9.28	−0.99	62	+6.908	−5.92
1892	10	22	13 31	2394.563	<10.5	±0.71	76	+0.409	F
1895	9	20	15 24	3457.642	9.31	+0.98	110	−0.848	+0.13
1896	9	30	11 42	3833.487	10.52	+0.71	122	−0.651	+0.06
	9	30	19 20	3833.806	<11.5	±0.43	122	−0.332	F
	10	1	13 35	3834.566	11.49	−0.43	122	+0.428	M
	10	31	12 15	3864.510	9.64	+0.90	123	−0.932	−0.03
	10	3	14 05	3864.587	10.02	+0.83	123	−0.855	−0.03
	12	3	12 47	3897.533	9.85	−0.86	124	+0.787	+0.07
1897	1	2	10 51	3927.452	10.85	+0.64	125	−0.598	+0.04
	4	6	20 59	4021.874	<11.7	0.00	128	−0.088	F
	5	7	19 34	4052.815	11.54	+0.36	129	−0.451	−0.09
	9	9	15 16	4177.636	10.02	−0.83	132	+0.846	+0.02
1898	1	13	11 02	4303.460	11.69	0.00	137	−0.238	M
	5	21	20 04	4431.836	9.28	−0.99	141	+2.922	−1.93
1899	8	30	12 46	4897.532	9.28	+0.99	156	−0.942	+0.05
1900	3	7	21 43	5086.905	10.95	−0.60	162	+0.607	+0.01
	8	9	20 13	5241.842	9.74	+0.89	167	−0.976	−0.09
1901	8	21	15 32	5618.647	<10.6	±0.69	179	+0.181	F
1902	4	28	17 44	5868.739	11.54	+0.36	187	−0.159	M
	4	28	18 18	5868.762	11.44	+0.46	187	−0.136	M
	4	28	19 07	5868.797	11.44	+0.46	187	−0.101	M
	4	28	19 37	5868.817	11.49	+0.44	187	−0.081	M
	4	28	20 20	5868.847	11.44	+0.46	187	−0.051	M
	4	28	20 34	5868.857	11.44	+0.46	187	−0.041	M

as yet discovered, and the duration of minimum, 2 days, is double that of S *Cancri*, the next in length. The period of S *Cancri* is 9.5 days, and the duration of minimum 0.9 days. The last minimum of $+43°$ 4101 occurred on April 28, 1902, at $21^h\ 33^m$ G. M. T. The predicted times of the next minima are May $30^d\ 4^h\ 51^m$, June $30^d\ 12^h\ 8^m$, July $31^d\ 19^h\ 26^m$, September $1^d\ 2^h\ 44^m$, and October $2^d\ 10^h\ 2^m$, 1902.

The anomalous variations of Schwab's variable, described in *Circular* No. 64, were explained by the period, 3.38 days, cabled by Professor Kreutz, and announced in our bulletin of February 3, which also stated that the careful watch of this star, proposed in the *Circular*, would therefore not be necessary.

EDWARD C. PICKERING.

May 6, 1902.

REVIEWS

Handbuch der Astronomischen Instrumentenkunde. L. AMBRONN. Berlin: Julius Springer, 1899.

THE development of research in astronomy and astrophysics has been so closely bound up with the invention and perfection of instruments that a treatise like that of Professor Ambronn cannot fail to be of great service. Through the aid of a most valuable series of illustrations, engraved in a manner which one would like to see as common in the United States as it is in Germany, the detailed construction of a host of instruments is rendered clear. Fortunately neither text nor illustrations is confined to the larger or more elaborate parts; the smaller but no less essential details, such as screws, clamps, axes, etc., are treated with the same care that is bestowed upon circles, levels, or escapements. Thus the book will be very useful to the designers and constructors of instruments, as well as to those who use them. At the same time it should be stated that the subject is not approached merely from the standpoint of the instrument maker: the theory of the instrument, as well as the methods of adjusting, testing, etc., are freely given.

Vol. I, which comprises 500 pages, opens with some remarks on the principles that underlie the construction and use of astronomical instruments. Bessel's statement that the purpose of an astronomical instrument is the determination of a star's position at a given time is taken as the motto of these introductory pages, and in some degree of the entire book, for the space and treatment accorded to astrophysical instruments, with the exception of photometers, is strikingly inadequate. Chap. i, which deals with screws, describes numerous types of screws used in the construction of instruments; micrometer screws, though far more important, are passed over lightly, though the ordinary methods of determining their errors are described. It is curious that no reference is made to Rowland's well-known article in the *Encyclopædia Britannica*, on his method of making a perfect screw. It would also seem natural at least to mention the fact that the errors of screws

can be directly measured with high precision by means of the interferometer.

The following chapter, on levels, gives in much detail the methods of making, filling, testing, and using precision levels. Chap. iii contains a good account of artificial horizons and collimators. The first section of the work is concluded with chap. iv, which discusses verniers and reading microscopes.

Sec. 2, comprising 115 pages, is devoted to clocks and chronometers. In view of the importance of Guillaume's work on the nickel-steel pendulum, it is strange that it is not referred to. Sec. 3, on the separate parts of instruments, devotes 209 pages to a discussion of axes, objectives and specula, tubes, reticles and their illumination, as well as methods of determining the focal length of lenses, variation of focal length with temperature and pressure, magnification, size of field, and light-gathering power.

The volume concludes with a chapter on divided circles, including an account of dividing engines and the investigation of division errors, methods of mounting circles on instruments, determination of eccentricity, procedure in observations, and clamps and slow motions.

Vol. II opens with a section of 117 pages on micrometers. Under double image micrometers, which occupy one of the two chapters of this section, the heliometer is fully illustrated and described. This should form one of the most useful portions of the book. Sec. 5 deals under chap. xii with instruments for special purposes, including photoheliographs, heliostats, photographic refractors, and machines for measuring stellar photographs. Chaps. xiii and xiv, respectively, describe photometers and spectroscopic apparatus. Professor Ambronn states that limitations of space prevent any adequate treatment of spectroscopes, and aims to give only the principles of construction and to describe a few typical instruments. It is, therefore, the more unfortunate that his discussion is not based upon the consideration of resolving power, and that much space is wasted on obsolete instruments. A description of the first Potsdam spectrograph is given, but nothing is said about the determination of stellar motion in the line of sight! It would be easy to point out many other serious defects in this chapter, which make it practically useless to spectroscopists and misleading to those who are unfamiliar with the subject.

After devoting a chapter to sextants and other reflection instruments, the author takes up universal instruments, altazimuths, vertical

circles and zenith telescopes. In the 223 pages allotted to this and the following chapter on transit instruments and meridian circles a large amount of valuable information is brought together. Here, in particular, the provision of numerous illustrations adds greatly to the value of the text. After a chapter on chronographs, equatorially mounted refractors are treated in a chapter of 109 pages. The description and illustrations of Repsold's mountings will be found very useful. It is interesting to compare these light and highly finished mountings with the much more massive mountings of the type evolved by Warner and Swasey. It is easy to see that the ideas which govern the design and construction of a microscope have strongly influenced the designers of these instruments. The American designers have approached the problem from the standpoint of the engineer, and it is a natural consequence that their mountings should differ widely from those of the German school. It is not improbable that this difference will be still further emphasized in the future, in such a way as to materially reduce the complexity of such a mounting as that of the 40-inch Yerkes refractor. In the case of large instruments it seems reasonable to suppose that the electric motors for producing the quick motions might advantageously be placed at or very near the point where their work is to be done, rather than within the telescope column, whence the power is transferred through long shafts and complicated trains of gearing. It should be said, however, that the mounting of the 40-inch telescope has proved entirely successful. The ease with which spectroscopic attachments weighing as much as 700 pounds are carried by the great tube, the steadiness of the driving clock, and the convenience of manipulation afforded by the electric motors, indicate that the designers knew how to meet the difficulties presented by this problem. All of the large Warner and Swasey mountings are illustrated, but Gautier's mountings are not adequately represented among the cuts.

Reflecting telescopes and special instruments, such as the equatorial coudé, are described in chap. xx, in which there is also a brief account of the errors of adjustment of equatorials. The work closes with a short section on observatory construction, giving the details of domes and shutter mechanism, piers, etc. The index is arranged with reference to both authors and subjects.

In spite of a few shortcomings, some of which are supplied by other works, Professor Ambronn's volumes will be indispensable to all astronomers who wish to keep in touch with the development of instruments.

The Cause of the Structure of Spectra. By WILLIAM SUTHERLAND. *Phil. Mag.* (6), **2**, 245–274, 1901.

The subject-matter of this paper may be divided into two parts. In the first part is included a method of generalizing Balmer's formula, so as to bring out the principle of harmonics in spectra; several examples of harmonic series; and observed facts in evidence of purely numerical relationships in the values of spectral parameters of different elements. In the second part is included a discussion of the above observed facts; a kinematical analysis of Balmer's formula, and Rydberg's laws, and an interpretation of the results as applied to the production of spectra.

BALMER'S FORMULA.

Balmer's formula, which represents with surprising accuracy the wave-lengths of the ordinary line-spectrum of hydrogen, is usually written

$$\lambda = \lambda_0 \frac{m^2}{m^2 - 4}, \qquad (1)$$

or

$$n = n_0 - Bm^{-2}, \qquad (2)$$

where λ is the wave-length; n, its reciprocal; λ_0, n_0, and B are constants; and m assumes integral values beginning with 3. The formula may also have the more general form given it by Rydberg, so as to apply to elements other than hydrogen; namely,

$$n = n_0 - \frac{B}{(m + \mu)^2}, \qquad (3)$$

where B is supposed to be constant for all substances, while n_0 and μ vary, and m has integral values as above. If now we can give m the general form, $p \pm \frac{r}{s}$, where p, r and s are integers, s being small, we shall have these formulæ so expressed as to represent harmonic relations in spectra.

Further, if λ is the wave-length in free ether and V, the velocity of light, then the time of vibration, which is equal to $\frac{\lambda}{V}$, in terms of equation (3) is

$$\tau = \frac{1}{2V\sqrt{n_0}} \left\{ \frac{1}{\sqrt{n_0} + \frac{\sqrt{B}}{(m + \mu)}} + \frac{1}{\sqrt{n_0} - \frac{\sqrt{B}}{(m + \mu)}} \right\}, \qquad (4)$$

where, it will be noticed, τ is expressed as the sum of two times. As

will be shown later, this equation admits of a purely kinematical interpretation, and this uneven vibration, of which τ is the time, will be found to be the result of the relative motion of the two parts of an "oscillator," the direct cause of the vibrations of light.

HARMONIC SERIES.

By solving equation (1) for m, using Professor Ames' values for 63 lines of the hydrogen spectrum, and also his value for λ_0, Sutherland found that, with few exceptions, all these wave-lengths can be quite well reproduced by a value of m of the form $p \pm \frac{1}{s}$, the exceptions being of the form $p \pm \frac{r}{s}$, where, as above, p and r are integers, and s is a small number, 1, 2, 3, - - - . This value of m having been put in the formula, the author sought other examples to justify the substitution.

The new series of hydrogen lines discovered by Pickering in the spectrum of the star ζ *Puppis* can be expressed by Balmer's formula written in the form

$$\lambda = \lambda_0 \frac{\left(m + \frac{1}{2}\right)^2}{\left(m + \frac{1}{2}\right)^2 - 4},$$

which besides giving three of Professor Ames' values of λ in air, gives the missing values for which $\frac{r}{s}$ has the value $\frac{1}{2}$.

Further, Rydberg considered this new series the "sharp series" of the hydrogen spectrum, and by applying his principles predicted that there ought to be a principal series which has in common with the new series the chief line $\lambda 4687.88$, which, in fact, almost exactly coincides with the line 4688 found in the spectra of stars of the fifth type. The important fact in the present connection is that this number was obtained from formula (3) by giving μ the value 0.5, which again shows the importance of this fraction.

Another example is furnished by a special series among the subordinate lines of magnesium. This may be represented by the formula

$$n = 39370 - \frac{107250}{\left(3 - \frac{2.343}{s}\right)^2},$$

where s has the integral values 3 to 8, and μ has a value nearly $2\frac{1}{3} \div s$.

NUMERICAL RELATIONS IN PARAMETERS.

Rydberg found that in the case of series of pairs, equation (3) gives one of the parallel series while the other is given by adding to n_0 a constant ν, the common difference between the pairs. In the case of series of threes, the other two series are given by increasing n_0 by ν_1, and ν_2, respectively, the constant differences in the threes.

For *Zn*, *Cd* and *Hg*, ν_1 has the values 386.4, 1159.4, 4633.3, where the second and third numbers are respectively very nearly 3 and 12 times the first number. The values of ν for *Na*, *K*, *Rb*, *Cs*, are 17.2, 56.8, 234.4, and 545.0, respectively, while 1, 3, 12 and 28 times 19.6 with 2 substracted are 17.6, 56.8, 233.2, 546.8.

The values of μ in the formula for the sharp series of the lithium family are 1.597, 1.66, 1.80, 1.667, 1.60, which are nearly $1\frac{3}{5}$, $1\frac{2}{3}$, $1\frac{4}{5}$, $1\frac{2}{3}$, $1\frac{3}{5}$, respectively, where the fractions are multiples of $\frac{1}{3}$ and $\frac{1}{5}$, whose difference is 0.133. The successive differences of μ in the principal series of the same family are 0.135, 0.133, 0.067, 0.0, two of these being equal to the characteristic difference in the sharp series, and the third equal to $\frac{1}{2}$ of it.

DISCUSSION OF OBSERVED FACTS.

The fact that the hydrogen lines can be so nearly represented by Balmer's formula by giving m the values $p \pm \frac{r}{s}$ indicates vibrations perfectly analogous to harmonic vibrations of a circle in which all possible modes of vibration, including fundamental, overtones and undertones, are represented by the fraction $\frac{2\pi q}{s}$, where q and s are whole numbers. Now, if the origin of all the main vibrations be taken at the same point, many of the nodes will coincide, and the most pronounced vibrations will be for wave-lengths of the form $p \pm \frac{1}{s}$, the fractional part being regarded of most importance when $s = 2$, next when $s = 3$, etc. As the examples given above seem to justify this comparison, we see that in series of lines we are dealing with modes of vibration analogous to harmonics.

The examples cited above show that the relation between the values of ν for different elements of a group is purely numerical, and is not

directly connected with atomic weights. This fact, together with the evidence of other purely numerical relations in spectral parameters, indicates that the relations among spectral lines are controlled by kinematical and not dynamical conditions, an idea which is also brought out by Balmer's formula.

KINEMATICAL ANALYSIS OF BALMER'S FORMULA AND RYDBERG'S LAWS.

Formula (4) suggested to the author the idea of an atom simplified to a circle round which a disturbance with angular velocity v travels in opposite directions. Calling its radius-vector the disturbance-vector, and another vector traveling with velocity u, the reference-vector, he found that there is established an uneven vibration whose period is

$$\tau = \frac{2\pi}{v}\left(\frac{1}{1+\frac{u}{v}} + \frac{1}{1-\frac{u}{v}}\right). \tag{5}$$

By comparison of (4) and (5) we have

$$\frac{2\pi}{v} = \frac{1}{Vn_0} = \frac{\lambda_0}{V} \equiv \tau_0,$$

$$\frac{n}{v} = \sqrt{\frac{B}{n_0}} \cdot \frac{1}{m+\mu},$$

where τ_0 is the natural period of vibration of the circle when the reference-vector is at rest. The ratio of the velocities has a series of values depending on $m+\mu$. Now substituting for n_0 its value as derived from Rydberg's laws as applied to spectra containing principal, diffuse and sharp series, we have

$$\frac{2\pi}{v} = \frac{(1+{}_s\mu)^2}{VB} \left(\text{or} = \frac{(1+{}_p\mu)^2}{VB}\right), \tag{6}$$

$$\frac{n}{v} = \frac{1+{}_s\mu}{m+{}_p\mu} \left(\text{or} = \frac{1+{}_p\mu}{m+{}_s\mu}\right), \tag{7}$$

where s and p refer to sharp and principal series respectively. This shows that "the principal and the sharp series in the spectra of the alkalies result, respectively, from the fundamental motion of the reference-vector with the motions of velocities, $1+{}_p\mu$, $2+{}_p\mu$, . . . $m+{}_p\mu$ of the disturbance-vector, and from the fundamental motion of the disturbance-vector with the motions of velocities $1+{}_s\mu$, $2+{}_s\mu$ . . . . $m+{}_s\mu$ of the reference-vector."

To carry this a step farther, Sutherland accepts the results of Stoney, who by a kinematical analysis shows that the origin of doublets and

triplets may be ascribed to the motions of electrons describing elliptic orbits. If these orbits be simplified to circles the angular velocities of the radius-vectors will form a series 1, 2, 3 . . . . , which, however, gives the ordinary series of harmonics. The harmonics analogous to a vibrating circle are supplied by the mechanically vibrating atom, whose fundamental frequency may be represented by $\frac{2\pi}{\Omega}$ and the frequency of its standing waves by

$$\left(p + \frac{r}{s}\right)\frac{2\pi}{\Omega}.$$

Now the electron is supposed to revolve round the atom in a nearly circular path, and to have its energy renewed by periodic collisions with the atom at the middle of a standing wave. In case the frequency of the electron is a multiple of the standing wave, there will be resonance, and the electron will absorb a maximum of energy. Assuming the angular velocity of the electron to be a constant ω, the periods between successive collisions of atom and electron will be $\left(p + \frac{r}{s}\right)\frac{2\pi}{\omega}$. In case of resonance, these must be multiples or submultiples of $\left(p + \frac{r}{s}\right)\frac{2\pi}{\Omega}$, the corresponding period for the standing wave in the atom. Whence we see that $\frac{2\pi}{\omega}$, the characteristic period for an electron, must be a multiple or submultiple of $\frac{2\pi}{\Omega}$, the fundamental mechanical period of the atom. But by the Kinetic Theory of Solids, the mechanical periods of the metals are calculated from their rigidities at absolute zero, and proved to exhibit simple harmonic relations. Hence the characteristic period of an electron is also harmonic.

This same theory brings out the result that the periods of the beryllium family are nearly equal to one-half the corresponding periods of the lithium family, which seems to indicate that there is a single fundamental period of vibration to which the periods of all the elements are related. This fundamental period is found to be of the order 3.5×10^{13} which is of the same order as that of light. The period of this common harmonic is probably the same as the period of the electron.

Now the positive and negative electrons may have different periods of vibration which we may write $(m + \mu_1)\frac{2\pi}{\omega}$ and $(m + \mu_2)\frac{2\pi}{\omega}$, respect-

ively, where μ_1 and μ_2 are generally harmonic fractions. The relative motion of the radius-vector then gives the type shown to be necessary by our kinematical analysis of Balmer's formula. Equations (6) and (7) then show "that a series in a spectrum arises from the several relative motions obtained from the motion of period $(1+\mu_1)\frac{2\pi}{\omega}$ of the one electron, and the motions of periods $(1+\mu_2)\frac{2\pi}{\omega}$, $(2+\mu_2)\frac{2\pi}{\omega}$ . . . . $(m+\mu_2)\frac{2\pi}{\omega}$ of the other." The fundamental angular velocities of the electrons round their center of inertia are $\frac{\omega}{(1+\mu_1)}$ and $\frac{\omega}{(1+\mu_2)}$.

From equation (6) it is seen that on comparing different elements, the angular velocity of the electron is of the form $\frac{A}{1+\mu}$, where A is an absolute constant. Its value equals that of $V\,B$, or 3×10^{10} times 109675, equal to 33×10^{14}. Hence the angular velocity of all electrons round the center of any atom of any element has a frequency of 33×10^{14} per second. This means that all spectra are produced by the same electrical apparatus, or arrangement of electrons, which receives energy from the vibrating atom at its various internodes.

GEO. W. MIDDLEKAUFF.

JOHNS HOPKINS UNIVERSITY.

May 12, 1902.

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The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

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